Joint Optimization of Lifetime and Transport Delay under Reliability Constraint Wireless Sensor Networks

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Abstract—This paper first presents an analysis strategy to meet requirements of a sensing application through trade-offs between the energy consumption (lifetime) and source-to-sink transport delay under reliability constraint wireless sensor networks. A novel data gathering protocol named Broadcasting Combined with Multi-NACK/ACK (BCMN/A) protocol is proposed based on the analysis strategy. The BCMN/A protocol achieves energy and delay efficiency during the data gathering process both in intra-cluster and inter-cluster. In intra-cluster, after each round of TDMA collection, a cluster head broadcasts NACK to indicate nodes which fail to send data in order to prevent nodes that successfully send data from retransmission. The energy for data gathering in intra-cluster is conserved and transport delay is decreased with multi-NACK mechanism. Meanwhile in inter-clusters, multi-ACK is returned whenever a sensor node sends any data packet. Although the number of ACKs to be sent is increased, the number of data packets to be retransmitted is significantly decreased so that consequently it reduces the node energy consumption. The BCMN/A protocol is evaluated by theoretical analysis as well as extensive simulations and these results demonstrate that our proposed protocol jointly optimizes the network lifetime and transport delay under network reliability constraint.

Index Terms—wireless sensor networks, network lifetime, transport delay, statistical reliability, cluster-radius.

1 INTRODUCTION

Wireless Sensor Networks (WSNs) are commonly used for environmental monitoring, surveillance operations, and home or industrial automation [1-4]. In cluster based WSNs, the cluster head (CH) performs aggregation of all received data from its cluster members and then forwards it to the sink in a multi hop manner. Due to the unreliability of WSNs, it results in large energy consumption for multiple retransmissions and gains unexpected network performance [4, 5]. For example, since the packet loss for the communication among nodes is up to 30%, the success rate becomes only 16.8% after five hops to the sink. Thus, the reliability achieves 90% only if at least two retransmissions have been done in each hop. In other words, network energy is supposed to be consumed twice more than expected at each hop. Thus, it is necessary to design a new protocol to guarantee QoS in WSNs such as lifetime, end-to-end reliability, and delay. Send and Wait Automatic Repeat-reQuest (ARQ) protocol (SW-ARQ) is commonly used to ensure the reliability by employing multiple retransmissions [5].

Due to the complexity of cluster based networks, there is little research efforts in achieving all of network reliability, transport delay, and lifetime optimization. The main contributions of our work are as follows:

(1) We theoretically analyze the node energy consumption and transport delay in cluster based networks under certain reliability δ . The theoretical analysis concludes that there exists the optimal cluster radius to maximize the network lifetime and minimize the transport delay. However, it is not necessarily the same value for the cluster radius which achieves maximum lifetime and minimum delay.

(2) A novel data gathering protocol named **B**roadcasting Combined with Multi-**N**ACK/**A**CK(BCMN/A, which NACK; standing for "Negative-Acknowledgment", ACK for "Acknowledgment") protocol is proposed, which jointly optimizes the energy and delay efficiency under statistically reliable constraint.

(3) We conduct extensive simulation experiments. Consistently with our theoretical results, simulation results demonstrate that the BCMN/A protocol is efficiency in both energy and delay under network reliability constraint, which on average improves the network lifetime by 8% and decreases the transport delay by 25%.

The remainder of the paper is organized as follows. Section 2 reviews related works comparing with our approach. Section 3 describes a network model and defines problem statements in the paper. In section 4, we give delay and lifetime analysis under Send and Wait Automatic Repeat-reQuest (SW-HBH ARQ) protocol for cluster based WSNs. In section 5, we propose a novel data gathering protocol named Broadcasting Combined with Multi-NACK/ACK (BCMN/A) protocol. Section 6 evaluates the performance of BCMN/A protocol and presents the results with some discussion. Finally, we present concluding remarks and outline the directions for future work in section 7.

2 RELATED WORK

In WSNs, lifetime, delay, and reliability are three important properties to be ensured and many efforts have been made in this research field [6, 7, 8].

Time division multiple access (TDMA) is an efficient MAC protocol, which plays an important role in the delay optimization. The authors of [8] take into account the number of packets being sent at every node, and provide the shortest schedule by eliminating the nodes without packets. However, algorithms proposed in [8, 9] are not scalable solutions because these require global topology information. To overcome this challenge, distributed slot assignment schemes have been proposed, such as DRAND [10], PACT [11], TRAMA [12].

For flat networks, a contention-free TDMA-based integrated MAC and routing protocol named DGRAM have been proposed in [13]. As for the specific data aggregation phenomenon in WSNs, Huang et al. [14] have proposed a centralized scheduling algorithm with the delay bound of $23R + \Delta + 18$ time slots, where R is the network radius and Δ is the maximum node degree. Yu et al. [15] have proposed a distributed scheduling method generating collision-free schedules with delay at most $24D + 6\Delta + 16$ time slots, where D is the network diameter. Xu et al. [16] have theoretically proved that the delay of the aggregation schedule generated by their algorithm is at most $16R + \Delta - 14$ time slots.

However, most of them have not taken the reliability into consideration. In WSNs, packets are forwarded via multiple wireless hops. On each wireless link, it is common that the packet error rates (PER) are around 10%-30% [5], which significantly decreases the end-to-end reliability. Therefore, retransmission protocols are quite effective to maintain the reliability of WSNs, but most of the proposed delay optimization strategies are not applicable for networks using retransmission protocols. This is beause in protocols based on TDMA, the node time slot is determined according to the number of data packets to be sent. Meanwhile in retransmission protocols [5, 7], the number of data packets to be sent is unknown such that a data packet may be successfully sent only at a time, or may be successfully sent at most *m* times (*m* is the maximum retransmission number). Thus, the time slot is unknown for sending one data packet. If the time slot is arranged according to the maximum retransmission number, the transport delay becomes very large. If the time slot is not arranged, many conflicts may occur in retransmission. Thus, it is critical to first consider the network reliability and then design the delay optimization strategy.

Network data gathering should not only consider the network delay, but also the network lifetime (energy consumption) as important metrics. In the literature, many research efforts in network delay and energy consumption have been made, such as in [17, 18].

Although there is much reliability research with ARQ retransmission protocols [4, 5, 7], in most studies, they aimed at linear networks, such as Zvi Rosberg and Changmian Wang et al [5, 7]. While other studies abstracted the routing path into linear network, such as Ref. [6], these studies are not suitable for multi-to-one networks. To the best of our knowledge, there is no analysis research of the data load and energy consumption with retransmission protocol in plane sensor networks, let alone analysis research with retransmission protocol in cluster based networks. Therefore, our first work is to analyze the lifetime and delay with ARQ protocol under certain reliability in cluster based networks. Liqi Shi et al [18] did a comprehensive study of network reliability, energy and delay. By properly setting the feedback period, Liqi Shi et al [18] considered energy cost of NACKs is negligible compared to that of data packets. In terms of the data retransmission number they considered that, given the packet loss rate p, the per-hop average number of total transmissions for a packet to be successfully received is $1/(1-p)$. In the actual application, the data retransmission number is relevant to the application requirements, and the required success times are not the same for each transmission, while the number of retransmissions take the value of $1/(1-p)$, which is the actual maximum transmission number, and the time needs to send NACKs as well as the energy consumption has much impact on the network delay and lifetime. Such research is instructive to our work.

3 System Model

3.1 Network Model

We employ a network model used in [16], which is described as fllows;

(1) *n* homogenous sensor nodes are deployed in a circular region with a sink situated at the centre. The node distribution follows a homogenous Poisson point process with a density of ρ nodes per unit area. The nodes in the network are divided into multiple clusters, each comprising a CH and cluster members that communicate via one hop to the CH. Data of CH is sent to the sink via multi-hop among CHs.

(2) The transmitting radius of a node is denoted with $\Re$, and the cluster radius is denoted with r. The transmission power of the node is adjustable, i.e. the node can adjust its transmission power according to the distance to a receiver, e.g., Berkeley Mote has 100 transmission power levels [19].

(3) For every node i, the probability that data transmission from node i to node $i+1$ is denoded by $1-p_i$ (denoted with $\overline{p_i}$) [5]. The probability that node *i* successfully receives acknowledgment or negative acknowledgment (ACK/NACK) from node $i+1$ is denoted by $1-q_i$ (denoted with $\overline{q_i}$) [5]. Assume that reception failures are spatial dependent but time independent.

(4) Deploying SW-HBH ARQ protocol [5], nodes within a cluster send data to their CH with TDMA mechanism, and the data reliability within the cluster is δ_1. Data is transmitted with pre-assigned different frequencies inter-clusters so that the data gathering inter-clusters can work simultaneously [1]. The CHs send data to the sink hop by hop with carrier sensing multiple access (CSMA) mechanism and the reliability is δ_2.

(5) Time is slotted and the slot time is fixed as Δs seconds corresponding to a single packet transmission. The transmitter serves new arrival packets on an FCFS (First Come First Serve) basis.

3.2 Energy Consumption Model and Related Definitions

In this paper, we adopt the topical energy consumption model in [7], where the transmission energy consumption E_t follows eq. 1 and energy consumption E_r for receiving follows eq. 2.

$$\begin{cases} E_t = lE_{elec} + l\varepsilon_{fs}d^2 \quad if \ d \le d_0 \\ E_t = lE_{elec} + l\varepsilon_{amp}d^4 \quad if \ d > d_0 \end{cases} \tag{1}$$

$$E_r = lE_{elec} \tag{2}$$

E_{elec} represents transmitting circuit loss. Both the free space (d^2 power loss) and the multi-path fading (d^4 power loss) channel models are used in the model, depending on the distance between the transmitter and receiver. ε_{fs} and ε_{amp} are respectively the energy required by power amplification in the two models. l represents the bits of data sent or received by nodes. The above parameter settings are adopted from Ref. [20].

3.3 Problem statement

Definition 1: Transport delay: The transport delay is defined as the time from a packet's first transmission until its successful arrival at the sink [6].

Definition 2: Network lifetime: The network lifetime is defined as the time when first node dies [4, 20].

In this paper, the main problems are: (1) In cluster based sensor networks, give the node energy consumption (network lifetime) and transport delay under reliability constraint from theoretical analysis; (2) How to further decrease the network delay and improve the network lifetime under the data reliability.

4 DELAY AND LIFETIME ANALYSIS UNDER SW HBH ARQ PROTOCOL

In this section, we present an analysis strategy to meet requirements of the application through trade-offs between the energy consumption and source-to-sink transport delay under SW HBH ARQ protocol for cluster based WSNs. Then, give an optimized Broadcasting combined with multi-NACK/ACK (BCMN/A) protocol in Section 5.

4.1 Analysis of node data load under SW HBH ARQ protocol

SW HBH ARQ is a data reliability protection protocol, its data gathering process is as the following: (1) Data gathering within the cluster, nodes within the cluster send data to the CH directly. If a transmitter receives an ACK from CH node before the preset timeout occurs, it transmits a new packet; otherwise, it retransmits the preceding packet. CH transmits an ACK for every packet it receives successfully including for duplicates; (2) Data inter-cluster heads is sent to the sink via multi-hop of CHs. The data reliability is assured in every hop. If a transmitter receives an ACK from its subsequent CH node before the preset timeout occurs, it transmits a new packet; otherwise, it retransmits the preceding packet. A receiver (CH or Sink) also transmits an ACK for every packet it receives successfully including for duplicates. In this section we analyze the node data load under SW HBH ARQ protocol.

