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Interrelationship Mining from a Viewpoint of Rough Sets on Two Universes

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***Abstract*—We discuss connections between the interrelationship mining, proposed by the authors, and rough sets on two universes. The interrelationship mining enable us to extract characteristics based on comparison between values of different attributes. Rough sets on two universes is an theoretical extension of the original rough sets by considering connection between two universes. In this paper, we point out that interrelationship between different attributes in the interrelationship mining is representable by a variant of rough sets on two universes.**

***Keywords*-interrelationship mining; rough set on two universes**

I. Introduction

Rough set theory [6], [7] provides a mathematical foundation of set-theoretical approximations of concepts and rule-extraction-based data mining. From a viewpoint of approximations, various extensions of rough set theory have been proposed; variable precision rough set [12], generalized rough set [9], covering-based rough set (e.g. [10], [13]), and rough sets on two universes [5], [8].

From a viewpoint of rule-extraction-based data mining, the authors have proposed a concept of interrelationship mining [1], [2], [3]. Almost approaches of decision rule extraction are based on comparison between values of the same attribute. On the other hand, the interrelationship mining enables us to extract characteristics based on comparison between values of different attributes in the framework of rough set theory. The interrelationships between attributes are defined by binary relations on the set of attribute values.

In this paper, we discuss a connection between the interrelationship mining and rough sets on two universes. In particular, we point out that interrelationship between different attributes in the interrelationship mining is representable by a variant of rough sets on two universes.

II. Rough Set

In this section, we briefly review Pawlak's rough set theory. The contents of this section is mainly [1], [4].

Information tables describe connections between objects and attributes by table-style format. In this paper, similar to the authors' previous manuscript [4], we use a general expression of information tables that was used by Yao et al. [11] defined by

$$S = (U, AT, \{V_{\mathsf{a}} \mid \mathsf{a} \in AT\}, \mathcal{R}_{AT}, \rho), \tag{1}$$

where U is a finite and nonempty set of objects, AT is a finite and nonempty set of attributes, V_{a} is a nonempty set of values for $\mathsf{a} \in AT$, $\mathcal{R}_{AT} = \{\{R_{\mathsf{a}}\} \mid \mathsf{a} \in AT\}$ is a set of families $\{R_{\mathsf{a}}\}$ of binary relations defined on each V_{a}, ρ is an information function $\rho : U \times AT \to V$ that assigns a value $\rho(x, \mathsf{a}) \in V_{\mathsf{a}}$ of the attribute $\mathsf{a} \in AT$ to each object $x \in U$, where $V = \bigcup_{\mathsf{a} \in AT} V_{\mathsf{a}}$ is the set of values of all attributes in AT.

The family $\{R_{\mathsf{a}}\}$ of binary relations for each attribute $\mathsf{a} \in AT$ can contain various binary relations; similarity, dissimilarity, ordering relation on V_{a} and usual information tables are implicitly assumed that the family $\{R_{\mathsf{a}}\}$ consists of only the equality relation $=$ on V_{a} [11]. We also assume that the equality relation $=$ is included into the family $\{R_{\mathsf{a}}\}$ for every attribute $\mathsf{a} \in AT$.

An information table is called a decision table if the set of attributes AT is partitioned into two disjoint sets; i.e., a set C of condition attributes and a set D of decision attributes. In this paper, without losing generality, we assume that D is a singleton, i.e., $D = \{\mathsf{d}\}$, and the attribute d is called the decision attribute.

Indiscernibility relations based on subsets of attributes provide classifications of objects in decision tables. For any set of attributes $B \subseteq AT$, the indiscernibility relation $IND(B)$ is the following binary relation on U:

$$IND(B) = \{(x, y) \mid \rho(x, \mathsf{a}) = \rho(y, \mathsf{a}), \forall \mathsf{a} \in B\}. \tag{2}$$

iAny indiscernibility relation $IND(B)$ provides a partition of U, i.e., the quotient set $U/IND(B)$.

