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<td>2次元フルベクトリアル有限要素解析におけるNRDガイドデバイスの性能評価と応用</td>
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Two-Dimensional Full-Vectorial Finite Element Analysis of NRD Guide Devices

Yasuhide Tsuji, Senior Member, IEEE, Keita Morimoto, Student Member, IEEE, Akito Iguchi, Member, IEEE, Tatsuya Kashiwa, Senior Member, IEEE, and Shinji Nishiwaki, Non-Member, IEEE

Abstract—Non-radiative dielectric waveguide (NRD guide) is a promising platform for realizing compact millimeter waveguide circuits. NRD guide devices have been simulated by several numerical simulation approaches so far and these approaches basically treat three dimensional structure because electric and magnetic fields in NRD guides vary in whole directions. In this paper, we propose a two-dimensional full-vectorial finite element analysis for efficient simulation of NRD guide devices.

Index Terms—Non-radiative dielectric waveguide (NRD guide), finite element method (FEM), full-vectorial analysis.

I. INTRODUCTION

In recent progress of wireless communication systems, such as 5G and beyond 5G, utilization of the higher frequency band is extensively explored. Non-radiative dielectric waveguide (NRD guide) is one of promising platforms, in which compact millimeter-wave circuit can be realized thanks to its non-radiative nature[1]-[12]. In order to design high performance devices with novel functions, several kinds of optimization technique for electromagnetic wave devices have been proposed and developed[13]-[16]. In such optimization techniques, numerical simulation is required to be iteratively carried out to optimize a lot of design variables, thus, improvement of the computational efficiency of the simulation is highly desired. NRD guide devices have been simulated by several numerical simulation approaches so far and these approaches basically treat three dimensional structure[16] because electric and magnetic fields of non-radiative modes in NRD guides vary in whole $x$-, $y$-, and $z$-directions even if the structure is uniform in the direction vertical to metal plates. These field variations are not the same for all the electromagnetic field components even along the uniform direction. Thus, the non-radiative modes in NRD guides are hybrid modes and mode coupling between different polarization modes occurs if the structure varies arbitrarily on the circuit plane. Therefore, three-dimensional analysis has been employed so far. In this paper, we first propose two-dimensional full-vectorial finite element analysis for efficient simulation of NRD guide devices with uniform structure in the vertical direction to metal plates. It is shown that our formulation can treat polarization coupling between LSM$_{01}$ and LSE$_{01}$ modes even in the two-dimensional analysis.

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\[ E_y = \phi_y(x, y) \sin \left( \frac{\pi}{a} z \right), \quad H_y = \psi_y(x, y) \cos \left( \frac{\pi}{a} z \right), \]
\[ E_z = \phi_z(x, y) \cos \left( \frac{\pi}{a} z \right), \quad H_z = \psi_z(x, y) \sin \left( \frac{\pi}{a} z \right). \]

In order to obtain finite element expression for waveguide analysis, discretizing computational domain into curvilinear edge/nodal hybrid elements \[17\]-[19] as shown in Fig. 2, approximating magnetic field component by
\[
\psi(x, y) = \{i_x(V)^T + i_y(V)^T\} \{\psi_x\} + i_z \{N\}^T \{\psi_z\}
\]
and applying variational principle, then, the following linear equation is derived:
\[
(K) \{\psi\} = \{u\} \tag{3}
\]
where \( \{U\} \), \( \{V\} \) are shape functions of edge element, \( \{N\} \) is a shape function of nodal element, and the submatrices are defined by
\[
[K_{tt}] = \sum_e \int_e \left[ p_x(V)\{V\}^T + p_y(U)\{U\}^T \right] \left( \frac{\pi}{a} \right)^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \] dx dy + \sum_e \int_e \left[ p_x(V)\{V\}^T + p_y(U)\{U\}^T \right] \left( \frac{\pi}{a} \right)^2 \frac{\partial^2}{\partial x^2} \left[ \frac{\partial N(\{V\})}{\partial y} \right] \frac{\partial N(\{U\})}{\partial y} \] dx dy \]
\[
[K_{tx}] = \sum_e \int_e \left[ p_x(V)\{V\}^T + p_y(U)\{U\}^T \right] \left( \frac{\pi}{a} \right)^2 \frac{\partial^2}{\partial x^2} \left[ \frac{\partial N(\{V\})}{\partial y} \right] \frac{\partial N(\{U\})}{\partial y} \] dx dy \]
\[
[K_{tx}] = \sum_e \int_e \left[ p_x(V)\{V\}^T + p_y(U)\{U\}^T \right] \left( \frac{\pi}{a} \right)^2 \frac{\partial^2}{\partial x^2} \left[ \frac{\partial N(\{V\})}{\partial y} \right] \frac{\partial N(\{U\})}{\partial y} \] dx dy \]
\[
[M_{tx}] = \sum_e \int_e \left[ q_x(U)\{U\}^T + q_y(V)\{V\}^T \right] \frac{\partial^2}{\partial z^2} \] dx dy \]
\[
[M_{tx}] = \sum_e \int_e \left[ q_x(U)\{U\}^T + q_y(V)\{V\}^T \right] \frac{\partial^2}{\partial z^2} \] dx dy

where \( \{U\} \), \( \{V\} \) are shape functions of edge element, \( \{N\} \) is a shape function of nodal element, and the submatrices are defined by
\[
\begin{bmatrix} K_{tt} & K_{tx} \\ K_{tx} & K_{xx} \end{bmatrix} \begin{bmatrix} K_{zz} \\ K_{xz} \end{bmatrix} = \begin{bmatrix} [M_{tt}] & [M_{tx}] \\ [M_{tx}] & [M_{xx}] \end{bmatrix} \begin{bmatrix} \{u\} \\ \{\psi\} \end{bmatrix}
\]