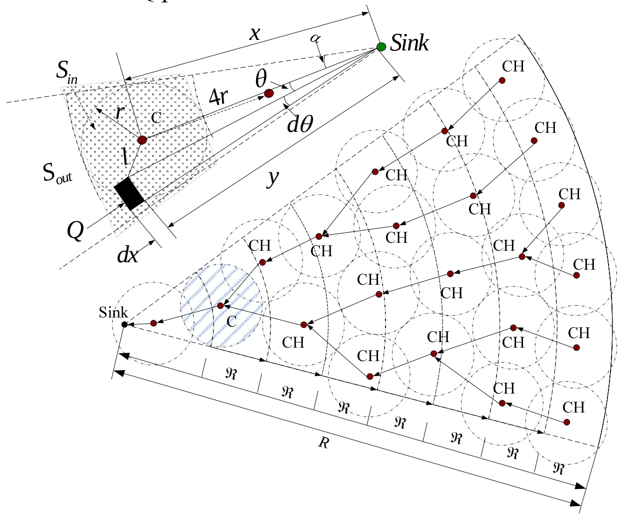

Fig. 1: the data transmission in cluster based networks

Theorem 1: Considering the system required data statistical reliability in intra-cluster is δ_1, then as for the node v_i within the cluster, the sent data amount is $X_i^{1,t}(\delta_1)$ and the number of ACK received is $Y_i^{1,r}(\delta_1)$, the received data amount of the cluster head within the cluster is $X_{ch}^{1,r}(\delta_1)$, and the ACK sent by the cluster head is $Y_{ch}^{1,t}(\delta_1)$, as the following.

$$\begin{cases} X_i^{1,t}(\delta_1) = \dfrac{1-(1-\overline{p_i}\,\overline{q_i})^{\zeta(\delta_1)}}{\overline{p_i}\,\overline{q_i}}, Y_i^{1,r}(\delta_1) = \dfrac{1-(1-\overline{p_i}\,\overline{q_i})^{\zeta(\delta_1)}}{\overline{p_i}\,\overline{q_i}}\overline{p_i}\,\overline{q_i} \\ X_{ch}^{1,r}(\delta_1) = n\dfrac{1-(1-\overline{p_i}\,\overline{q_i})^{\zeta(\delta_1)}}{\overline{p_i}\,\overline{q_i}}\overline{p_i}, Y_{ch}^{1,t}(\delta_1) = n\dfrac{1-(1-\overline{p_i}\,\overline{q_i})^{\zeta(\delta_1)}}{\overline{p_i}\,\overline{q_i}}\overline{p_i}, \zeta(\delta_1) = \left\lceil \dfrac{\log(1-\delta_1)}{\log(p_i)} \right\rceil \end{cases} \tag{3}$$

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 1.

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CH undertakes not only the data amount of intra-cluster, but also the data forwarding among the CHs. Its data and ACK load is as the following theorem 2.

Theorem 2: Considering the distance from cluster head C_l to the sink is l, $l = hr + x$, the required statistical reliability of data to the sink is δ_2, then the data load and ACK load D_l and ACK load M_l of C_l is:

$$\begin{cases} D_l^{1,t} = n(l,0)X_{h+0}^{h}(\delta_2) + n(l,1)X_{h+1}^{h}(\delta_2) + \ldots + n(l,z)X_{h+1}^{h}(\delta_2) \\ D_l^{1,r} = 0 + n(l,1)X_{h+1}^{h+1}(\delta_2)\overline{p_{h+1}} + \ldots + n(l,z)X_{h+z}^{h+1}(\delta_2)\overline{p_{h+z}} \\ M_l^{1,r} = (D_l^t\,\overline{p_h}\,\overline{q_{h-1}}) / N_{clusternode} \\ M_l^{1,t} = D_l^{1,r} / N_{clusternode},\ n(l,i) = \phi\rho\alpha(4lr + 8ir^2) \\ X_{h+j}^{h}(\delta_2) = \dfrac{1-(1-\overline{p_h}\,\overline{q_h})^{S_{h+j}(\delta_2)}}{\overline{p_h}\,\overline{q_h}}, S_h(\delta_2) = \left\lceil \dfrac{\log(1-\sqrt[h+1]{\delta_2})}{\log(p_h)} \right\rceil, \\ N_{clusternode} = \rho\pi r^2 \end{cases} \tag{4}$$

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 2.

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4.2 Analysis of node average energy consumption under SW HBH ARQ protocol

Since the cluster head and common node work in alternate way in cluster based networks, the energy consumption calculation is relatively complicated. The following theorem 3 gives the average energy consumption calculation.

Theorem 3: If the distance from cluster head to the sink is l, then the total energy consumption for data of all nodes in intra-cluster sent to the cluster head is:

$$E_{l,total}^{1,in} = 2X_i^{1,t}(\delta_1)\rho\alpha\{2(E_{elec}+\varepsilon_{fs}l^2)lr + 2lr\varepsilon_{fs}(l^2+r^2)\}$$
$$-4X_i^{1,t}(\delta_1)\rho\varepsilon_{fs}l\sin\alpha(2l^2r+\frac{2}{3}r^3)+4Y_i^{1,r}(\delta_1)\rho\alpha\cdot E_{elec}lr \quad (5)$$

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 3.

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Then, after one round data gathering, the node average energy consumption is as theorem 4.

Theorem 4: In multi-hop cluster based networks, considering the cluster radius is r, after one round data gathering of the entire network, the average energy consumption of node whose distance from the sink is $l=hr+x$, $E_l^{1,avg}$ is as the following:

$$\begin{cases} E_l^{1,avg} = \{E_{ch}^{1,in} + (D_l^{1,t}+M_l^{1,t})(E_{elec}+\varepsilon l^a) + (D_l^{1,r}+M_l^{1,r})E_{elec} + (n-1)\frac{E_{l,total}^{1,in}}{n}\}/n \ \ if\ l\le 2r \\ E_l^{1,avg} = \{E_{ch}^{1,in} + (D_l^{1,t}+M_l^{1,t})(E_{elec}+\varepsilon (2r)^a) + (D_l^{1,r}+M_l^{1,r})E_{elec} + (n-1)\frac{E_{l,total}^{1,in}}{n}\}/n \ \ if\ l> 2r \\ E_{ch}^{1,in} = X_{ch}^{1,r}(\delta_1)E_{elec} + E_l^{1,ack}, M_{l,ch}^{1,t} = \frac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}\overline{p_i} \\ E_l^{1,ack} = 2M_{l,ch}^{1,t}(\delta_1)\rho\alpha\{2(E_{elec}+\varepsilon_{fs}l^2)lr+2lr\varepsilon_{fs}(l^2+r^2)\} - 4M_l^{1,t}(\delta_1)\rho\varepsilon_{fs}l\sin\alpha(2l^2r+\frac{2}{3}r^3) \end{cases} \quad (6)$$

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 4.

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Corollary 1: The network lifetime can be calculated as:

$$life_1 = Einit/\max(E_l^{1,avg}) \mid l \in \{l_{\min}, R)$$

Proof: Obviously, the network lifetime is determined by the lifetime of the node which has the maximum energy consumption, that is $\max(E_l^{1,avg}) \mid l \in \{l_{\min}, R)$, so the network lifetime is $life_1 = Einit/\max(E_l^{1,avg}) \mid l \in \{l_{\min}, R)$

■

4.3 Transport delay of multi-hop cluster based network under SW HBH ARQ protocol

Considering the round-trip time for common nodes send data to CH is t_{rtt} (RTT), the retransmission time out is t_{rto} (RTO), obviously, $t_{rto} > t_{rtt}$. The time for data gathering within the cluster is as the theorem 5.

Theorem 5: Considering $m_1 = \zeta(\delta_1)$, the then time for data gathering in intra-cluster is

$$t_i^{1,in} = (n-1)((m_1-1)t_{rto} + t_{rtt}/2)$$

(7)

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 5.

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SW-Hop By Hop ARQ protocol is used in data transmission among CHs, as shown in Fig. 2. The delay at the transmitter includes the queuing delay t_q and transport delay t. t_q is the queuing time for transmission after receiving, t is the time from sending to receiving. The total delay among CHs is shown in theorem 6.

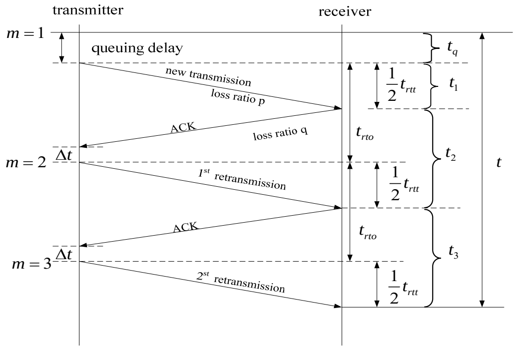

Fig.2: traditional transmission among CHs

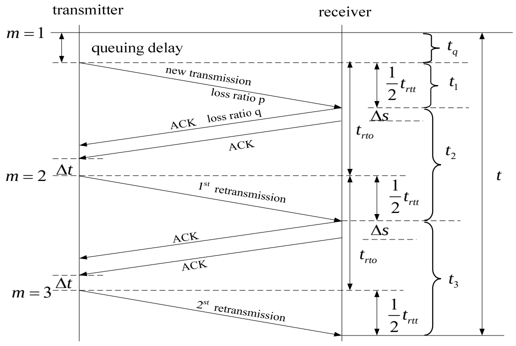

Fig.3: transmission among CHs under BCMN/A

Theorem 6: The delay $t_{l,CH}^1$, queuing delay t_q^1 and transport delay $E(t_t^1)$ of CH C_l at $l=hr+x$ from the sink is as following:

$$t_{l,CH}^1 = t_q^1 + E(t_t^1) = \frac{\rho^2}{(1-\rho)\lambda_1} + \sum_{k=1}^{\zeta(\delta_2)}\{(\frac{1}{2}t_{rtt}+(k-1)t_{rto})(1-p)p^{k-1}\}$$

(8)

note: $t_q^1 = \frac{\rho^2}{(1-\rho)\lambda_1}$, $\rho = \frac{\lambda_l}{\mu_1}$, $\mu_1 = \frac{T}{t_{rto}\overline{pq}\sum_{k=1}^{\zeta(\delta_2)}\{k(1-\overline{pq})^{k-1}\}}$

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 6.

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Theorem 6 gives the delay at the CH, and the total delay to the sink includes the data gathering delay in intra-cluster and the total delay at each CH in the routing path. Considering node v_j belongs to the CH C_l whose distance from the sink is $l=hr+x$. C_{l-2ir} denotes the i hop of C_l to the sink, then the routing path of C_l to the sink is $R_j = \{l, l-2r, l-4r, ...l-2ir, ..., r+x, x\}$, redefine the routing path with distance, that is $R_j = \{l, l-2r, l-4r, ...l-2ir, ..., r+x, x\}$, then we can get corollary 2.

Corollary 2: Considering node v_j belongs to the CH C_l whose distance from the sink is $l = hr + x$, the routing path of C_l to the sink is $R_j = \{l, l-2r, l-4r, ...l-2ir, ..., r+x, x\}$. Then the transport delay of node v_j is:

$$t^1_{j,total} = \sum_{s \in R_j} t^1_{s,CH} + n((\zeta(\delta_1) - 1)t_{rto} + t_{rtt}/2) \qquad (9)$$

Note:

$$t^1_{s,ch} = \frac{\rho_s^{\ 2}}{(1-\rho_s)\lambda_s} + \sum_{k=1}^{\zeta_j(\delta_2)} \{(\frac{1}{2}t_{rtt} + (k-1)t_{rto})(1-p)p^{k-1}\},$$

$$\rho_s = \frac{\lambda_s}{\mu}, \mu = \frac{T_l}{t_{rto}\overline{pq}\sum_{k=1}^{\zeta_j(\delta_2)}\{k(1-\overline{pq})^{k-1}\}}$$

Proof: The data gathering time of node v_j is $n((\zeta(\delta_1) - 1)t_{rto} + t_{rtt}/2)$, where *n* is the node number in the cluster. According to theorem 6, the delay at CH with distance s from the sink is $t^1_{s,CH}$, while the routing path of v_j to the sink is R_j, then the average delay of v_j in the entire path is the total forwarding delay of each CH.

$$t^{ch}_{total} = \sum_{s \in R_j} t^1_{s,CH}$$

Therefore, the total transport delay is derived as follows.

$$t^1_{j,total} = \sum_{s \in R_j} t^1_{s,CH} + n((\zeta(\delta_1) - 1)t_{rto} + t_{rtt}/2) \qquad ■$$

Corollary 3: Network delay $delay_1 = \max(t^1_{j,total}) || j \in \{1, n\}$.