Example 1 ([4]): Table I is an example of a decision table S. The decision table consists of a set of eight users of sample products; $U = \{u_1, \cdots, u_8\}$ as the set of objects, the set of attributes AT that is divided into the set of condition attributes $C = \{\mathsf{Member}, \mathsf{Sex}, \mathsf{Before}, \mathsf{After}\}$ that represents users' membership, sex, evaluation before/after using a sample product, respectively, and the set of decision attribute $D = \{\mathsf{Purchase}\}$ that represents users' answer to

Table I
DECISION TABLE

U	Member	Sex	Before	After	Purchase
$u1$	yes	female	good	v.g.	yes
$u2$	yes	female	normal	good	yes
$u3$	yes	male	good	v.g.	yes
$u4$	no	female	good	good	yes
$u5$	yes	male	good	good	maybe
$u6$	no	male	normal	normal	maybe
$u7$	yes	male	normal	bad	no
$u8$	no	female	good	normal	no

the question about purchase. Each attribute $\mathsf{a} \in AT$ has the following family of binary relations on the set V_{a} of values:

$$\begin{gathered}\mathsf{Member}: \{=\},\ \ \mathsf{Sex}: \{=\},\\ \mathsf{Before}: \{=, \succ_{\mathsf{Before}}, \succeq_{\mathsf{Before}}\}, \mathsf{After}: \{=, \succ_{\mathsf{After}}, \succeq_{\mathsf{After}}\},\\ \mathsf{Purchase}: \{=, \succ_{\mathsf{Purchase}}, \succeq_{\mathsf{Purchase}}\},\end{gathered}$$

where each relation $\succ_{\mathsf{a}}$ is a preference relation that is defined as follows:

$$\begin{array}{ll} \succ_{\mathsf{Before}}: & \text{v.g.} \succ \text{good} \succ \text{normal} \succ \text{bad} \succ \text{v.b.},\\ \succ_{\mathsf{After}}: & \text{v.g.} \succ \text{good} \succ \text{normal} \succ \text{bad} \succ \text{v.b.},\\ \succ_{\mathsf{Purchase}}: & \text{yes} \succ \text{maybe} \succ \text{no}. \end{array}$$

III. INTERRELATIONSHIP MINING

In this section, we review the concept of interrelationship mining [1], [2], [3].

Suppose a decision table S by (1) with $AT = C \cup \{\mathsf{d}\}$ is given and let $\mathsf{a}, \mathsf{b} \in C$ be two condition attributes with ranges V_{a} and V_{b}, respectively, and $R \subseteq V_{\mathsf{a}} \times V_{\mathsf{b}}$ be a binary relation from V_{a} to V_{b}. We call that attributes a and b are interrelated by R if and only if there exists an object $x \in U$ such that $(\rho(x, \mathsf{a}), \rho(x, \mathsf{b})) \in R$ holds.

We denote the set of objects that those values of attributes a and b satisfy the relation R as follows:

$$R(\mathsf{a}, \mathsf{b}) = \{x \in U \mid (\rho(x, \mathsf{a}), \rho(x, \mathsf{b})) \in R\}, \tag{3}$$

and we call the set $R(\mathsf{a}, \mathsf{b})$ the support set of the interrelation between a and b by R.

An interrelationship between two attributes by a binary relation provides a formulation of comparison of attribute values between different attributes. However, to simplify the formulation, we allow the interrelationship between the same attribute.

We can introduce indiscernibility relations in a given decision table by interrelationships between attributes. Let S be a decision table, and suppose that condition attributes $\mathsf{a}, \mathsf{b} \in C$ are interrelated by a binary relation $R \subseteq V_{\mathsf{a}} \times V_{\mathsf{b}}$, i.e., $R(\mathsf{a}, \mathsf{b}) \neq \emptyset$ holds. The indiscernibility relation on U based on the interrelationship between a and b by R is defined by

$$IND(\mathsf{a}R\mathsf{b}) = \left\{ (x, y) \;\middle|\; \begin{array}{l} x \in U, y \in U, \text{ and} \\ x \in R(\mathsf{a}, \mathsf{b}) \text{ iff } y \in R(\mathsf{a}, \mathsf{b}) \end{array} \right\}. \tag{4}$$

For any objects x and y, $(x, y) \in IND(\mathsf{a}R\mathsf{b})$ means that x is not discernible from y from the viewpoint of whether the interrelationship between the attributes a and b by the relation R holds. Any binary relation $IND(\mathsf{a}R\mathsf{b})$ on U defined by (4) is an equivalence relation, and we can construct equivalence classes from an indiscernibility relation $IND(\mathsf{a}R\mathsf{b})$.

To explicitly treat interrelationships between attributes, we need to reformulate the information table S by (1) by using the given binary relations between values of different attributes. This reformulation is based on revising the set $\mathcal{R}_{AT}$ of families of binary relations for comparing attribute values and expression of interrelationships by new condition attributes.

Definition 1 ([3]): Let S be an information table by (1). The information table S_{int} for interrelationship mining with respect to S is defined as follows:

$$S_{int} = (U, AT_{int}, V \cup \{0, 1\}, \mathcal{R}_{int}, \rho_{int}), \tag{5}$$

where U and $V = \bigcup_{\mathsf{a} \in AT} V_{\mathsf{a}}$ are identical to S.

