A simplification method for reflective and rotational symmetry model in electromagnetic field analysis

A Simplification Method for Reflective and Rotational Symmetry Model in Electromagnetic Field Analysis

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***Abstract*—In this paper, a simplification method for reflective and rotational symmetry model, is proposed. Using spatial eigenmodes, the coefficient matrix of the final simultaneous equations for the time and storage capacity can be reduced. The proposed method can adapt to not only a integral-equation-method model but also finite element method model.**

***Index Terms*—Model simplification, reflective symmetry.**

I. Introduction

FOR PRACTICAL electromagnetic field problems, the size of the simultaneous equations of numerical models becomes increasingly large. Therefore, simplification methods of the numerical models are required. In a rotational symmetry model, the simplification methods are proposed [1], [2], [4]. However, these simplification methods can not adapt to the model that has the reflective symmetry. In this paper, the authors propose the new simplification method that can adapt to the reflective symmetry model and the reflective and rotational symmetry model. This method can adapt to both the finite element method (FEM) and integral-equation-method (IEM). In this method, since the coefficient matrix of the simultaneous equations is transferred to the block diagonal matrix , the computation time and storage capacity are substantially reduced.

II. Formulation

A. Integral Equation Method

A reflective and rotational model is shown in Fig. 1. The model has a reflective symmetry between upper group and lower group. There is another reflective symmetry, which includes rotational symmetry, in both upper and lower groups. Each group is called real or imaginary group. Each symmetry boundary A, B and Z are shown in Fig. 1. Fig. 1(a) is the example, which N (number of subregions in each group) is three.

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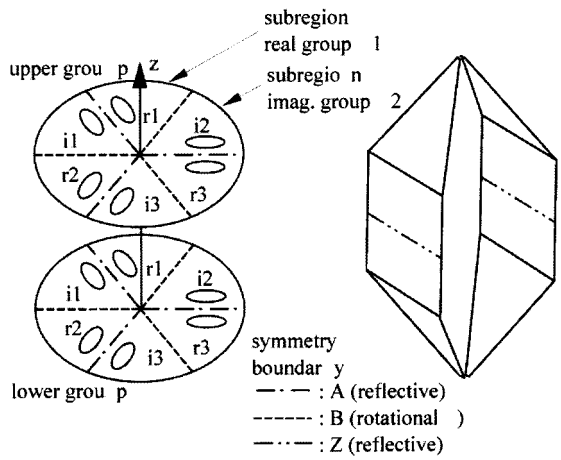

Fig. 1. Symmetrical model.

Firstly, neglecting the connection of each subregion, the simultaneous equations are introduced as follows:

$$\begin{bmatrix} S & S' & M & M' \\ S' & S & M' & M \\ M & M' & S & S' \\ M' & M & S' & S \end{bmatrix} \begin{Bmatrix} \boldsymbol{F_{u,r}} \\ \boldsymbol{F_{u,i}} \\ \boldsymbol{F_{l,r}} \\ \boldsymbol{F_{l,i}} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{B_{u,r}} \\ \boldsymbol{B_{u,i}} \\ \boldsymbol{B_{l,r}} \\ \boldsymbol{B_{l,i}} \end{Bmatrix}, \quad (1)$$

where

$\boldsymbol{F}$	is unknown variables vector,
$\boldsymbol{B}$	the constant vector,
the matrices $[S]$, $[S]'$, $[M]$, and $[M]'$	the coefficient matrices, and
the suffices u, l, r, i	denote upper, lower, real, imaginary group.

Moreover, the vectors and the matrices of each group are divided by the subregions as follow:

$$\mathbf{F} = \{\mathbf{F}_1 \quad \mathbf{F}_2 \quad \cdots \quad \mathbf{F_N}\}^{\mathbf{T}},$$

$$\mathbf{B} = \{\mathbf{B}_1 \quad \mathbf{B}_2 \quad \cdots \quad \mathbf{B_N}\}^{\mathbf{T}}$$

$$[S] = \begin{bmatrix} S_0 & S_1 & \cdots & S_{N-1} \\ S_{N-1} & S_0 & \cdots & \cdots \\ \cdots & \cdots & \cdots & S_1 \\ S_1 & \cdots & S_{N-1} & S_0 \end{bmatrix},$$

