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Xiaoheng Deng*†, You Wu*, Mianxiong Dong‡, Chenglong Li* and Yan Pan*
*School of Information Sci & Eng, Central South University, Changsha 410083, China
†Email:dxh@csu.edu.cn
‡Department of Information and Electronic Engineering
Muroran Institute of Technology 27-1 Mizumoto-cho, Muroran, Hokkaido, 050-8585, Japan
Email: mx.dong@csse.muroran-it.ac.jp

***Abstract*—Many complex networks in practice can be described by weighted network models, and the BBV model is one of the most classical ones. In this paper, by introducing the concept of correlation degree between nodes, a new weighted network model based on the BBV model is proposed. The model takes the both node strength and node correlation into consideration during the network evolution, which better reveals the evolving mechanisms behind various real-world networks. Results from theoretical analysis and numerical simulation have demonstrated the scale-free property and small-world property of the network model, which have been widely observed in many real-world networks. Compared with the BBV model, the added correlation preferential attachment rule in the model leads to a faster network propagation velocity.**

***Index Terms*—BBV model; self attribute; correlation degree; scale-free property; propagation velocity;**

I. Introduction

Recently, a great deal of effort has been devoted to the study of complex networks due to their important role in understanding basic mechanisms of many real complex systems in a wide variety of fields, including the World Wide Web [1], metabolic networks [2], worldwide airport networks [3], scientific collaboration networks [4] and social networks [5], etc [6], [7]. In a bid to comprehend these complex networks, scientists proposed numerous unweighted network models in early time, like ER random graph model [8], WS small-world model [9], NW small-world model [10] and scale-free network model [11]. In unweighted networks, all the links are considered equivalent. However, the connections in many real networks are not homogeneous [12], which naturally calls for a typical measurement of the edge weight. Therefore, real systems are best described by weighted growing networks with nonuniform strengths of the links.

As a result, various weighted network models have been proposed to describe and explain the real-world complex systems, such as Yook-Jeong-Barabasi-Tu (YJBT) model [13], Zheng-Trimper-Zheng-Hui (ZTZH) model [14], Antal-Krapivsky (AK) model [15] and etc [16], [17]. The BBV network model [18] was introduced by Barrat *et. al.* in 2004, where the evolutions of degree and weight are coupled in time. With the growth of a network, the BBV network's node degree, node strength and link weight all display the scale-free property. The BBV model laid a good foundation for the research on weighted networks, and a series of network models are introduced based on it, such as traffic-driven growth [19], spatial constraints [20], group-based preferential attachment [21] and accelerating growth [22], [23].

However, the majority of existing weighted network models merely consider the node strength in the evolution rule, but without referring to the effect caused by the correlation between nodes. Take the social network for examples, user B can friend user A for the reason that A has a large amount of fans, and user B can also make a friend with user C on the consideration of the correlation between them in the same breath. Specifically, the correlation between them can be the same nationality, same interests or even just the same friends. The strength preferential attachment mechanism based on the node strength is somewhat patchy in establishing the rule for network evolution. Concentrating on this aspect, we promote the idea of node correlation in our model for creating a more pragmatic weighted network.

The rest of this paper is organized as follows. In section 2, the related work about weighted network models are presented. Section 3 contains the description of the proposed weighted network model. Section 4 is devoted to the theoretical analysis. Numerical simulations of the model are presented in Section 5. Section 6 draws the conclusion.

II. Related work

As it mentioned in section 1, there are two kinds of network models which are presented to describe the complex networks: the unweighted network model and the weighted network model. In unweighted networks, the edge only represents the presence or absence of interaction. In other words, all the edges in unweighted networks have equal weights. However, many real network systems display different interaction strengths between nodes. Therefore, weighted growing network models with non-uniform strengths of the links are better models since they can well formulate practical architectures of more realistic complex networks, and in so many weighted network models, the BBV model [18] is one of the most classic ones.

In this section, we mainly introduce some weighted network models improved on the BBV model [18].

In the BBV model [18], only the weights of the edges departing from the vertex i will obtain an increase, but the weights of the other edges will keep unchanged, that means the weights are rearranged locally. However, much empirical evidence has demonstrated that the establishment of new edges will introduce variations of the existing weights across the network in most real networks. So in this paper [24], the authors let the emergences of new edges promote a total increase of traffic, that is proportionally distributed among all the edges in accordance with their own weights, which can rapidly spurs the expansion of networks.

By studying the real directional social network and analyzing the dynamic evolution of international import and export trade network, this paper [25] proposed the topology generation algorithm of weighted directed network based on the triad formation rule.In the algorithm, directed edges were added to the network by using weight preferential attachment rule and triad formation rule. Simulation results show that the algorithm can generate the network topology consistent with real network environment and has good controllability of the clustering coefficient.