The set $\mathcal{R}_{int}$ of families of binary relations is defined by

$$\begin{aligned}\mathcal{R}_{int} = \mathcal{R}_{AT} \cup \left\{ \{R_{\mathsf{a}_i \times \mathsf{b}_i}\} \;\middle|\; \begin{array}{l} R_{\mathsf{a}_i \times \mathsf{b}_i} \subseteq V_{\mathsf{a}_i} \times V_{\mathsf{b}_i}, \\ \exists \mathsf{a}_i, \mathsf{b}_i \in C \end{array} \right\} \\ \cup \{\{=\} \mid \text{For each } \mathsf{aRb}\},\end{aligned} \tag{6}$$

where each family $\{R_{\mathsf{a}_i \times \mathsf{b}_i}\} = \{R^1_{\mathsf{a}_i \times \mathsf{b}_i}, \cdots, R^{n_i}_{\mathsf{a}_i \times \mathsf{b}_i}\}$ consists of n_i ($n_i \geq 0$) binary relation(s) defined on $V_{\mathsf{a}_i} \times V_{\mathsf{b}_i}$. The expression aRb is defined below.

The set AT_{int} is defined by

$$AT_{int} = AT \cup \{\mathsf{aRb} \mid \exists R \in \{R_{\mathsf{a} \times \mathsf{b}}\}, R(\mathsf{a}, \mathsf{b}) \neq \emptyset\}, \tag{7}$$

and each expression aRb is called an interrelated condition attribute. $AT = C \cup \{\mathsf{d}\}$ is identical to S.

The information function ρ_{int} is defined by

$$\rho_{int}(x, \mathsf{c}) = \begin{cases} \rho(x, \mathsf{c}), & \text{if } \mathsf{c} \in AT, \\ 1, & \mathsf{c} = \mathsf{aRb} \text{ and } x \in R(\mathsf{a}, \mathsf{b}), \\ 0, & \mathsf{c} = \mathsf{aRb} \text{ and } x \notin R(\mathsf{a}, \mathsf{b}). \end{cases} \tag{8}$$

Each interrelated condition attribute aRb represents whether each object $x \in U$ supports the interrelationship between the attributes $\mathsf{a}, \mathsf{b} \in C$ by the binary relation $R \subseteq V_{\mathsf{a}} \times V_{\mathsf{b}}$. For every interrelated condition attribute, we only treat the equality relation for comparing attribute values of different objects. This is because interrelated condition attributes are nominal attributes.

Indiscernibility of objects by an interrelationship between two attributes a and b by a binary relation R in the original decision table S is representable by an indiscernibility relation by the singleton $\{\mathsf{aRb}\}$ in S_{int}, i.e., the following equation holds [1], [3]:

$$IND_S(\mathsf{a}R\mathsf{b}) = IND_{S_{int}}(\{\mathsf{aRb}\}), \tag{9}$$

where $IND_S(\mathsf{a}R\mathsf{b})$ is the indiscernibility relation in S with respect to the interrelationship between a and b by R defined by (4), and $IND_{S_{int}}(\{\mathsf{aRb}\})$ is the indiscernibility relation in S_{int} constructed from the singleton $\{\mathsf{aRb}\}$ defined by (2).

IV. Interrelationship Mining on Two Universes

In this section, we formulate the interrelationship mining on two universes.

A. Approximation Space for Interrelationship Mining

Let U and V be two finite and non-empty universes. In general, a binary relation $R \subseteq U \times V$ is considered for connecting two universes [5], [8], we use a finite set F of mappings from U to V, i.e.,

$$F = \{\mathsf{a} \mid \mathsf{a} : U \to V\}. \tag{10}$$

Suppose that, for each mapping $\mathsf{a} \in F$, the set V_{a} denotes the range of the mapping a and $V_{\mathsf{a}} \subseteq V$ holds.

We call the triple (U, V, F) an approximation space for interrelationship mining. The two universes U and V and the set of mapping F are able to be regarded as the set of objects, the set of attribute values, and the set of attributes in a given information table, respectively.

Let $\mathsf{a} \in F$ be a mapping. For each element $v \in V_{\mathsf{a}}$, the inverse image of the singleton $\{v\}$ is a subset of U, i.e.,

$$\mathsf{a}^{-1}(\{v\}) = \{x \in U \mid \mathsf{a}(x) = v\}. \tag{11}$$

It is easily observed that the set of all non-empty inverse images of singletons $\{v\}$ for $v \in V_{\mathsf{a}}$ provides a partition of U.