$$[S]' = \begin{bmatrix} S_0 & S_1 & \cdots & S_{N-1} \\ S_1 & \cdots & S_{N-1} & S_0 \\ \cdots & \cdots & \cdots & \cdots \\ S_{N-1} & S_0 & \cdots & S_{N-2} \end{bmatrix}',$$

$$[M] = \begin{bmatrix} M_0 & M_1 & \cdots & M_{N-1} \\ M_{N-1} & M_0 & \cdots & \cdots \\ \cdots & \cdots & \cdots & M_1 \\ M_1 & \cdots & M_{N-1} & M_0 \end{bmatrix},$$

$$[M]' = \begin{bmatrix} M_0 & M_1 & \cdots & M_{N-1} \\ M_1 & \cdots & M_{N-1} & M_0 \\ \cdots & \cdots & \cdots & \cdots \\ M_{N-1} & M_0 & \cdots & M_{N-2} \end{bmatrix}'. \quad (2)$$

Secondly the connections of subregions are treated as boundary conditions. The boundary conditions are as follows:

$$\boldsymbol{F_{n,z}} = -\boldsymbol{F_{l,z}},\ \boldsymbol{F_{r,a}} = -[K_a]\boldsymbol{F},\ \boldsymbol{F_{r,b}} = -[K_b]\boldsymbol{F_{i,b}}, \quad (3)$$

where

$$[K_a] = \begin{bmatrix} E & 0 & \cdots & 0 \\ 0 & \cdots & 0 & E \\ \cdots & \cdots & \cdots & \cdots \\ 0 & E & 0 & \cdots \end{bmatrix},$$

$$[K_b] = \begin{bmatrix} 0 & E & 0 & 0 & 0 \\ E & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & E \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & E & 0 & \cdots \end{bmatrix}, \quad (4)$$

and $[E]$ is the unit matrix, $[0]$ the null matrix, the suffices z, a, b, denote the Z, A, B boundaries, and the variable is the normal vector to the boundary.

Then, according to the boundary, the coefficient matrix and the constant vector of each subregion are expressed as follows:

$$[S]_0 = \begin{bmatrix} S_{zz} & S_{za} & S_{zb} & S_{zf} \\ S_{az} & S_{aa} & S_{ab} & S_{af} \\ S_{bz} & S_{ba} & S_{bb} & S_{bf} \\ S_{fz} & S_{fa} & S_{fb} & S_{ff} \end{bmatrix}_0,$$

$$\boldsymbol{B} = \{\, \boldsymbol{B_z} \quad \boldsymbol{B_a} \quad \boldsymbol{B_b} \quad \boldsymbol{B_f} \,\}^T, \quad (5)$$

where, the suffix f denotes the region except the boundary.

Using the boundary condition (3), the coefficient matrix and the constant vector are modified as follows:

$$[S_{zz}]_0 = [S_{zz}]_0 - [M_{zz}]_0, \quad [S_{aa}]_0 = [S_{aa}]_0 - [S_{aa}]'_0$$
$$[S_{bb}]_0 = [S_{bb}]_0 - [S_{bb}]'_1.$$
$$\boldsymbol{B_{u,z}} = \boldsymbol{B_{u,z}} - \boldsymbol{B_{l,z}}, \quad \boldsymbol{B_{l,z}} = \boldsymbol{B_{l,z}} - \boldsymbol{B_{u,z}},$$
$$\boldsymbol{B_{r,a}} = \boldsymbol{B_{r,a}} - [K_a]\boldsymbol{B_{i,a}}, \quad \boldsymbol{B_{i,a}} = \boldsymbol{B_{i,a}} - [K_a]\boldsymbol{B_{r,a}},$$
$$\boldsymbol{B_{r,b}} = \boldsymbol{B_{r,b}} - [K_b]\boldsymbol{B_{i,b}}, \quad \boldsymbol{B_{i,b}} = \boldsymbol{B_{i,b}} - [K_b]\boldsymbol{B_{r,b}} \quad (6)$$

where the matrices and the vectors of right term are quantities before modification.

Since the equations, that are modified with boundary condition, have the same mathematical form as (1), we discuss about (1).