Fig. 3. Dispersion relation of guided modes in an NRD guide and field distribution of LSM01 mode at frequency of 60 GHz.

where
\[
\begin{align*}
\psi_{\Gamma} &= i_x\{N\}^T \{\psi_x\}_{\Gamma} + i_y\{V\}^T \{\psi_y\}_{\Gamma} + i_z\{N\}^T \{\psi_z\}_{\Gamma} \\
[M_{tx}] &= \sum_e \int_e \frac{1}{\pi} \frac{\partial(N)}{\partial y} dy
\end{align*}
\]

\[
\begin{align*}
\phi_x &= \frac{1}{j \omega \varepsilon_0 \varepsilon_r} \left( \frac{\pi}{a} \right) \frac{\partial \psi_z}{\partial y} \\
\phi_y &= \frac{1}{j \omega \varepsilon_0 \varepsilon_r} \left( \frac{\pi}{a} \right) \frac{\partial \psi_z}{\partial y} \\
\phi_z &= \frac{1}{j \omega \varepsilon_0 \varepsilon_r} \left( \frac{\pi}{a} \right) \frac{\partial \psi_z}{\partial y}
\end{align*}
\]

where \( \varepsilon_0 \) is the permittivity of vacuum and \( \omega \) is the angular frequency.

Although wave equation for LSE mode is given by
\[
\frac{\partial}{\partial y} \left( \frac{\partial \phi_z}{\partial y} \right) + \left( \varepsilon_r k_0^2 - \left( \beta^2 + \left( \frac{\pi}{a} \right)^2 \right) \right) \phi_z = 0
\]
both LSE and LSM modes can be treated by (3) because (3) is derived from full-vectorial expression (1).

### III. NUMERICAL EXAMPLES

#### A. Guided Mode Analysis

In order to confirm the validity of two-dimensional finite element analysis, first, we calculate the guided modes in the NRD guide with height of \( a = 2.25 \) mm and width of \( b = a \). The relative permittivity of the dielectric material is assumed to be \( \varepsilon_r = 2.2 \), and surrounding material is assumed to be air with relative permittivity of \( \varepsilon_{\text{air}} = 1 \). Figure 3 shows the results obtained by two-dimensional (2D)
analysis and three-dimensional (3D) analysis. In this analysis, the results in 2D analysis are analytically calculated and those in 3D analysis are obtained by FEM with edge/nodal hybrid triangular elements [19]. We can see that both results are in good agreement.

B. Wave Propagation Analysis

Next, we consider a waveguide bend as shown in Fig. 4. The height and width of the input and output waveguides are \( a = 2.25 \) mm and \( b = a \), respectively. The inner and outer radius of the waveguide bend are \( R_1 = 3.875 \) mm and \( R_2 = 6.125 \) mm, respectively. We consider the case that the LSM\(_{01}\) mode with operating frequency of 60 GHz is launched into port 1. Figure 5 shows the propagating fields obtained by 2D FEM and 3D FEM [22]. We can see that both results are in good agreement and the normalized transmission power as LSM\(_{01}\) is only 0.21 in both analyses due to mode conversion from LSM\(_{01}\) to LSE\(_{01}\). In this analysis, only 0.4 GB of memory and 4.5 seconds of computational time are required in 2D analysis, whereas about 20 GB of memory and 109 seconds of computational time are required in 3D analysis using a PC with Intel(R) Xeon(R) CPU E5-2660 v4 @ 2.00GHz. In 2D analysis, we can get sufficient numerical convergence using even fewer discretization. In this 3D analysis, the numerical accuracy is within relative error of about 0.5% in the normalized transmission power. To obtain more accurate results, further computational cost is required.

Finally, we consider a waveguide bend as shown in Fig. 6. The shape of this bend is represented by 20 control points \((N = 10)\) placed on the material boundary and these control points are optimized by the simple gradient method where the sensitivities with respect to the design variables are calculated by forward difference formula. In this example, the control points are placed at equal intervals in the angular direction and only the distance from the center is designed. Figure 7 shows the propagating fields in the designed waveguide bend obtained by 2D FEM and 3D FEM. We can see that both results are in good agreement and the normalized transmission power as LSM\(_{01}\) is improved up to 0.97 because mode conversion to LSE\(_{01}\) mode can be suppressed. In Table I, S-parameters are listed for two bends shown in Fig. 4 and optimized bend shown in Fig. 6.

<table>
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<tr>
<th>S-parameter</th>
<th>Output mode</th>
<th>Bend in Fig. 4</th>
<th>Bend in Fig. 6</th>
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<tr>
<td>(</td>
<td>S_{21}</td>
<td>) dB</td>
<td>LSE(_{01})</td>
</tr>
<tr>
<td>LSE(_{01})</td>
<td>-1.04</td>
<td>-1.05</td>
<td>-16.1</td>
</tr>
<tr>
<td>(</td>
<td>S_{11}</td>
<td>) dB</td>
<td>LSE(_{01})</td>
</tr>
<tr>
<td>LSE(_{01})</td>
<td>-34.1</td>
<td>-34.2</td>
<td>-23.0</td>
</tr>
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IV. Conclusion

We proposed two-dimensional finite element analysis of NRD guide devices with uniformity along the vertical direction and derived the formulation by using edge/nodal hybrid triangular elements. To show the validity and usefulness of this approach, two kinds of waveguide bends were calculated by the proposed 2D FEM and the conventional 3D FEM and the good agreement between both results was confirmed. We think this 2D FEM is very useful in future topology optimal design of NRD guide devices.

References