Proposition 1: Let $\mathsf{a} \in F$ be a mapping from U to V_{a}. The set P_{a} of all non-empty inverse images of singletons $\{v\}$ for $v \in V_{\mathsf{a}}$ defined by

$$P_{\mathsf{a}} = \{\mathsf{a}^{-1}(\{v\}) \mid v \in V_{\mathsf{a}}, \mathsf{a}^{-1}(\{v\}) \neq \emptyset\} \tag{12}$$

is a partition of U, i.e., P_{a} satisfies the following properties:
(1) $\mathsf{a}^{-1}(\{v_i\}) \cap \mathsf{a}^{-1}(\{v_j\}) = \emptyset$ if $v_i \neq v_j$ holds, and
(2) $\bigcup_{\mathsf{a}^{-1}(\{v\}) \in P_{\mathsf{a}}} \mathsf{a}^{-1}(\{v\}) = U$.

It is well-known that a partition P on U generates a unique equivalence relation R on U such that the quotient set U/R is identical to the partition P. Therefore, each mapping $\mathsf{a} \in F$ provides an equivalence relation R_{a} on U by

$$R_{\mathsf{a}} = \{(x, y) \mid \exists \mathsf{a}^{-1}(\{v\}) \in P_{\mathsf{a}} \text{ s. t. } x, y \in \mathsf{a}^{-1}(\{v\})\}. \tag{13}$$

Moreover, it is also well-known that the intersection of equivalence relations is also an equivalence relation. We then have an equivalence relation $R_G = \bigcap_{\mathsf{a} \in G} R_{\mathsf{a}}$ for any non-empty subset $G \subseteq F$, and we are also able to consider lower and upper approximations of any subset $X \subseteq U$ with respect to R_G.

Example 2: We construct an approximation space (U, V, F) for interrelationship mining from Table I in Example 1.

Let $S = (U, AT, \{V_{\mathsf{a}} \mid \mathsf{a} \in AT\}, \mathcal{R}_{AT}, \rho)$ be the decision table in Table I. We treat the set U of objects as a set U in the approximation space for interrelationship mining. As the set V in the approximation space, we use the union of the sets of values, i.e., $V = \bigcup_{\mathsf{a} \in AT} V_{\mathsf{a}}$.

For each attribute $\mathsf{a} \in AT$, we define a mapping a from U to V by $\mathsf{a}(x) \stackrel{\text{def}}{=} \rho(x, \mathsf{a})$. We used the same symbol a to denote a mapping $\mathsf{a} : U \to V$ generated from the attribute $\mathsf{a} \in AT$ in the table I. Therefore, the set F of mappings is

$$F \stackrel{\text{def}}{=} \{\mathsf{a} : U \to V \mid \mathsf{a} \in AT, \mathsf{a}(x) = \rho(x, \mathsf{a})\}.$$

Suppose (U, V, F) be the approximation space we constructed from Table I and $\mathsf{Member} \in F$ be the mapping $\mathsf{Member} : U \to V$. For two elements $\text{yes}, \text{no} \in V$, the inverse image of each singleton by Member is

$$\begin{aligned} \mathsf{Member}^{-1}(\{\text{yes}\}) &= \{u1, u2, u3, u5, u7\}, \\ \mathsf{Member}^{-1}(\{\text{no}\}) &= \{u4, u6, u8\}. \end{aligned}$$

We then have a partition P_{Member} induced by the mapping Member.

B. Representatio of interrelationships on the approximation space (U, V, F)

In the previous subsection, we constructed partitions from mappings from U to V and elements $v \in V$. In this section, we consider adding some conditions to the elements $v \in V$ by introducing binary relations on V.

Suppose R is a binary relation on V and $v \in V$ is an element of V. We define a set $l_R(v)$ as follows and call the set $l_R(v)$ the *left-concerned set* of R with respect to v.

$$l_R(v) \stackrel{\text{def}}{=} \{w \in V \mid wRv\}. \tag{14}$$

It is easily observed that we can treat various conditions of elements in V by introducing a binary relation R on V.

Proposition 2: Let $\mathsf{a} \in F$ be a mapping, R be a binary relation on V, and $v \in V$ be an element of V. The union of sets $\mathsf{a}^{-1}(\{w\})$ for elements w in the left-concerned set of R with respect to v satisfies the following equation:

$$\bigcup_{w \in l_R(v)} \mathsf{a}^{-1}(\{w\}) = \{x \in U \mid (\mathsf{a}(x), v) \in R\}. \tag{15}$$

If the binary relation R on V is the equality $=$ on V, the union of the left-concerned sets of an element $v \in V$ defined by (15) is identical to the inverse image of the singleton $\{v\}$ defined by (11), i.e.,

$$\begin{aligned} \bigcup_{w \in l_{=}(v)} \mathsf{a}^{-1}(\{w\}) &= \{x \in U \mid \mathsf{a}(x) = v\} \\ &= \mathsf{a}^{-1}(\{v\}). \end{aligned}$$

Therefore, this property indicates that (11) is a special case of (15).