And then, using two reflective symmetries, the variables transform are introduced as follows:

$$\mathbf{F_{u,r}} = \mathbf{F_I} + \mathbf{F_{II}} + \mathbf{F_{III}} + \mathbf{F_{IV}},$$
$$\mathbf{F_{u,i}} = \mathbf{F_I} - \mathbf{F_{II}} + \mathbf{F_{III}} - \mathbf{F_{IV}},$$
$$\mathbf{F_{l,r}} = \mathbf{F_I} + \mathbf{F_{II}} - \mathbf{F_{III}} - \mathbf{F_{IV}},$$
$$\mathbf{F_{l,i}} = \mathbf{F_I} - \mathbf{F_{II}} - \mathbf{F_{III}} + \mathbf{F_{IV}} \quad (7)$$

where $\boldsymbol{F_I}$, $\boldsymbol{F_{II}}$, $\boldsymbol{F_{III}}$, $\boldsymbol{F_{IV}}$ are new variables.

The transform (7) gives four one-fourth size models as follows:

$$[S + M + S' + M']\mathbf{F_I} = \mathbf{B_I},$$
$$[S + M - S' - M']\mathbf{F_{II}} = \mathbf{B_{II}},$$
$$[S - M + S' - M']\mathbf{F_{III}} = \mathbf{B_{III}},$$
$$[S - M - S' + M']\mathbf{F_{IV}} = \mathbf{B_{IV}}, \quad (8)$$

where $\boldsymbol{B_I}$, $\boldsymbol{B_{II}}$, $\boldsymbol{B_{III}}$, $\boldsymbol{B_{IV}}$ are the new constant vectors. The mathematical form of four equations in (8) are same. To simplify the explanation, we treat only $\boldsymbol{F_I}$ and the suffix I is omitted.

Then, to utilize the rotational symmetry, the following variables transform is used [2].

$$\boldsymbol{F_K} = \sum_{\boldsymbol{n}=0}^{N-1} \boldsymbol{a}^{(k-1)\boldsymbol{n}} \boldsymbol{f_n}, \qquad (k = 1, 2, \ldots, N) \quad (9)$$

where $\boldsymbol{f}$ is the new variable vector,

$$a = \varepsilon^{j2\pi/N}.$$

Using the transform (9), the equation is simplified as follows:

$$\left([\tilde{S}]_0 + [\tilde{S}]'_0\right)\boldsymbol{f}_0 = \boldsymbol{b}_0,$$
$$[\tilde{S}]_k \boldsymbol{f}_k + [\tilde{S}]'_{N-k}\boldsymbol{f}_{N-k} = \boldsymbol{b}_k,$$
$$[\tilde{S}]'_k \boldsymbol{f}_k + [\tilde{S}]_{N-k}\boldsymbol{f}_{N-k} = \boldsymbol{b}_{N-k}, \quad (k = 1, 2, \ldots, K)$$
$$\left([\tilde{S}]'_{N/2} + [\tilde{S}]'_{N/2}\right)\boldsymbol{f}_{N/2} = \boldsymbol{b}_{N/2}: \quad N \text{ even},$$

where $\boldsymbol{b}$ is a new constant vector,

$$[\tilde{S}]_{\mathrm{k}} = \sum_{n=0}^{N-1} a^{kn}[S + M]_n, \quad [\tilde{S}]'_{\mathrm{k}} = \sum_{n=0}^{N-1} a^{kn}[S + M]'_n$$
$$K = (N-1)/2: \quad N \text{ add}$$
$$K = (N-2)/2: \quad N \text{ even}. \quad (11)$$

B. Finite Element Method

Firstly, to simplify the treatment of boundary condition on A, we renumber the subregions as Fig. 2. Neglecting the connection between subregions, we introduce the equations. In this case, the only $[S]_0$ is not the null matrix in the coefficient matrices. Then, we assume that the boundary A, B and Z have not the cross point to each other. And the matrices $[S_{za}]$, $[S_{zb}]$ and $[S_{ab}]$ become the null matrices.