Proof: Obviously, the network delay is the delay of node with the maximum delay, so $delay_1 = \max(t^1_{j,total})$, note j is the number of v_j | $j \in \{1, n\}$. ■

5 BROADCASTING COMBINED WITH MULTI-ACK FOR OPTIMIZATION LIFETIME AND DELAY

5.1 The idea of Broadcasting combined with multi-NACK/ACK (BCMN/A)

To further improve the performance of cluster based networks, this paper presents Broadcasting combined with multi-NACK/ACK (BCMN/A) protocol. The improvement of BCMN/A protocol is mainly reflected in the following two aspects.

1) As for data gathering within the cluster, ACK is send by broadcasting and the data gathering mechanism is as the following. (A) The CH allocates time slot for each node in intra-cluster and the time slot is $\frac{1}{2}t_{rtt}$, each node sends data to the CH during its time slot. (B) After each node has send data to the CH, CH broadcast a message which employ NACK indicate those nodes whose data is not send to CH successful and the time slot scheduling sequence for next round.

(C) Nodes receive broadcast a message and determine whether CH has received its data according, if received, node sleeps and this data gathering round is completed, if not, node retransmits data according to the arranged time slot. Node will keep silent if it does not receive the broadcasting and wait for the next broadcasting. (D) After the second data gathering round, similar with the first cycle, the ID of nodes which CH did not receive data from and the time sequence is broadcasted, and nodes in intra-cluster adopt the same mechanism and continue until the reliability meets the requirement δ_1.

2) As for data transmission of inter-clusters, the improved protocol is that the CH returns *n* ACK for each data packet it receives (this is called muti-ACK, see Fig. 3).

5. 2 Analysis of the lifetime for BCMN/A protocol

Under the BCMA protocol, after each data gathering in intra-cluster and the broadcasting by CH (one broadcasting or mutil-broadcastings), such process is called one round. The following theorem gives the data load of node in one round.

Theorem 7: If only one broadcasting is processed for each data gathering for intra-cluster, to meet the reliability δ_1, the number of retransmissions in onde roud should be

$$m_2 = \left\lceil \frac{\log(1-\delta_1)}{\log(1-(1-q)(1-p))} \right\rceil$$

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 7.

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Corollary 4: As for the data gathering in intra-cluster, if there are z broadcastings for each data gathering process, then the number of rounds needed is:

$$m_3 = \left\lceil \frac{\log(1-\delta_1)}{\log(1-(1-q^z)(1-p))} \right\rceil$$

Proof: Obviously, if z broadcastings are processed for each data gathering process, the reliability after k rounds is $1-(1-(1-q^z)(1-p))^k$, set the reliability $1-(1-(1-q^z)(1-p))^{m_3} > \delta_1$.

$$\Rightarrow m_3 = \left\lceil \frac{\log(1-\delta_1)}{\log(1-(1-q^z)(1-p))} \right\rceil$$

■

The following gives the data load under BCMA protocol.

Theorem 8: Under BCMA protocol, if ACK is only broadcasted only once each time, the node data load intra-cluster is as the following. Note, $D^{2,t}_i$ is the total number of data packets, $M^{2,r}_i$ is the node received ACK bits amount, $M^{2,t}_{ch}$ is the CH sent ACK bits amount, the number of received data packets is $D^{2,r}_{ch}$. m_2 is the number of gathering rounds, *n* is the number of nodes intra-cluster, *c* is the needed bits amount of each node ID.

$$\begin{cases} D_i^{2,t} = (1-q)\sum_{k=1}^{m_2}(1-\overline{pq})^{k-1},\ M_i^{2,r} = \sum_{k=1}^{m_2}(1-q)(1-\overline{pq})^{2(k-1)}(n-1)^2 c \\ D_{ch}^{2,r} = (n-1)D_i^{2,t}\overline{p},\ M_{ch}^{2,t} = (n-1)c\sum_{k=1}^{m_2}(1-\overline{pq})^{k-1} \end{cases} \tag{10}$$

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 8.

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Under BCMA protocol, if the ACK broadcasting is repeated z times after each data gathering, namely the multi-ACK, the data load and ACK load of intra-cluster is as theorem 9.

Theorem 9: If the ACK broadcasting is repeated z times after each data gathering round, the data load is as the following. Note, $D_i^{3,t}$ is the number of total data packets sent by nodes within the cluster, $M_i^{3,r}$ is the ACK bits amount received by nodes, $M_{ch}^{3,t}$ is the ACK bits amount sent by the CH, $D_{ch}^{3,r}$ is the number of data packets received, m_3 is the number of data gathering round, n is the number of nodes within the cluster, c is the node ID bits amount of each node.

$$\begin{cases} D_i^{3,t} = (1-q^z)\sum_{k=1}^{m_3}(1-\overline{p}(1-q^z))^{k-1},\ M_i^{3,r} = \sum_{k=1}^{m_3}(1-q^z)(1-\overline{p}(1-q^z))^{2(k-1)}(n-1)^2 c \\ D_{ch}^{3,r} = (n-1)D_i^{2,t}\overline{p},\ M_{ch}^{3,t} = (n-1)c\sum_{k=1}^{m_3}(1-\overline{p}(1-q^z))^{k-1} \end{cases} \tag{11}$$

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 9.

■

Theorem 10: Under BCMN/A protocol, CH returns z ACK for each data it receives, assuming the distance from CH C_l to the sink is $l = hr + x$, the required statistical reliability is δ_2, then the data load and ACK load of is:

$$\begin{cases} D_l^{4,t} = n(l,0)X_{h+0}^{h}(\delta_2) + n(l,1)X_{h+1}^{h}(\delta_2) + ... + n(l,z)X_{h+1}^{h}(\delta_2) \\ M_l^{4,t} = 0 + n \cdot n(l,1)X_{h+1}^{h+1}(\delta_2)\overline{p} + ... + n \cdot n(l,z)X_{h+z}^{h+1}(\delta_2) \\ D_l^{4,r} = M_l^{t}/n,\ M_l^{4,r} = nD_l^{t}\overline{pq},\ n(l,i) = \phi\rho\alpha(4lr + 8ir^2) \\ X_{h+j}^{h}(\delta_2) = \dfrac{1-(1-\overline{p}(1-(1-\overline{q})^z))^{S_{h+j}(\delta_2)}}{\overline{p}(1-\overline{q})^z)}, S_h(\delta_2) = \left\lceil \dfrac{\log(1-\sqrt[h+1]{\delta_2})}{\log(p)} \right\rceil \end{cases} \tag{12}$$

Proof: Under Send and wait hop to hop one data n ACK protocol, the maximum retransmission number is the same with theorem 4, the transmission times of the source node sends its one data packet is a truncated geometrically distributed r.v. with a success probability of $\overline{p_i}(1-(1-\overline{q_i})^z)$ taking values in the set $\{1, ..., S_h(\delta_2)\}$. Its expected value is given by:

$$X_{h+j}^{h}(\delta_2) = \frac{1-(1-\overline{p}(1-(1-\overline{q})^z)^{S_{h+j}(\delta_2)}}{\overline{p}(1-\overline{q})^z)}.$$

Similar with theorem 4, we can get theorem 10. ■

Theorem 11: Under multi-hop cluster based protocol, considering the cluster radius is r, as for the data gathering intra-cluster, NACK is broadcasted once for each data gathering round and z ACK is returned for each received data inter-clusters, after an entire data gathering, the average energy consumption $E_l^{2,avg}$ for node whose distance from the sink is $l = hr + x$ is as the following.

$$\begin{cases} E_l^{2,avg} = \{E_{ch}^{2,in} + (D_l^{4,t} + M_l^{4,t})(E_{elec} + \varepsilon l^a) + (D_l^{4,r} + M_l^{4,r})E_{elec} + (n-1)\dfrac{E_{l,total}^{2,in}}{n}\}/n \ \ if\ l < 2r \\ E_l^{2,avg} = \{E_{ch}^{2,in} + (D_l^{4,t} + M_l^{4,t})(E_{elec} + \varepsilon(2r)^a) + (D_l^{4,r} + M_l^{4,r})E_{elec} + (n-1)\dfrac{E_{l,total}^{2,in}}{n}\}/n \ \ if\ l > 2r \\ E_{ch}^{2,in} = D_{ch}^{2,r}E_{elec} + M_{ch}^{2,t}(E_{elec} + \varepsilon r^a) \\ E_{l,total}^{2,in} = 2D_i^{2,t}(\delta_1)\rho\alpha\{2(E_{elec} + \varepsilon_{fs}l^2)lr + 2lr\varepsilon_{fs}(l^2 + r^2)\} - \\ \quad 4D_i^{2,t}(\delta_1)\rho\varepsilon_{fs}l\sin\alpha(2l^2 r + \frac{2}{3}r^3) + 4M_i^{2,r}\rho\alpha \cdot E_{elec}lr \end{cases} \tag{13}$$

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 11.

■

Corollary 5: Under multi-hop cluster based protocol, assuming the cluster radius is r, when NACK is broadcasted z times in each data gathering round, z ACK is returned for each data packet received inter-clusters, after an entire data gathering round, the average energy consumption $E_l^{3,avg}$ for node whose ditance from the sink is $l = hr + x$ is as the following.

$$\begin{cases} E_l^{3,avg} = \{E_{ch}^{3,in} + (D_l^{4,t} + M_l^{4,t})(E_{elec} + \varepsilon l^a) + (D_l^{4,r} + M_l^{4,r})E_{elec} + (n-1)\dfrac{E_{l,total}^{3,in}}{n}\}/n \ \ if\ l < 2r \\ E_l^{3,avg} = \{E_{ch}^{3,in} + (D_l^{4,t} + M_l^{4,t})(E_{elec} + \varepsilon(2r)^a) + (D_l^{4,r} + M_l^{4,r})E_{elec} + (n-1)\dfrac{E_{l,total}^{3,in}}{n}\}/n \ \ if\ l > 2r \\ E_{ch}^{3,in} = D_{ch}^{3,r}E_{elec} + M_{ch}^{3,t}(E_{elec} + \varepsilon r^a) \\ E_{l,total}^{3,in} = 2D_i^{3,t}(\delta_1)\rho\alpha\{2(E_{elec} + \varepsilon_{fs}l^2)lr + 2lr\varepsilon_{fs}(l^2 + r^2)\} \\ \quad -4D_i^{3,t}(\delta_1)\rho\varepsilon_{fs}l\sin\alpha(2l^2 r + \frac{2}{3}r^3) + 4M_i^{3,r}\rho\alpha \cdot E_{elec}lr \end{cases} \tag{14}$$

Proof: Similar with theorem 11, it can be proved (omitted).

■

5. 3 Analysis of the transport delay for BCMN/A protocol

The following analyzes the transport delay under BCMN/A protocol, similarly, the node transport delay includes the delay of data gathering within the cluster and the delay of data sent to the sink. The following gives the analysis results.

Theorem 12: Under BCMN/A protocol, if the NACK is broadcasted only once after the first data gathering round, the data delay is

$$t_i^{2,in} = \frac{1}{2}(n-1)t_{rtt} + (n-1)\sum_{k=2}^{m_2}(1-\overline{pq})^{k-1}\frac{1}{2}t_{rtt} + \frac{1}{2}m_2 t_{rtt}$$

If the broadcasting is repeated z times in one data gathering round, the delay is

$$t_i^{3,in} = \frac{1}{2}(n-1)t_{rtt} + (n-1)\sum_{k=2}^{m_3}(1-\overline{p}(1-q^z))^{k-1}\frac{1}{2}t_{rtt} + \frac{1}{2}m_3 t_{rtt}$$

.

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 12.

■

Theorem 13: Under BCMA/A protocol, the delay is

$t^2_{l,CH}$ for CH C_l whose distance from the sink is $l = hr + x$, the queuing delay t^2_q and transport delay $E(t^2_t)$ are as the following:

$$t^2_{l,CH} = t^2_q + E(t^2_t) = \frac{\rho_l^2}{(1-\rho_l)\lambda_l} + (\frac{1}{2}t_{rtt})(1-p) + \sum_{k=2}^{\zeta(\delta_2)}\{(k-1)t^A_{rto}(1-p)p^{k-1}\}$$

Note: $t^2_q = \frac{\rho_l^2}{(1-\rho_l)\lambda_l}, \rho_l = \frac{\lambda_l}{\mu_2}, \mu_2 = \frac{T_l}{t^A_{rto}\,\overline{p}(1-q^z)\sum_{k=1}^{\zeta(\delta_2)}\{k(1-\overline{p}(1-q^z))^{k-1}\}}$.