Example 3: This example is continuation of Example 2. Suppose (U, V, F) be the approximation space we constructed from Table I and $\mathsf{After} \in F$ be the mapping

$\mathsf{After} : U \to V$. Suppose a binary relation $\succ$ on V provides the following order relationship:

$$\text{v.g.} \succ \text{good} \succ \text{normal} \succ \text{bad} \succ \text{v.b..}$$

We then have the left-concerned set $l_{\succ}(\text{normal})$ for the value $\text{normal} \in V$ as follows:

$$l_{\succ}(\text{normal}) = \{\text{v.g.}, \text{good}\}.$$

Therefore, the union of sets $\mathsf{a}^{-1}(\{w\})$ for elements in the left-concerned set $l_{\succ}(\text{normal})$ is

$$\begin{aligned} & \bigcup_{w \in l_{\succ}(\text{normal})} \mathsf{After}^{-1}(\{w\}) \\ = & \{x \in U \mid \mathsf{After}(x) \succ \text{normal}\} \\ = & \{u1, u2, u3, u4, u5\}, \end{aligned}$$

i.e., the set of elements that the value by the mapping After is v.g. or good.

Note that the value $v \in V$ used for the left-concerned set $l_R(v)$ does not need to be in the range of the mapping to consider the inverse images. Then, we can use elements in the range of another mapping for the left-concerned set.

Let a and b be mappings from U to V, $v \in V_{\mathsf{a}} \subseteq V$ and $p \in V_{\mathsf{b}} \subseteq V$ be elements in the range of a and b, respectively. The intersection of the inverse images of singletons $\{v\}$ and $\{p\}$, i.e.,

$$\mathsf{a}^{-1}(\{v\}) \cap \mathsf{b}^{-1}(\{p\}) = \{x \in U \mid \mathsf{a}(x) = v, \mathsf{b}(x) = p\} \tag{16}$$

is the set of objects that satisfy both conditions of mappings a and b. Therefore, similar to the cases of equations (11) and (15), we can show that the following property holds.

Proposition 3: Let a and b be mappings from U to V, and $v \in V_{\mathsf{a}}$ and $p \in V_{\mathsf{b}}$ be elements in the range of a and b, respectively. The following equation holds:

$$\bigcup_{p \in V_{\mathsf{b}}} \bigcup_{v \in l_R(p)} \left(\mathsf{a}^{-1}(\{v\}) \cap \mathsf{b}^{-1}(\{p\})\right) \tag{17}$$

$$= \{x \in U \mid (\mathsf{a}(x), \mathsf{b}(x)) \in R\}. \tag{18}$$

This property shows that the equation (17) corresponds to the support set $R(\mathsf{a}, \mathsf{b})$ of the interrelationship between a and b by the binary relation R defined by (3). Support sets are essential for the interrelationship mining, and therefore, this property concludes that the interrelationship mining is also representable from a viewpoint of rough sets on two universe.

Example 4: This example is continuation of Example 3 and Suppose $\mathsf{Before} \in F$ be the mapping $\mathsf{Before} : U \to V$. We consider left-concerned set for each element $p \in V_{\mathsf{Before}}$. We then construct the following set of objects by the two mappings After and Before and the binary relation $\succ$:

$$\begin{aligned} & \bigcup_{p \in V_{\mathsf{Before}}} \bigcup_{v \in l_{\succ}(p)} \left(\mathsf{After}^{-1}(\{v\}) \cap \mathsf{Before}^{-1}(\{p\})\right) \\ = & \{x \in U \mid \mathsf{After}(x) \succ \mathsf{Before}(x)\} \\ = & \{u1, u2, u3\}. \end{aligned}$$

We then have the set $\{u1, u2, u3\}$, which corresponds to the support set of the interrelationship between After and Before by the binary relation $\succ$ in Table I.

V. Conclusion

In this paper, we discussed that the interrelationship mining is representable from a viewpoint of rough set on two universes. The concept of the approximation space for interrelationship mining is a general framework and it contains the Pawlak's approximation space as a special case, and support sets, essential components of interrelationship mining, is representable in the approximation space for interrelationship mining.

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