There is an another model [26] which also considers the triad formation. The most evolution mechanisms just describe interactions between the newly added node and the old ones. Actually, such interactions also exist between old nodes. Such interactions more easily occur between neighbors(friends of friends), so-called Triad Formation. Furthermore, some interactions are generated randomly representing the small-world effect of networks.

Another drawback of the BBV model [18] is also pretty conspicuous, and it is that the model barely illustrates interactions between newly added node and the old ones, totally ignoring interactions between old nodes. Additionally, the rearrangement of weights is local, but we need a model that allows the flow to be widely updated. A truly novel network model is the one that has widely weighted dynamics, which promotes a general mechanism for the occurrence of varying power-law behaviors without resorting to more complicated topological rules and variations of the basic preferential attachment rule [27].

This paper [28] proposes a directed weighted network model based on BBV model by culminating with directivity and characteristic of network evolution. It introduces parameters p, q, the strength of a node is divided into in-strength and out-strength, picks over and evolution of this model based on BBV building thought [18]. Theory analysis and numerical value simulation results show that node distribution of out-strength and in-strength with the exponent of $[2, 3]$ in this model. Average path and clustering coefficient which are adjusted by parameter can consistent with the characteristics of complex network.

Considering the node attraction, a new and realistic weighted evolving complex network model [29] was proposed based on the network model with limited node strength. Through the research it is found that the distribution of node strength of this model is changed and its more realistic in the network comparing with the BBV model [18]. By adjusting the parameters of the relevant property that a more optimal state of the network can be gained. It can guide the evolution of the actual network, reduce the networks load and enhance its performance.

However, the above weighted network models merely consider the node strength in the evolution rule, but without referring to the effect caused by the correlation between nodes. In this paper, we introduce the concept of correlation degree in our model for creating a more pragmatic weighted network.

III. Network model

A. Preliminary

A weighted network can be described by a weighted adjacency matrix with entries that are equal to the weights on the edges, namely

$$W = (w_{ij}) \tag{1}$$

where $i, j = 1, 2, 3 \ldots, \ldots N$. If there is no edge between node i and node j, we have $w_{ij} = 0$.

The node degree of a node i in a weighted network, namely node strength s_i(or node weight), is defined as

$$s_i = \sum_{j \in v(i)} w_{ij} \tag{2}$$

where $v(i)$ is the set of neighbors of node i. The node strength distribution function $p(s)$ represents the probability that a node's strength value is s.

B. The correlation degree between nodes

The evolution rules of most previous weighted network models are based on node strength preferential attachment, which means that the nodes with greater node strength would be more likely to be chosen as "friend". But these kind of evolving rules are inadequate to describe the network evolution scenario shown as follows. A new user who joins in an online social network for the first time may not want to connect to the most popular user, instead he is more likely to choose the ones who have same hobbies or come from same place to be his neighbors.

To address this practical challenge, we propose a new weighted evolving network model with additional consideration of the correlation between nodes in the growth of a network, and the related parameters are defined as follows.

Self attribute is a newly defined node feature which represents node inherent self attributes. For an example of an online social network, a user's self attribute could be his interests, profession, hometown, and even the graduated college. The different users who have same self attribute could become friends on a certain probability. In the paper, the self attribute of node n is denoted by β_n. In the process of network evolution, the values of all the nodes self attributes are randomly assigned to a numeric between 0 and 1.

Correlation degree is a newly defined network feature which represents the correlation degree between any two nodes in the network. We assume that the closer values of self attributes, the higher correlation degree between nodes. The correlation degree between node n and node i is denoted by τ_{ni}, and the calculation formula of parameter τ_{ni} is defined as follows:

$$\tau_{ni} = 1 - |\beta_n - \beta_i| \tag{3}$$

According to the (3), we can see that closer value of self attributes between nodes is an indicator of high correlation degree.

C. Evolution Algorithm of the Model

1) Network Initialization: We start from a small number m_0 of fully connected nodes, and fix a self attribute value $\beta(0 \leq \beta \leq 1)$ to every node. The weight of each edge is assigned to w_0.

2) Topological Growth: In every time step

i. With probability α, we add a new node n with the self attribute value of β_n to the network. The new node develops m links to the existing nodes in the network. According to the strength preferential attachment rule, the probability of an existing node i being selected for connection is dependent on the strength of node i.

$$\prod_{n \to i} = \frac{s_i}{\sum_l s_l} \tag{4}$$

where function $\sum_l s_l$ represents the sum of node strength of the whole network, and probability parameter $\alpha \in [0, 1]$.

ii. With probability $1-\alpha$, we add a new node n with the self attribute value of β_n to the network. The new node develops m links to the existing nodes in the network. According to the correlation preferential attachment rule, the probability of an existing node i being selected for connection is dependent on the correlation degree between node i and node n.

$$\prod_{n \to i} = \frac{1 - |\beta_n - \beta_i|}{\sum_l (1 - |\beta_n - \beta_l|)} = \frac{\tau_{ni}}{\sum_l \tau_{nl}} \tag{5}$$

where function $\sum_l \tau_{nl}$ represents the sum of correlation degree between node n and any other nodes in the whole network.