Proof: Please refer to Section 1 of the online supplementary file for the proof of theorem 13.

■

If the data gathering delay of intra-cluster and the transport delay inter-clusters are obtained, the transport delay from node v_j generates data to data received by the sink can be obtained as the corollary 6.

Corollary 6: Under BCMA/A protocol, assuming node v_j belongs to the CH C_l whose distance from the sink is $l = hr + x$, then the routing path of C_l to the sink is $R_j = \{l, l-2r, l-4r, \ldots l-2ir, \ldots, r+x, x\}$. Then the transport delay of v_j is:

$$t^2_{j,total} = \sum_{s\in R_j} t^2_{s,CH} + t_i^{2,in} \quad \text{or} \quad t^2_{j,total} = \sum_{s\in R_j} t^2_{s,CH} + t_i^{3,in},$$

where $t^2_{s,ch} = t^2_{l,ch}$ when $s = l$

Proof: Please refer to Section 1 of the online supplementary file for the proof of corollary 6.

■

6 Analysis of Experimental Results

OMNET++ is used for experimental verification. OMNET++ is an open network simulation platform that provides open source, component-based, modular simulation platform for large network, which has been widely recognized by academics [22]. Without loss of generality, the parameters are $p = q$ =0.3, $\delta_1 = \delta_1 = 0.9$, R =500m, 1000 nodes are deployed. The other parameter settings refer to table 1 of the online supplementary file.The retransmission protocol in Section 4 is called SW-ARQ for short, Broadcasting combined with one-NACK or ACK deployed within the cluster is called BCON/A for short, Broadcasting combined with multi-NACK or ACK deployed both in the cluster and among CHs is called BCMN/A for short.

6.1 Comparison of theoretical and experimental result of node data load

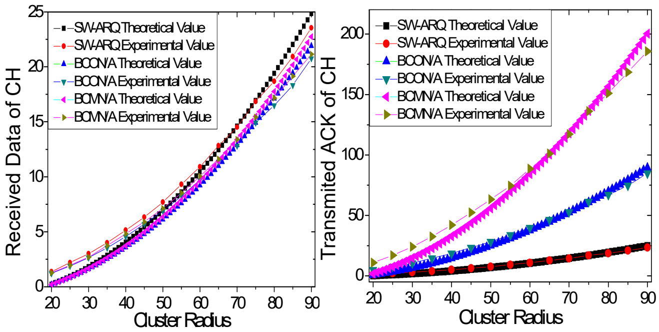


Fig.4 data load in the cluster of CH

Fig.5 ACK amount in the cluster of CH

The main purpose of the experiment in this section is to verify whether the analysis in this paper of data load under retransmission protocol in cluster based networks matches the actual situation. Fig. 4 and Fig. 5 respectively shows the data load and ACK amount under different cluster radius under SW-ARQ, BCON/A, BCMN/A protocol, from which we can see the analysis model describes the data load of the network well since experimental results and theoretical analysis results are very consistent.

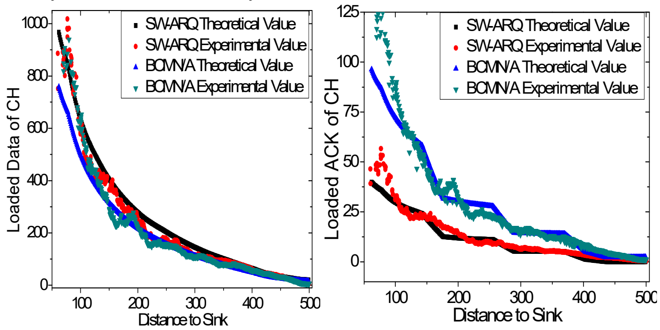


Fig.6 total data load of CH Fig.7 total ACK load of CH

Fig. 6 and Fig. 7 shows the data load under ACK load of CH under SW-ARQ, BCMN/A, the same, as can be seen that the theoretical results are consistent with experimental results. Therefore, as can be seen from the above experiments: (1) BCMN/A bears more ACK and less data load of CH than SW-ARQ. (2) CH near the sink bears more data load and ACK load than CH away from the sink.

6.2 Comparison of theoretical and experimental result of node energy consumption

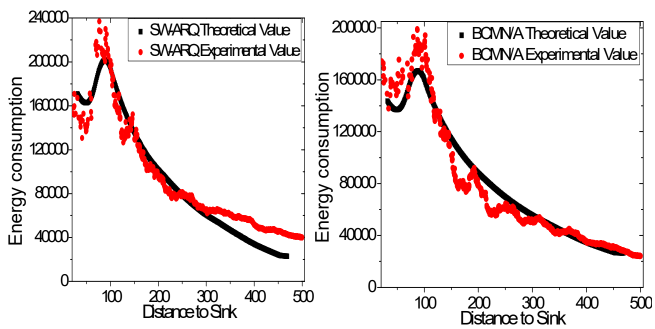
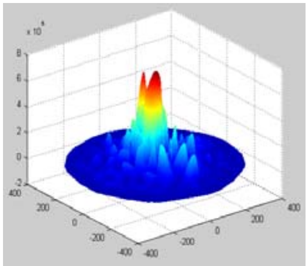


Fig.8 energy consumption under SW-ARQ

Fig.9 energy consumption under BCMN/A

Fig. 8 and 9 respectively shows the node average energy consumption with different distances from the sink under

SW-ARQ and BCMN/A. As can be seen from the experimental results, the theoretical model better reflects the network's energy consumption situation, which has good guidance. Fig. 10 and 11 give the three-dimensional map of network node energy consumption under SW-ARQ, BCMN / A protocol.



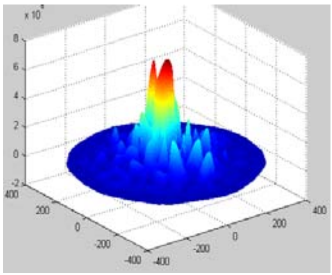
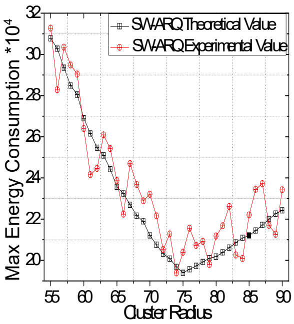


Fig.10 3D map of energy consumption under SW-ARQ

Fig.11 3D map of energy consumption under BCMN/A

Fig. 12 and Fig. 13 respectively shows the energy consumption of node with maximum energy consumption under different cluster radius under SW-ARQ and BCMN/A protocol, as can be seen, the maximum energy consumption is different under different cluster radius r . Therefore, we can choose optimized r to achieve maximum lifetime. Fig. 14 shows the maximum energy consumption comparison under different cluster radius under SW-ARQ and BCMN/A protocol, as can be seen from the results, the improved BCMN/A can reduce the energy consumption nearly by 20%. Fig. 15 shows the network lifetime comparison under SW-ARQ and BCMN/A, we can see that the BCMN/A protocol in this paper can improve the network lifetime effectively.



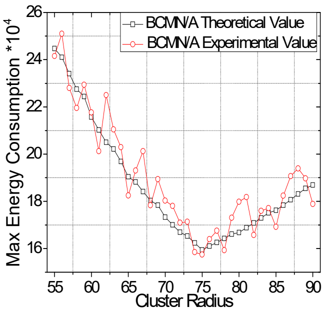
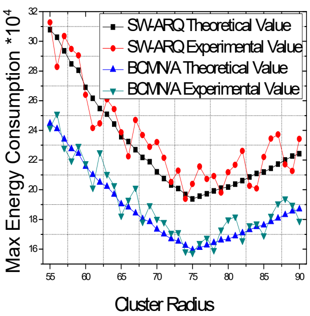


Fig.12 the maximum energy consumption of node under different cluster radius under SW-ARQ

Fig.13 the maximum energy consumption of node under different cluster radius under BCMN/A



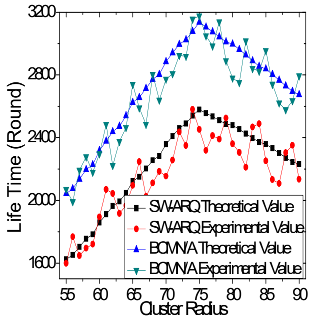
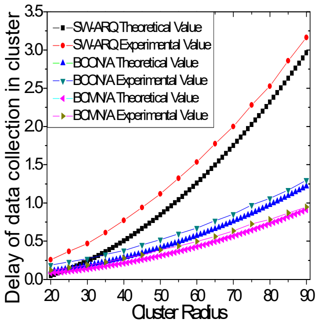


Fig.14 max energy consumption comparison under SW-ARQ and BCMN/A

Fig.15 lifetime comparison under SW-ARQ and BCMN/A

6.3 Comparison of theoretical and experimental result of network delay



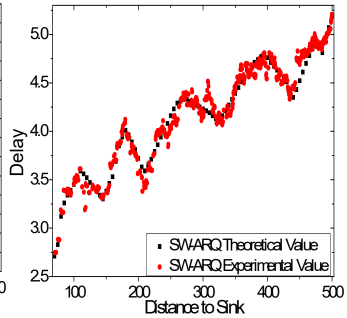
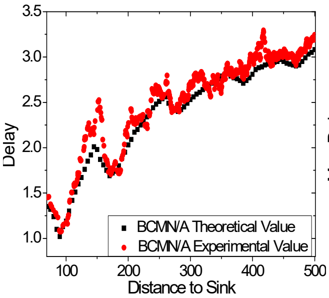


Fig.16 data gathering delay under different cluster radius

Fig.17 delay of different distances from the sink under SW-ARQ

Fig. 16 shows the data gathering delay of intra-cluster under SW-ARQ and BCMN/A, as can be seen that BCMN/A protocol can better reduce the delay. Fig. 17 and Fig. 18 respectively shows the node average delay with different distances from the sink under SW-ARQ and BCMN/A, as can be seen that the farther from the sink, the greater is the node delay. However, the average delay is not a line growth, but the undulating rise. The reason is: the average delay is mainly proportional to the number of hops, and for nodes with the same hops from the sink, the data load decreases as the distance from the sink, so the average delay decreases, and the data load increases in the next hop, so the average delay increases.



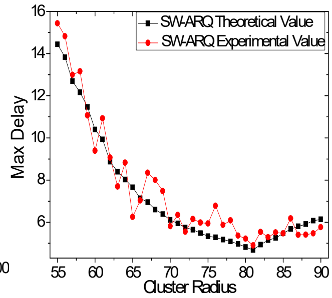
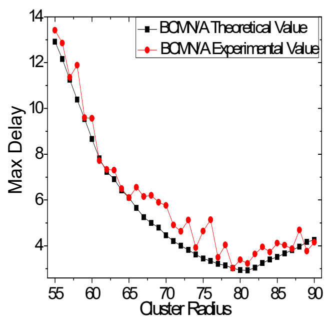


Fig.18 delay of different distances from the sink under BCMN/A

Fig.19 Max delay under different cluster radius under SW-ARQ

Fig.19 and Fig. 20 respectively shows the maximum network delay with different cluster radius under SW-ARQ and BCMN/A protocol, as can be seen that the maximum average delay is not the same when the cluster radius is different, but there must be an optimal cluster radius r to achieve the minimum network delay.