Secondary, the boundary conditions are given as follows:

$$\boldsymbol{F_{u,z}} = \boldsymbol{F_{l,z}},\ \boldsymbol{F_{r,a}} = \boldsymbol{F_{i,a}},\ \boldsymbol{F_{r,b}} = [K_b]'\boldsymbol{F_{i,b}} \quad (12)$$

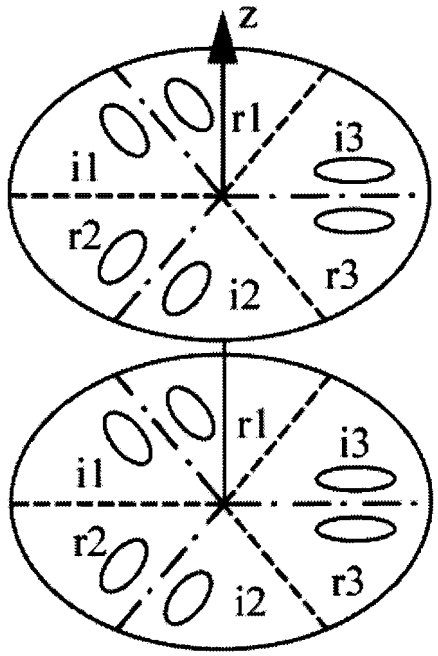

Fig. 2. Renumbering of subregions.

where

$$[K_b]' = \begin{bmatrix} 0 & \cdots & 0 & E \\ E & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & 0 & E & 0 \end{bmatrix}.$$

And since subregions are renumbered, the matrix $[S]'$ becomes a differential form with the case of IEM as follows:

$$[S]' = \begin{bmatrix} S_0 & 0 & \cdots & 0 & S_1 \\ S_1 & S_0 & 0 & \cdots & 0 \\ 0 & S_1 & S_0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & S_1 & S_0 \end{bmatrix}'. \tag{13}$$

Using boundary conditions (12), the submatrices of coefficient matrices and constant vectors are modified as follows:

$$\begin{aligned} &[S_{zz}]_0 = 2[S_{zz}]_0, \quad [S_{aa}]_0 = 2[S_{aa}]_0, \\ &[S_{bb}]_0 = 2[S_{bb}]_0, \quad [M_{zf}]_0 = [S_{zf}]_0, \\ &[S_{af}]'_0 = [S_{af}]_0, \quad [S_{bf}]'_1 = [S_{bf}]_0, \\ &\boldsymbol{B_{r,b}} = \boldsymbol{B_{r,b}} + [K_b]'\boldsymbol{B_{i,b}}, \quad \boldsymbol{B_{i,b}} = \boldsymbol{B_{i,b}} + ([K_b]')^{\boldsymbol{T}}\boldsymbol{B_{r,b}}, \\ &\boldsymbol{B_{r,a}} = \boldsymbol{B_{r,a}} + \boldsymbol{B_{i,a}}, \qquad \boldsymbol{B_{i,a}} = \boldsymbol{B_{i,a}} + \boldsymbol{B_{r,a}}, \\ &\boldsymbol{B_{u,z}} = \boldsymbol{B_{u,z}} + \boldsymbol{B_{l,z}}, \boldsymbol{B_{l,z}} = \boldsymbol{B_{l,z}} + \boldsymbol{B_{u,z}}. \end{aligned} \tag{14}$$

Firstly, using the same procedure as the case of IEM, the model become four one-fourth models with variables transform (8). Moreover, using transform (9), the model is more simplified. And, to make the symmetric matrix, the coefficient matrix and the constant vector are modified. Then, the simplified equations are got as follows:

$$\begin{bmatrix} \boldsymbol{S_{zz}} & 0 & 0 & \boldsymbol{S_{zf}} \\ 0 & \boldsymbol{S_{aa}} & 0 & \boldsymbol{S_{af}} \\ 0 & 0 & \boldsymbol{S_{bb,n}} & \boldsymbol{S_{bf}} \\ \boldsymbol{S_{fz}} & \boldsymbol{S_{fa}} & \boldsymbol{S_{fb}} & \boldsymbol{S_{ff}} \end{bmatrix} \begin{Bmatrix} \mathrm{f}_{z,n} \\ \mathrm{f}_{a,n} \\ \mathrm{f}_{b,n} \\ \mathrm{f}_{f,n} \end{Bmatrix} = \begin{Bmatrix} \tilde{\mathrm{b}}_{z,n} \\ \tilde{\mathrm{b}}_{a,n} \\ \tilde{\mathrm{b}}_{b,n} \\ \tilde{\mathrm{b}}_{f,n} \end{Bmatrix},$$
$$(n = 0, 1, \ldots, N-1) \tag{15}$$

where

$$[S_{bb,n}] = \left\{2/1\left(1 + a^{-\boldsymbol{n}}\right)\right\}[S_{bb}], \quad \tilde{\boldsymbol{b}}_{\boldsymbol{b,n}} = \boldsymbol{b_{b,n}}/\left(1 + a^{-\boldsymbol{n}}\right),$$
$$\tilde{\boldsymbol{b}}_{\boldsymbol{z,n}} = \boldsymbol{b_{z,n}}/2, \quad \tilde{\boldsymbol{b}}_{\boldsymbol{a,n}} = \boldsymbol{b_{a,n}}/2.$$

Finally, $4N$ small simultaneous equations are given. Since each model size is one-$4N$th, the required storage capacity and computation time are extremely decreased. Table I shows the effect of the proposed method.