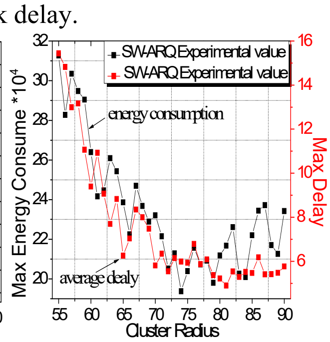
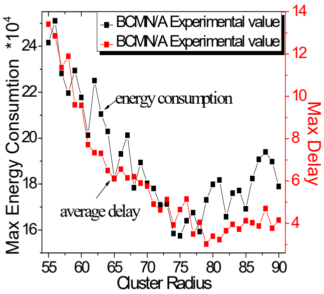


Fig.20 max delay with different cluster radius under BCMN/A

Fig.21 max energy consumption and max delay with different cluster radius under SW-ARQ

6.4 Overall optimization of network lifetime and delay



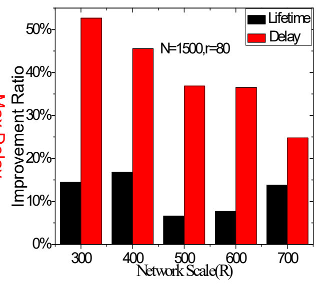


Fig.22 max energy consumption and max delay with different cluster radius under BCMN/A

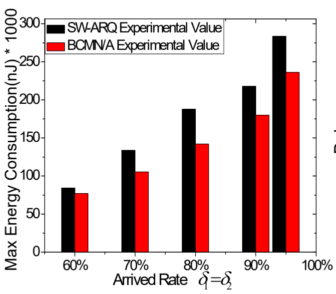
Fig.23 improved ratio of network lifetime and delay with different network scale under BCMN/A

Fig. 21 and Fig. 22 respectively shows the maximum energy consumption and network average delay with different cluster radius under SW-ARQ and BCMN/A. As can be seen, when the cluster radius r is small, with the growth of cluster radius, the network energy consumption and average delay decrease, but when the r grows to a certain degree, the energy consumption and average delay also increase, and the optimal r for network delay and energy consumption are not entirely consistent. Obviously, it is easy to obtain the optimal r to optimize the lifetime and delay under application requirements. With optimized r under different network scale R, Fig. 23 shows the improved ratio of network lifetime and delay under BCMN/A compared with SW-ARQ, showing BCMN/A protocol improves the network lifetime by average 8% above and optimizes the delay by more than 25%.

6.5 Effects of network parameters

(1): Effects of reliability constraint

Fig. 24 and Fig. 25 show the effect of reliability constraint on the network energy consumption and delay. As can be seen, the reliability δ_1 and δ_2 (respectively refers to the reliability of intra-cluster and reliability inter-clusters) have a very big impact on the energy consumption and delay. When $\delta_1=\delta_2$=90%, the energy consumption and delay is four times of that when $\delta_1=\delta_2$=60%. Therefore, in the evaluation of sensor network performance, the fact that the reliability in WSNs is relatively low should be taken into consideration, and there is often big difference between the result under the optimal network without packet loss and that of the actual network.



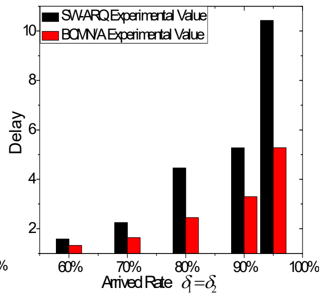
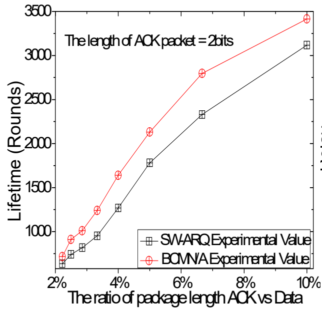


Fig.24 max energy consumption under different reliability requirements

Fig.25 delay under different reliability requirements

(2): Effects of different length of ACK and data packets



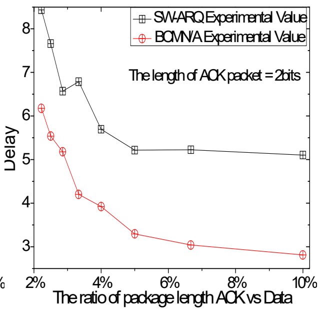
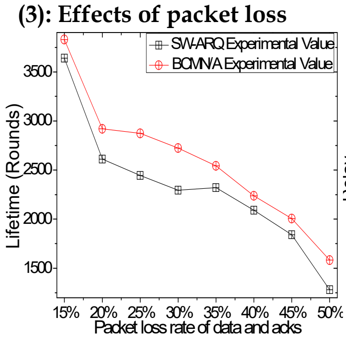


Fig.26 lifetime under different ratios of ACK length and data packet length

Fig.27 delay under different ratios of ACK length and data packet length

Fig. 26 and Fig. 27 show the lifetime and delay under different ratios of ACK length and data packet length. As can be seen, when the ACK length is fixed, the bigger is ratio of ACK length and data packet length, indicts the smaller packet length, so the network lifetime is bigger, and the delay is smaller. Meanwhile, as can be seen from the result, ACK has big effect on the network lifetime and delay, which cannot be ignored.

(3): Effects of packet loss



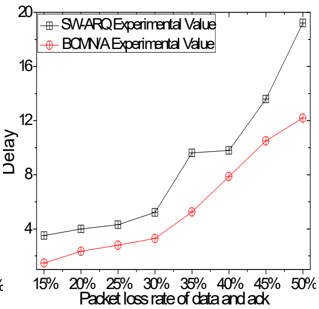
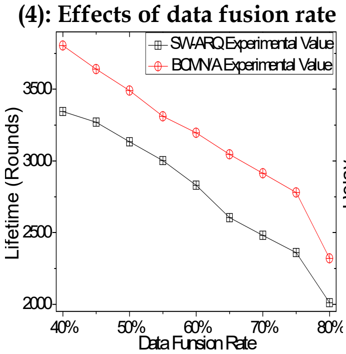


Fig.28 lifetime under different packet loss rates

Fig.29 delay under different packet loss rates

Fig. 28 and Fig. 29 show the effects of different packet loss rates on the network lifetime and delay. As can be seen, when the application required reliability is fixed, the higher is the network packet loss rate, the more retransmissions are required, and this causes the increase of data load and energy consumption, and thus the network performance of delay and lifetime and are deteriorating.

(4): Effects of data fusion rate



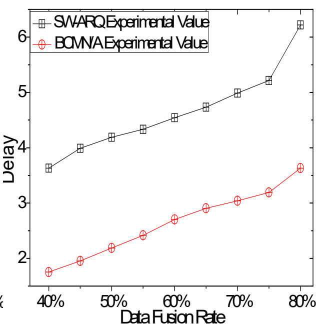
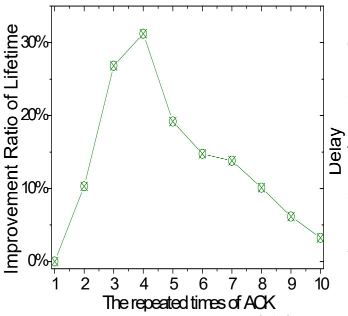


Fig.30 lifetime under different data fusion rates

Fig.31 delay under different data fusion rates

Fig. 30 and Fig. 31 show the effects of different data fusion rates on the network lifetime and delay. As can be seen, when the data fusion rate is higher, since the data load is decreased, the network lifetime is improved and the network delay is reduced.



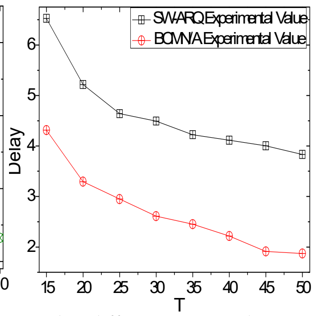


Fig.32 improvement ratio of lifetime under different repeated times of ACK

Fig.33 delay under different data gathering cycles T

Fig. 32 shows the effect of more ACK transmissions after receiving one data packet on the lifetime under BCMN/A protocol. In previous experiments, two ACK are returned for each data received under BCMN/A, and the result in Fig. 32 shows that 3-4 ACK can better improve the lifetime. Fig. 33 shows the longer time period of data collection cycle, the less network transport delay. The reason is that when T increases, although the node data load is the same, the packet arrival rate is decreased, thus reducing the queuing delay, thus reducing the delay.

7 Conclusion

Please refer to Section 3 of the online supplementary file for the conclusion and possible future work.

ACKNOWLEDGMENTS

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Supplementary File

Joint Optimization of Lifetime and Transport Delay under Reliability Constraint Wireless Sensor Networks

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1 SECTION 1

Theorem 1: Considering the system required data statistical reliability in intra-cluster is δ_1, then as for the node v_i within the cluster, the sent data amount is $X_i^{1,t}(\delta_1)$ and the number of ACK received is $Y_i^{1,r}(\delta_1)$, the received data amount of the cluster head within the cluster is $X_{ch}^{1,r}(\delta_1)$, and the ACK sent by the cluster head is $Y_{ch}^{1,t}(\delta_1)$, as the following.

$$\begin{cases} X_i^{1,t}(\delta_1)=\dfrac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}, Y_i^{1,r}(\delta_1)=\dfrac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}\overline{p_i q_i} \\ X_{ch}^{1,r}(\delta_1)=n\dfrac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}\overline{p_i}, Y_{ch}^{1,t}(\delta_1)=n\dfrac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}\overline{p_i}, \zeta(\delta_1)=\left\lceil \dfrac{\log(1-\delta_1)}{\log(p_i)} \right\rceil \end{cases} \tag{3}$$

Proof: Considering the required data reliability within the cluster is δ_1, then if the data reliability of each node is δ_1, the data reliability of whole cluster can be δ_1. And when the node reliability is δ_1, the maximum retransmission number can be calculated as the following: Suppose that each sensed data has *N* backups. By the time independency assumption, the number of sensed data successfully delivered across wireless link having failure probabilities of $P = \{ p_i \}$, is binomially distributed with *N* and success probability of $\prod_{i=0}^{k} \overline{p_i}$; thus,

$$\delta_1(N,P) \cong P(X_{k,p} > 1) = 1-(1-\prod_{i=0}^{k}\overline{p_i})^N$$

So the maximum retransmission number within the cluster is: $\zeta(\delta_1) = \left\lceil \dfrac{\log(1-\delta_1)}{\log(p_i)} \right\rceil$

According to probability theory, the expected transmission number can be calculated as:

$$X_i^{1,t}(\delta_1) = \frac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}$$

The expected number of received ACK return by the CH can be easily calculated as the following.

$$Y_i^{1,r}(\delta_1) = \frac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}\overline{p_i q_i}$$

Since the expected value of each node sends data to the CH is $\dfrac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}$, and the success probability is $\overline{p_i}$, then the number of data packets received by CH is $\dfrac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}\overline{p_i}$, since there are n nodes within the cluster, the total number of data packets received by Ch is $n\dfrac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}\overline{p_i}$, since ACK is returned for each received data packet, thus the number of ACK sent equals the data amount received, thus proved. ■

Theorem 2: Considering the distance from cluster head C_l to the sink is l, $l = hr + x$, assuming the required statistical reliability of data to the sink is δ_2, then the data load and ACK load D_l and ACK load M_l of C_l is:

$$\begin{cases} D_l^{1,t} = n(l,0)X_{h+0}^{h}(\delta_2) + n(l,1)X_{h+1}^{h}(\delta_2) + \ldots + n(l,z)X_{h+1}^{h}(\delta_2) \\ D_l^{1,r} = 0 + n(l,1)X_{h+1}^{h+1}(\delta_2)\overline{p_{h+1}} + \ldots + n(l,z)X_{h+z}^{h+1}(\delta_2)\overline{p_{h+z}} \\ M_l^{1,r} = (D_l^{t}\overline{p_h q_{h-1}}) / N_{clusternode} \\ M_l^{1,t} = D_l^{1,r} / N_{clusternode}, \ n(l,i) = \phi\rho\alpha(4lr + 8ir^2) \\ X_{h+j}^{h}(\delta_2) = \dfrac{1-(1-\overline{p_h q_h})^{S_{h+j}(\delta_2)}}{\overline{p_h q_h}}, S_h(\delta_2) = \left\lceil \dfrac{\log(1-\sqrt[h+1]{\delta_2})}{\log(p_h)} \right\rceil, \\ N_{clusternode} = \rho\pi r^2 \end{cases} \tag{4}$$

Proof: The data load of CH C_l whose distance from the sink is l can be calculated as following. As shown in Fig.1, the responsible area of C_l is the fan-shaped region whose angle is 2α. Then, after data of nodes whose distance is $[l+r, l+3r]$ from CH is sent to the cluster head, there is still one hop to C_l, the data amount of this region is $\rho\dfrac{\alpha}{\pi}(\pi(l+3r)^2 - \pi(l+r)^2)$, the responsible data amount of 2 hops is $\rho\dfrac{\alpha}{\pi}(\pi(l+5r)^2 - \pi(l+3r)^2)$, then the responsible data amount of i hops is $\rho\alpha(4lr + 8ir^2)$. Set $n(l,i) = \phi\rho\alpha(4lr + 8ir^2)$, where ϕ is the data aggregation rate, $n(l,i)$ represents the origin data amount forwarded by

C_l for CH at i hop.

Consider the data retransmission and packet loss. With SW-HBH, assuming CH is h hop to the sink, if we want to ensure the data is to the sink under reliability after $h+1$ hops, then we must ensure the reliability δ_2 of each hop $\geq \sqrt[h+1]{\delta_2}$, similar with theorem 1, the maximum retransmission number at each hop can be obtained, which is also the maximum number of C_l needs to send its data, that is:

$$S_h(\delta_2) = \left\lceil \frac{\log(1-\sqrt[h+1]{\delta_2})}{\log(p_h)} \right\rceil$$

Similarly, the expected retransmission number of CH whose distance from the sink is h hops is calculated as:

$$X_{h+0}^{h}(\delta_2) = \frac{1-(1-\overline{p_h q_h})^{S_h(\delta_2)}}{\overline{p_h q_h}}$$

The retransmission number of C_l for data from $h+j$ hop to h hop is as the following.

$$X_{h+j}^{h}(\delta_2) = \frac{1-(1-\overline{p_h q_h})^{S_{h+j}(\delta_2)}}{\overline{p_h q_h}}$$

The data amount of nodes at $h+j$ hop is $\phi\rho\alpha(4lr+8jr^2)$. Then the data amount needs to be forwarded by C_l for CH whose distance from C_l is j hops can be calculated as: $n(l,j)\ \dfrac{1-(1-\overline{p_h q_h})^{S_{h+j}(\delta_2)}}{\overline{p_h q_h}}$

Then the transmission data amount of nodes at C_l is the total amount of all CHs whose distance from C_l is 0 to z hops, as the following:

$$D_l^{1,t} = n(l,0)X_{h+0}^{h}(\delta_2) + n(l,1)X_{h+1}^{h}(\delta_2) + \ldots + n(l,z)X_{h+1}^{h}(\delta_2)$$

The received data amount is the expected sent data of the previous CH multiply by $\overline{p}$:

$$D_l^{1,r} = M_l^{1,t} = 0 + n(l,1)X_{h+1}^{h+1}(\delta_2)\overline{p_{h+1}} + \ldots + n(l,z)X_{h+z}^{h+1}(\delta_2)\overline{p_{h+z}}$$

ACK is sent for each data received from a cluster head, thus $M_l^{1,t} = D_l^{1,r} / N_{clusternode}$, the number of ACK received is: $M_l^{1,r} = (D_l^{1,t}\,\overline{p_h q_{h-1}}) / N_{clusternode}$, $N_{clusternode} = \rho\pi r^2$ denotes the number of nodes within a cluster, thus proved. ■

Theorem 3: If the distance from cluster head to the sink is l, then the total energy consumption for data of all nodes in intra-cluster sent to the cluster head is:

$$E_{l,total}^{1,in} = 2X_i^{1,t}(\delta_1)\rho\alpha\{2(E_{elec}+\varepsilon_{fs}l^2)lr + 2lr\varepsilon_{fs}(l^2+r^2)\}$$
$$-4X_i^{1,t}(\delta_1)\rho\varepsilon_{fs}l\sin\alpha(2l^2r+\frac{2}{3}r^3) + 4Y_i^{1,r}(\delta_1)\rho\alpha\cdot E_{elec}lr \quad (5)$$

Proof: Take any location within the cluster, its distance to the network centre O is $y \mid y \in \{l-r\,..\,l+r\}$, take a fan-shape ring with angle $d\vartheta$ on the ring whose width is dy, as Q in Fig.1. There are $y \times d\theta \times dy \times \rho$ nodes in this region, nodes within the cluster send data directly to the cluster head, the distance is calculated as:

$$L^2 = l^2 + y^2 - 2ly\cos\theta$$

According to the energy consumption Formula 1, the energy consumption of this fan-shaped ring is:

$$\widehat{E} = X_i^{1,t}(\delta_1)\left\{y\cdot d\theta\cdot dy\cdot\rho\cdot E_{elec} + y\cdot d\theta\cdot dy\cdot\rho\cdot\varepsilon_{fs}\cdot L^2\right\}$$
$$+Y_i^{1,r}(\delta_1)y\cdot d\theta\cdot dy\cdot\rho\cdot E_{elec}$$

Integration to this region, the total energy consumption of all common nodes send data to the cluster head is:

$$E_{l,total}^{1,in} = 2\int_{l-r}^{l+r}\int_0^{\alpha}\widehat{E} =$$
$$= 2X_i^{1,t}(\delta_1)\rho\alpha\{2(E_{elec}+\varepsilon_{fs}l^2)lr + 2lr\varepsilon_{fs}(l^2+r^2)\}$$
$$-4X_i^{1,t}(\delta_1)\rho\varepsilon_{fs}l\sin\alpha(2l^2r+\frac{2}{3}r^3) + 4Y_i^{1,r}(\delta_1)\rho\alpha\cdot E_{elec}lr$$

■

Theorem 4: In multi-hop cluster based networks, assuming the cluster radius is r, after one round data gathering of the entire network, the average energy consumption of node whose distance from the sink is $l = hr + x$, $E_l^{1,avg}$ is as the following:

$$\begin{cases} E_l^{1,avg} = \{E_{ch}^{1,in} + (D_l^{1,t}+M_l^{1,t})(E_{elec}+\varepsilon l^a) + (D_l^{1,r}+M_l^{1,r})E_{elec} + (n-1)\dfrac{E_{l,total}^{1,in}}{n}\}/n \ \ if\ l \leq 2r \\ E_l^{1,avg} = \{E_{ch}^{1,in} + (D_l^{1,t}+M_l^{1,t})(E_{elec}+\varepsilon(2r)^a) + (D_l^{1,r}+M_l^{1,r})E_{elec} + (n-1)\dfrac{E_{l,total}^{1,in}}{n}\}/n \ \ if\ l > 2r \\ E_{ch}^{1,in} = X_{ch}^{1,r}(\delta_1)E_{elec} + E_l^{1,ack}, M_{l,ch}^{1,t} = \dfrac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}\overline{p_i} \\ E_l^{1,ack} = 2M_{l,ch}^{1,t}(\delta_1)\rho\alpha\{2(E_{elec}+\varepsilon_{fs}l^2)lr + 2lr\varepsilon_{fs}(l^2+r^2)\} - 4M_l^{1,t}(\delta_1)\rho\varepsilon_{fs}l\sin\alpha(2l^2r+\frac{2}{3}r^3) \end{cases} \quad (6)$$

Proof: Take any node v_i with distance from l the sink, if it is the cluster head, according to theorem 1, the data amount of all nodes within the cluster is $X_{ch}^{1,r}(\delta_1)$, and the energy consumption is $X_{ch}^{1,r}(\delta_1)\ E_{elec}$. The ACK amount sent to the nodes within the sink is $M_{ch,in}^{1,t} = \dfrac{1-(1-\overline{p_i q_i})^{\zeta(\delta_1)}}{\overline{p_i q_i}}\overline{p_i}$. Similar with theorem 2, ACK is sent to any shadow area within the cluster such as Q (see Fig.1), there are $y \times d\theta \times dy \times \rho$ nodes in Q. The CH transmission distance can be calculated as $L^2 = l^2 + y^2 - 2ly\cos\theta$. Integration to this region, the total energy consumption for ACK to all nodes within the sink is:

$$E_l^{1,ack} = 2\int_{x-r}^{x+r}\int_0^{\alpha} M_l^{1,t}\left\{y\cdot\rho\cdot E_{elec} + y\cdot\rho\cdot\varepsilon_{fs}\cdot\left(y^2+l^2-2ly\cos\theta\right)\right\}d\theta dy$$
$$= 2M_{l,ch}^{1,t}(\delta_1)\rho\alpha\{2(E_{elec}+\varepsilon_{fs}l^2)lr + 2lr\varepsilon_{fs}(l^2+r^2)\}$$
$$-4M_l^{1,t}(\delta_1)\rho\varepsilon_{fs}l\sin\alpha(2l^2r+\frac{2}{3}r^3)$$

Thus, the energy consumption for data gathering within the cluster is: $E_{ch}^{1,in} = X_{ch}^{1,r}(\delta_1)E_{elec} + E_l^{1,ack}$

Besides, CH forwards data from itself and other CHs, the data and ACK amount is shown as theorem 2, then the energy consumption is as the following.

$$\begin{cases} E_{ACK}^{1} = (D_l^{1,t} + M_l^{1,t})(E_{elec} + \varepsilon l^a) + (D_l^{1,r} + M_l^{1,r})E_{elec} & if\ x \leq 2r \\ E_{ACK}^{1} = (D_l^{1,t} + M_l^{1,t})(E_{elec} + \varepsilon(2r)^a) + (D_l^{1,r} + M_l^{1,r})E_{elec} & if\ x > 2r \end{cases}$$

If it is common node, the energy consumption for all nodes send data to the CH is shown as theorem 3: $E_{l,total}^{1,in}$, then the average energy consumption in one round is $E^{1,in}_{l,total}/n$, each node works as the CH once and as common node n-1 times in one cycle, therefore, the average energy consumption is the energy consumption of once as CH and n-1 times as common node divided by n, thus proved.

■

Theorem 5: Considering $m_1=\zeta(\delta_1)$, the then time for data gathering in intra-cluster is

$$t_i^{1,in}=(n-1)((m_1-1)t_{rto}+t_{rtt}/2) \tag{7}$$

Proof: Theorem 1 has proved that when the data reliability is δ_1, the maximum retransmission number is $\zeta(\delta_1)$, TDMA mechanism is used for data transmission within the cluster, then the time slot needed is as the following.

First, the time slot should be assigned according to the maximum time slot of all nodes. Since it has to wait time of t_{rto} to know whether retransmission is necessary, and the maximum retransmission number is $m_1=\zeta(\delta_1)$, so the time slot needed is $m_1 t_{rto}$. It is not necessary for the ACK returned for the last transmission because this does not impact on the result, in order to reduce the delay, the cluster head does not return ACK for the last $\zeta(\delta_1)$ transmission , so the time slot for each node should be:

$(m_1\text{-}1)t_{rto}+t_{rtt}/2$

If there are n nodes in the cluster, the number of common nodes needs time slot is n-1, and the time for all data gathering is $t_i^{1,in}=(n-1)((m_1-1)t_{rto}+t_{rtt}/2)$

■

Theorem 6: The delay $t^1_{l,CH}$, queuing delay t^1_q and transport delay $E(t^1_t)$ of CH C_l at $l=hr+x$ from the sink is as following:

$$t^1_{l,CH}=t^1_q+E(t^1_t)=\frac{\rho^2}{(1-\rho)\lambda_l}+\sum_{k=1}^{\zeta(\delta_2)}\{(\frac{1}{2}t_{rtt}+(k-1)t_{rto})(1-p)p^{k-1}\} \tag{8}$$

note: $t^1_q=\frac{\rho^2}{(1-\rho)\lambda_l}$, $\rho=\frac{\lambda_l}{\mu_1}$, $\mu_1=\frac{T}{t_{rto}\overline{pq}\sum_{k=1}^{\zeta(\delta_2)}\{k(1-\overline{pq})^{k-1}\}}$

Proof: The sending (service) time needs to be calculated in order to calculate t^1_q, since the maximum retransmission number can be $\zeta(\delta_2)$, and the time at k retransmission is $t_k=kt_{rto},\ k\in 1..\zeta(\delta_2)$. Thus the expected service time is $t=\sum_{k=1}^{m}t_k p_k\,|\,t_k=kt_{rto},\ k=1..\zeta(\delta_2)$

The success probability at the k retransmission is $(1-\overline{pq})^{k-1}\overline{pq}$ The average process(service) time is:

$$E(t_s)=\sum_{k=1}^{\zeta(\delta_2)}t_k p_k=\sum_{k=1}^{\zeta(\delta_2)}\{kt_{rto}(1-\overline{pq})^{k-1}\overline{pq}\}=t_{rto}\overline{pq}\sum_{k=1}^{\zeta(\delta_2)}k\{(1-\overline{pq})^{k-1}\}$$