TABLE I
REDUCTION EFFECT OF CALCULATING TIME

modeling	coefficient matrix	Gauss elimination	
		N: odd	N:even
IEM	1/(4N)	(4N-3)/(16N³)	(2N-3)/(8N³)
FEM		1/(16N²)	

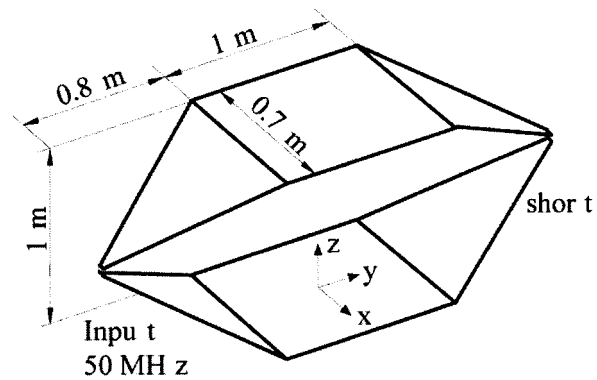

Fig. 3. TEM cell method.

III. Computation Results

Fig. 3 shows a TEM(transverse-electromagnetic) cell model. The problem is solved by the triangular-patch moment method [3]. Using the simplification method, the model is simplified to a one-8th model. Fig. 4 shows the computation result of current distributions. The results agree with the results which do not use the simplification method.

IV. Conclusion

A new simplification method for reflective and rotational symmetry model was proposed. The method can adapt to not only IEM model but also FEM model. Using spatial eigenmodes, the model is simplified and the computation time is

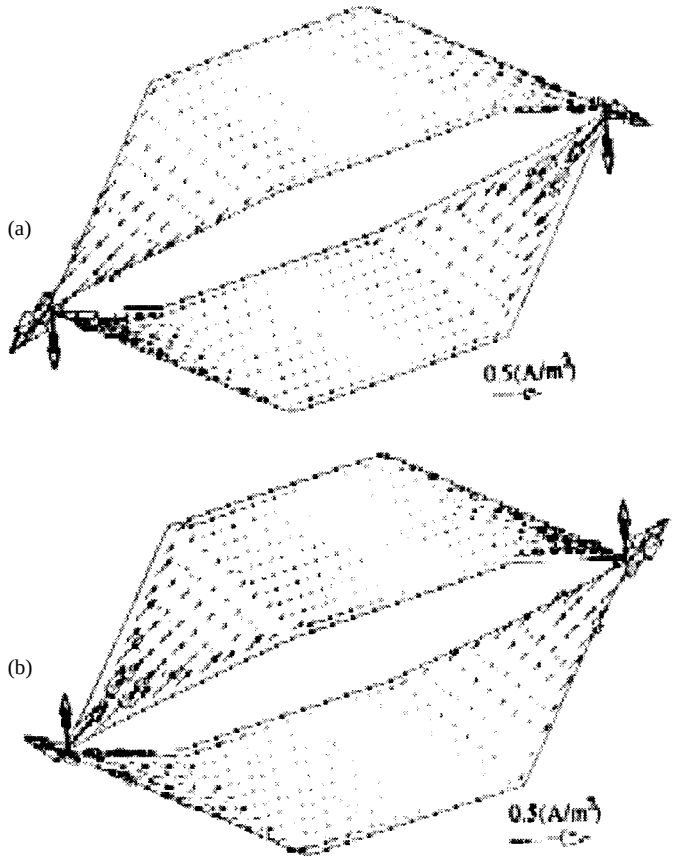

Fig. 4. Calculated distribution of current density vectors, (a) real part, (b) imaginary part.

reduced. Applicability of the proposed method was verified by TEM cell model.

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