Considering data packers is Poisson stream, the number of data packets arrive at $T_l=T+\sum_{k=h+1}^{z}t^1_{k,CH}$ is:

$$n(l)=\rho\frac{\alpha}{\pi}(\pi R^2-\pi(l-r)^2)\delta_1\delta_2=\rho\alpha(R^2-(l-r)^2)\delta_1\delta_2$$

The data arrive rate is $\lambda_l=n(l)/T_l$

Considering the node data processing time follows a negative exponential distribution, the average service time under SW-ARQ is $E(t_s)$, then the number of data packets(service productivity) can be processed in time T_l is $\mu_1=T_l/E(t_s)$, and the service strength is $\rho=\frac{\lambda_l}{\mu_1}$. Based on the queuing theory, the queuing time t^1_q is

$$t^1_q=W_q=\frac{L_q}{\lambda_l}=\frac{\rho L}{\lambda_l}=\frac{\rho\frac{\rho}{1-\rho}}{\lambda_l}=\frac{\rho^2}{(1-\rho)\lambda_l}\,|\,\rho=\frac{\lambda_l}{\mu_1},\mu_1=\frac{T_l}{t_{rto}\overline{pq}\sum_{k=1}^{\zeta(\delta_2)}k\{(1-\overline{pq})^{k-1}\}}$$

Then consider the calculation of average transport delay. It is the transmission time at k multiplied by the transmission probability at k, as the following.

$$t^1_t=\sum_{k=1}^{\zeta(\delta_2)}t_k p_k\quad t_k=\frac{1}{2}t_{rtt}+(k-1)t_{rto},\ \ k\in 1..\zeta(\delta_2)$$

Since the once success probability is 1- p, then the success probability at k is:

$$P(m_1=k)=(1-p)p^{k-1}\quad 1\le k\le\zeta(\delta_2)$$

Therefore, the mean transport delay $E(t^1_t)$, is derived as follows.

$$E(t^1_t)=\sum_{k=1}^{\zeta(\delta_2)}\{(\frac{1}{2}t_{rtt}+(k-1)t_{rto})(1-p)p^{k-1}\}$$

Therefore, the total delay of a CH is the sum of queuing delay and transport delay, thus proved.

■

Theorem 7: If only one broadcasting is processed for each data gathering for intra-cluster, to meet the reliability δ_1, the number of retransmissions in onde roud should be

$$m_2=\left\lceil\frac{\log(1-\delta_1)}{\log(1-(1-q)(1-p))}\right\rceil$$

Proof: There are $1-q$ nodes can receive the time slot information when the CH broadcasts for the first time, and there is $1-p$ data sent to the CH after the first round from nodes which receive the broadcasting, so, after the first data gathering round, there are $1-(1-q)(1-p)$ nodes fail, therefore, the reliability of the first round is $1-(1-(1-q)(1-p))$. After the CH broadcasting the information for the first data gathering, the probability of receiving the broadcasting for nodes which fail to send data is $1-q$, and then succeed with probability of $1-p$, so fail probability for the remaining nodes is $1-(1-q)(1-p)$, then after the first two rounds, there are still

$(1-(1-q)(1-p))^2$ nodes fail, so the reliability after the first two rounds is $1-(1-(1-q)(1-p))^2$. Therefore, the reliability after k rounds is $1-(1-(1-q)(1-p))^k$, set the reliability $1-(1-(1-q)(1-p))^k > \delta_1$.

$$\Rightarrow m_2 = \left\lceil \frac{\log(1-\delta_1)}{\log(1-(1-q)(1-p))} \right\rceil$$

■

Theorem 8: Under BCMA protocol, if ACK is only broadcasted only once each time, the node data load intra-cluster is as the following. Note, $D_i^{2,t}$ is the total number of data packets, $M_i^{2,r}$ is the node received ACK bits amount, $M_{ch}^{2,t}$ is the CH sent ACK bits amount, the number of received data packets is $D_{ch}^{2,r}$. m_2 is the number of gathering rounds, *n* is the number of nodes intra-cluster, *c* is the needed bits amount of each node ID.

$$\begin{cases} D_i^{2,t} = (1-q)\sum_{k=1}^{m_2}(1-\overline{pq})^{k-1},\ M_i^{2,r} = \sum_{k=1}^{m_2}(1-q)(1-\overline{pq})^{2(k-1)}(n-1)^2 c \\ D_{ch}^{2,r} = (n-1)D_i^{2,t}\overline{p},\ M_{ch}^{2,t} = (n-1)c\sum_{k=1}^{m_2}(1-\overline{pq})^{k-1} \end{cases} \quad (10)$$

Proof: (1) The calculation of node sent data amount in intra-cluster. In the first round, the probability of node receives the time slot broadcasting is $1-q$, then the total number of data packets sent is $(1-q)(n-1)$. After the first round, data packets of $(1-(1-p)(1-q))(n-1)$ nodes are not received by the CH, then the time slot is broadcasted with probability $1-q$, the number of data packets sent in the second round is $(1-q)(1-(1-p)(1-q))(n-1)$. Similarly, the number of data packets sent in the m_2 round is $(1-q)(1-(1-p)(1-q))^{m_2-1}(n-1)$. Therefore, the total number of data packets sent is

$$\begin{pmatrix}(1-q)+(1-q)(1-(1-p)(1-q))+(1-q)(1-(1-p)(1-q))^2 \\ +\ldots+(1-q)(1-(1-p)(1-q))^{m_2-1}\end{pmatrix}(n-1)$$
$$=\left((1-q)\sum_{k=1}^{m_2}(1-\overline{pq})^{k-1}\right)(n-1)$$

Therefore, the average amount of data sent by every node is $D_i^{2,t} = \left((1-q)\sum_{k=1}^{m_2}(1-\overline{pq})^{k-1}\right)$.

(2) The calculation of ACK bits amount received by nodes in intr-cluster. The data amount sent in the first round is $(n-1)$, and $(1-p)(n-1)$ successfully received, there are $p\ (n-1)$ unsuccessful after the first round, the number of active nodes is $(n-1)$. The ACK received in the next broadcasting is the number of active nodes $*(1-q)*(n-1)$.

Similarly, the number of received broadcasting in the k round is $(1-\overline{pq})^{k-1}(1-q)(n-1)$, and the length of ACK at each time is

$$(1-(1-q)(1-p))^{k-1}(n-1)c = (1-\overline{pq})^{k-1}(n-1)c$$

. The total length of ACK at each time is

$$(1-\overline{pq})^{k-1}(1-q)(n-1)(1-\overline{pq})^{k-1}(n-1)c = (1-q)(1-\overline{pq})^{2(k-1)}(n-1)^2 c.$$

Therefore, the total received ACK bits amount is $\sum_{k=1}^{m_2}(1-q)(1-\overline{pq})^{2(k-1)}(n-1)^2 c$, then $M_i^{2,r}$ is obtained.

(3): The calculation of ACK bits amount sent by the CH. According to theorem 7, the number of ACK broadcasting is m_2, namely, the ID message of each node is a constant c, so the total length of ID message sent in the first broadcasting is $(n-1)c$, n is the number of nodes of intra-cluster. After the first round, there are $1-(1-q)(1-p)$ nodes need to be sent, so the length of the second broadcasting is $(1-(1-q)(1-p))(n-1)c$, if the data needs m_2 times transmissions, then the length of broadcasting at the last time is $(1-(1-q)(1-p))^{m_2-1}\cdot(n-1)c$. Therefore, the length of total broadcasted data packets is

$$(n-1)c+(1-(1-q)(1-p))(n-1)c+(1-(1-q)(1-p))^2(n-1)c$$
$$+\ldots+(1-(1-q)(1-p))^{m_2-1}(n-1)c = (n-1)c\sum_{k=1}^{m_2}(1-\overline{pq})^{k-1}.$$

Then the ACK bits amount sent by the CH is $(n-1)c\sum_{k=1}^{m_2}(1-\overline{pq})^{k-1}$.

(4) The data amount received by the CH is the data amount sent by all nodes of intra-cluster multiplied by $\overline{p}$, then $D_{ch}^{2,r} = (n-1)D_i^{2,t}\overline{p}$, thus proved.

■

Theorem 9: If the ACK broadcasting is repeated z times after each data gathering round, the data load is as the following. Note, $D_i^{3,t}$ is the number of total data packets sent by nodes within the cluster, $M_i^{3,r}$ is the ACK bits amount received by nodes, $M_{ch}^{3,t}$ is the ACK bits amount sent by the CH, $D_{ch}^{3,r}$ is the number of data packets received, m_3 is the number of data gathering round, *n* is the number of nodes within the cluster, *c* is the node ID bits amount of each node.

$$\begin{cases} D_i^{3,t} = (1-q^z)\sum_{k=1}^{m_3}(1-\overline{p}(1-q^z))^{k-1},\ M_i^{3,r} = \sum_{k=1}^{m_3}(1-q^z)(1-\overline{p}(1-q^z))^{2(k-1)}(n-1)^2 c \\ D_{ch}^{3,r} = (n-1)D_i^{2,t}\overline{p},\ M_{ch}^{3,t} = (n-1)c\sum_{k=1}^{m_3}(1-\overline{p}(1-q^z))^{k-1} \end{cases} \quad (11)$$

Proof: (1) The calculation of node sent data amount of intra-cluster. In the first round, the probability of node receives the time slot broadcasting is $1-q^z$, then the total number of data packets sent is $(1-q^z)(n-1)$. After the first round, data packets of $(1-(1-p)(1-q^z))(n-1)$ nodes are not received by the CH, then the time slot is broadcasted with probability $1-q^z$, the number of data packets sent in the second round is $(1-q^z)(1-(1-p)(1-q^z))(n-1)$. Similarly, the number of data packets sent in the m_3 round is $(1-q^z)(1-(1-p)(1-q^z))^{m_3-1}(n-1)$. Therefore, the total number of data packets sent is

$$\{(1-q^z)+(1-q^z)(1-(1-p)(1-q^z))+(1-q^z)(1-(1-p)(1-q^z))^2+ \\ ...+(1-q^z)(1-(1-p)(1-q^z))^{m_3-1}\}(n-1)=(1-q^z)\sum_{k=1}^{m_3}(1-\overline{p}(1-q^z))^{k-1}(n-1)$$

Therefore, the average amount of data sent by every node is

$$D_i^{3,t}=(1-q^z)\sum_{k=1}^{m_3}(1-\overline{p}(1-q^z))^{k-1}.$$

(2) The ACK bits amount received by the nodes within the cluster can be obtained with similar method in theorem 8, that is

$$M_i^{3,r}=\sum_{k=1}^{m_3}(1-q^z)(1-\overline{p}(1-q^z))^{2(k-1)}(n-1)^2c.$$

(3): The calculation of ACK bits amount sent by the CH. According to corollary 4, the number of ACK broadcasting is m_3, namely, the ID message of each node is a constant c, so the total length of ID message sent in the first broadcasting is $(n-1)c$, n is the number of nodes of intra-cluster. After the first round, there are $1-(1-q^z)(1-p)$ nodes need to be sent, so the length of the second broadcasting is $(1-(1-q^z)(1-p))(n-1)c$, if the data needs m_3 times transmissions, then the length of broadcasting at the last time is $(1-(1-q^z)(1-p))^{m_3-1}(n-1)c$. Therefore, the length of total broadcasted data packets is

$$(n-1)c+(1-(1-q^z)(1-p))(n-1)c+(1-(1-q^z)(1-p))^2(n-1)c \\ +.....+(1-(1-q^z)(1-p))^{m_3-1}(n-1)c=(n-1)c\sum_{k=1}^{m_3}(1-\overline{p}(1-q^z))^{k-1}$$

. Then the ACK bits amount sent by the CH is $(n-1)c\sum_{k=1}^{m_3}(1-\overline{p}(1-q^z))^{k-1}$.

(4) The data amount received by the CH is the data amount sent by all nodes within the cluster multiplied by $\overline{p}$, then $D_{ch}^{3,r}=(n-1)D_i^{3,t}\overline{p}$, thus proved. ■

Theorem 11: Under multi-hop cluster based protocol, assuming the cluster radius is r, as for the data gathering intra-cluster, NACK is broadcasted once for each data gathering round and z ACK is returned for each received data inter-clusters, after an entire data gathering, the average energy consumption $E_l^{2,avg}$ for node whose distance from the sink is $l=hr+x$ is as the following.

$$\begin{cases} E_l^{2,avg}=\{E_{ch}^{2,in}+(D_l^{4,t}+M_l^{4,t})(E_{elec}+\varepsilon l^a)+(D_l^{4,r}+M_l^{4,r})E_{elec}+(n-1)\frac{E_{l,total}^{2,in}}{n}\}/n \ \ if\ l<2r \\ E_l^{2,avg}=\{E_{ch}^{2,in}+(D_l^{4,t}+M_l^{4,t})(E_{elec}+\varepsilon (2r)^a)+(D_l^{4,r}+M_l^{4,r})E_{elec}+(n-1)\frac{E_{l,total}^{2,in}}{n}\}/n \ \ if\ l>2r \\ E_{ch}^{2,in}=D_{ch}^{2,r}E_{elec}+M_{ch}^{2,t}(E_{elec}+\varepsilon r^a) \\ E_{l,total}^{2,in}=2D_i^{2,t}(\delta_1)\rho\alpha\{2(E_{elec}+\varepsilon_{fs}l^2)lr+2lr\varepsilon_{fs}(l^2+r^2)\}- \\ \qquad 4D_i^{2,t}(\delta_1)\rho\varepsilon_{fs}l\sin\alpha(2l^2r+\frac{2}{3}r^3)+4M_i^{2,r}\rho\alpha\cdot E_{elec}lr \end{cases} \quad (13)$$

Proof: Take node v_i whose distance from the sink is l. It has two states, as the cluster head and as the common node. If it is the cluster head, the energy consumption includes the energy consumption for data gathering form intra-cluster and inter-clusters. First calculate the energy consumption intra-cluster. According to theorem 2, the received data amount is $D_{ch}^{2,r}$, and the energy consumption is $D_{ch}^{2,r}E_{elec}$, the ACK sent by CH to nodes in the cluster is $M_{ch}^{2,t}$, and this energy consumption is $M_{ch}^{2,t}(E_{elec}+\varepsilon r^a)$.

Besides, CH forwards data of itself and other cluster heads. The data amount and ACK amount is shown in theorem 10, if the distance from the sink is less than $2r$, then the transmission distance is

$$\begin{cases} (D_l^{4,t}+M_l^{4,t})(E_{elec}+\varepsilon l^a)+(D_l^{4,r}+M_l^{4,r})E_{elec} & if\ x\le 2r \\ (E_{elec}+\varepsilon(2r)^a)+(D_l^{4,r}+M_l^{4,r})E_{elec}(D_l^{4,t}+M_l^{4,t}) & if\ x>2r \end{cases}$$

If it is the common node, the energy consumption of node sends data to CH can be obtained similar with theorem 3,

$$E_{l,total}^{2,in}=2D_i^{2,t}(\delta_1)\rho\alpha\{2(E_{elec}+\varepsilon_{fs}l^2)lr+2lr\varepsilon_{fs}(l^2+r^2)\} \\ -4D_i^{2,t}(\delta_1)\rho\varepsilon_{fs}l\sin\alpha(2l^2r+\frac{2}{3}r^3)+4M_i^{2,r}\rho\alpha\cdot E_{elec}lr$$

Then the node average energy consumption in one round is $E_{l,total}^{2,in}/n$, nodes intra-cluster work as the cluster once in one cycle, and n-1 times as common node, therefore, the average energy consumption is the sum of energy consumption as CH once and energy consumption as common node n-1 times divided by n, then theorem 11 is obtained. ■

Theorem 12: Under BCMN/A protocol, if the NACK is broadcasted only once after the first data gathering round, the data delay is

$$t_i^{2,in}=\frac{1}{2}(n-1)t_{rtt}+(n-1)\sum_{k=2}^{m_2}(1-\overline{pq})^{k-1}\frac{1}{2}t_{rtt}+\frac{1}{2}m_2t_{rtt}$$

If the broadcasting is repeated z times in one data gathering round, the delay is

$$t_i^{3,in}=\frac{1}{2}(n-1)t_{rtt}+(n-1)\sum_{k=2}^{m_3}(1-\overline{p}(1-q^z))^{k-1}\frac{1}{2}t_{rtt}+\frac{1}{2}m_3t_{rtt}$$

.

Proof: (1): If the NACK is broadcasted only once after the first data gathering round, each node needs time slot $\frac{1}{2}t_{rtt}$ to send data, CH needs $\frac{1}{2}t_{rtt}$ time to broadcast NACK, therefore, the time needed for the first round is

$(n-1)\frac{1}{2}t_{rtt}+\frac{1}{2}t_{rtt}=\frac{1}{2}(n-1)t_{rtt}+\frac{1}{2}t_{rtt}$, n is the number of nodes within the cluster.

See theorem 8, since the broadcasting can be lost in the first round when CH sends the time slot information, so nodes that did not receive the time slot information will keep silent and wait slot, then the data amount successfully received by the CH in the first round is $(n-1)(1-\overline{p}\overline{q})$. Similarly, the time slot needed in the k_{th} round is $(n-1)(1-\overline{p}\overline{q})^{k-1}\frac{1}{2}t_{rtt}+\frac{1}{2}t_{rtt}$. There are total m_2 rounds, so the total time slot is

$$t_i^{2,in}=\frac{1}{2}(n-1)t_{rtt}+(n-1)\sum_{k=2}^{m_2}(1-\overline{p}\overline{q})^{k-1}\frac{1}{2}t_{rtt}+\frac{1}{2}m_2t_{rtt}.$$

(2) If the broadcasting is processed z times in one data gathering round. The time arranged for common node is $\frac{1}{2}(n-1)t_{rtt}$, the CH needs $\frac{1}{2}t_{rtt}+(z-1)\Delta s$. According to theorem 8, in the k round of data gathering, time arranged for common node is:

$$(n-1)(1-\overline{p}(1-q^z))^{k-1}\frac{1}{2}t_{rtt}+\frac{1}{2}t_{rtt}.$$

Therefore, the total delay is

$$t_i^{3,in}=\frac{1}{2}(n-1)t_{rtt}+(n-1)\sum_{k=2}^{m_3}(1-\overline{p}(1-q^z))^{k-1}\frac{1}{2}t_{rtt}+\frac{1}{2}m_3t_{rtt}$$

■

Theorem 13: Under BCMA/A protocol, the delay is $t^2_{l,CH}$ for CH C_l whose distance from the sink is $l=hr+x$, the queuing delay t^2_q and transport delay $E(t^2_t)$ are as the following:

$$t^2_{l,CH}=t^2_q+E(t^2_t)=\frac{\rho_l^2}{(1-\rho_l)\lambda_l}+(\frac{1}{2}t_{rtt})(1-p)+\sum_{k=2}^{\zeta(\delta_2)}\{(k-1)t^A_{rto}(1-p)p^{k-1}\}.$$

Note: $t^2_q=\frac{\rho_l^2}{(1-\rho_l)\lambda_l},\rho_l=\frac{\lambda_l}{\mu_2},\mu_2=\frac{T_l}{t^A_{rto}\overline{p}(1-q^z)\sum_{k=1}^{\zeta(\delta_2)}\{k(1-\overline{p}(1-q^z))^{k-1}\}}.$

Proof: First calculate the node average service time, the time needed for CH to send data in the k round is $t_k=kt^A_{rto}$, $k=1..\zeta(\delta_2)$. $t^A_{rto}=t_{rto}+(z-1)\Delta s$, if node returns z ACK for each data it receives (see Fig.3). Since the success probability of the first time is $(1-p)(1-q^z)=\overline{p}(1-q^z)$, then the success probability of the second time is $(1-\overline{p}(1-q^z))\overline{p}(1-q^z)$. Therefore, the success probability of the k time is $(1-\overline{p}(1-q^z))^{k-1}\overline{p}(1-q^z)$. Since the maximum retransmission number is $\zeta(\delta_2)$, then the average service time is:

$$E(t)=\sum_{k=1}^{\zeta(\delta_2)}t_kp_k=t^A_{rto}\overline{p}(1-q^z)\sum_{k=1}^{\zeta(\delta_2)}\{k(1-\overline{p}(1-q^z))^{k-1}\}.$$

According to theorem 6, the data arrival rate is $\lambda_l=n(l)/T_l,T_l=T+\sum_{k=h+1}^{z}t^2_{k,CH}$,. Then the node service rate is $\mu_2=T_l/E(t)$. Thus the service strength is $\rho_l=\frac{\lambda_l}{\mu_2}$. Therefore,

$$t^2_q=\frac{\rho_l^2}{(1-\rho_l)\lambda_l},\rho_l=\frac{\lambda_l}{\mu_2},\mu_2=\frac{T_l}{t^A_{rto}\overline{p}(1-q^z)\sum_{k=1}^{\zeta(\delta_2)}\{k(1-\overline{p}(1-q^z))^{k-1}\}}$$

The calculation of the transport delay:

$$t^A_t=\sum_{k=1}^{m}t^A_kp_k,t^A_k=\frac{1}{2}t_{rtt}+(k-1)t^A_{rto}=\frac{1}{2}t_{rtt}+(k-1)(t_{rto}+(z\text{-}1)\Delta s)$$

$$,t^A_k=\frac{1}{2}t_{rtt}+(k-1)t^A_{rto}=\frac{1}{2}t_{rtt}+(k-1)(t_{rto}+(z\text{-}1)\Delta s)$$

The same, the success probability at the k time is:

$P(m_1=k)=(1-p)p^{k-1}\quad 1\le k\le m_1$

Therefore, the mean transport delay $E(t)$ is derived

■

If the data gathering delay of intra-cluster and the transport delay inter-clusters are obtained, the transport delay from node v_j generates data to data received by the sink can be obtained as the corollary 6.

Corollary 6: Under BCMA/A protocol, assuming node v_j belongs to the CH C_l whose distance from the sink is $l=hr+x$, then the routing path of C_l to the sink is $R_j=\{l,l-2r,l-4r,...l-2ir,...,r+x,x\}$. Then the transport delay of v_j is:

$$t^2_{j,total}=\sum_{s\in R_j}t^2_{s,CH}+t_i^{2,in}\quad\text{or}\quad t^2_{j,total}=\sum_{s\in R_j}t^2_{s,CH}+t_i^{3,in},$$

where $t^2_{s,ch}=t^2_{l,ch}$ when $s=l$

Proof: Data generated by node v_j has two stages, including data gathering of intra-cluster and transmission inter- clusters. The data gathering time is $t_i^{2,in}$ or $t_i^{3,in}$, depending on whether single NACK or multi-ACK is deployed. Since there is delay $t^2_{s,CH}$ at s of the routing path, the total data transport delay is $t^2_{j,total}$.

■

2 EXPERIMENTAL PARAMETERS

Table 1 network parameters

Parameter	Value
Threshold distance (d_0) (m)	87
Sensing range r_s (m)	15
E_{elec} (nJ/bit)	50
e_{fs} (pJ/bit/m^2)	10
e_{amp} (pJ/bit/m^4)	0.0013
Initial energy (J)	0.5

3 CONCLUSION

As for the high unreliability in the transmission link of WSNs, using ARQ protocol to enhance network reliability is an effective method. Therefore, this paper obtains the energy consumption and delay under SW-ARQ protocol with theoretical analysis to obtain the optimal cluster radius r , which provides a theoretical guidance. Then we propose the advanced BCMN/A protocol, BCMN/A protocol broadcasts intra-cluster and returns multi-ACK for each data received from clusters, by sending more ACK which has smaller load, fewer data packets with heavier load are sent, and thus improve the network lifetime and decrease the network delay. This paper gives detailed theoretical analysis results of BCMN/A protocol. Finally, a large number of experiments confirm the correctness of our theoretical analysis, as well as the validity of BCMN/A, which can increase the network lifetime by more than 8% and reduce network delay by more than 25%.