3) Weighted Dynamics: The weight of each new edge (n, i) is initially set to a predefined value w_0. For the sake of simplicity, we limit the weight evolving condition to the case where the introduction of a new link on node i will trigger only local rearrangements of weights on node i's existing neighbors $j \in \tau(i)$, According to the following rules:

$$w_{ij} \to w_{ij} + \Delta w_{ij} \tag{6}$$

$$w_{ij} = \delta_i \cdot \frac{w_{ij}}{s_i} \tag{7}$$

constant δ_i represents the extra information flow of node i, which is brought by the new edge (n, i). Since the connected edge will share some flow according to w_{ij}, so the strength of node i will change dynamically according to the following rule:

$$s_i \to s_i + w_0 + \delta_i \tag{8}$$

IV. Theoretical analysis

When a new node is added into the network, the strength of an existing node i in the network might be affected in the following two cases: (1) a new edge connects to node i directly; (2) a new edge connects to one of node i's neighbors.

The weight of each new edge is fixed to $w_0 = 1$. The evolution equation for $s_i(t)$ is thus given by

$$\frac{ds_i}{dt} = \alpha \cdot m \cdot \frac{s_i}{\sum_l s_l} \cdot (1+\delta) + \alpha \cdot \sum_{j \in v(i)} m \cdot \frac{s_i}{\sum_l s_l} \cdot \delta \frac{w_{ij}}{s_i} + (1-\alpha) \cdot m \cdot \frac{\tau_{ni}}{\sum_l \tau_{nl}} \cdot (1+\delta) + (1-\alpha) \cdot \sum_{j \in v(i)} m \cdot \frac{\tau_{ni}}{\sum_l \tau_{nl}} \cdot \delta \frac{w_{ij}}{s_i} \tag{9}$$

The new edge increases the total strength of the whole network by an amount equal to $2 + 2\delta$, implying that $\sum_l s_l \approx 2m(1+\delta)t$. Because we fix the node self attribute $\beta \sim U(0, 1)$, the correlation degree also follows uniform distribution, thus implying that $\sum_l \tau_{nl} \approx \frac{1}{2} \cdot m \cdot t$, and

$$\frac{ds_i}{dt} = \alpha \cdot \frac{2\delta+1}{2\delta+2} \cdot \frac{s_i}{t} + 2(1-\alpha) \cdot (\delta+1) \cdot \frac{\tau_{ni}}{t} \tag{10}$$

So we have

$$\frac{ds_i}{dt} = A \cdot \frac{s_i}{t} + B \cdot \frac{\tau_{ni}}{t} \tag{11}$$

where $A = \alpha \cdot \frac{2\delta+1}{2\delta+2}$, and $B = 2(1-\alpha)(\delta+1)$. With the initial condition $s_i(t = i) = m$, we can integrate (10) to obtain

$$s_i(t) = (m + \frac{B\tau_{ni}}{A}) \cdot (\frac{t}{i})^A - \frac{B\tau_{ni}}{A} \tag{12}$$

The time evolution equation for k_i is:

$$\frac{dk_i}{dt} = (1-\alpha) \cdot m \cdot \frac{\tau_{ni}}{\sum_l \tau_{nl}} + \alpha \cdot m \cdot \frac{s_i}{\sum_l s_l} \tag{13}$$

We can obtain

$$k_i(t) \approx \frac{\alpha}{2(2\delta+1) \cdot A} s_i = \frac{\delta+1}{(2\delta+1)^2} s_i \tag{14}$$

Considering all these above, k_i and s_i are considered to be linear.

We set t_i to be the time that a node i enters the network, so the probability $P(s_i(t) < s)$ that a node's strength $s_i(t)$ is smaller than s can be written as

$$\begin{aligned} P(s_i(t) < s) &= P\{t_i > t(\frac{mA + B\tau_{ni}}{sA + B\tau_{ni}})^{\frac{1}{A}}\} \\ &= 1 - P\{t_i \leq t(\frac{mA + B\tau_{ni}}{sA + B\tau_{ni}})^{\frac{1}{A}}\} \end{aligned} \tag{15}$$

Then the probability density of $P(s)$ can be given by

$$P(s_i) = \frac{\partial P(s_i < s)}{\partial s} = \frac{t}{m_0 + t} \cdot \frac{(mA + B\tau_{ni})^{\frac{1}{A}}}{(sA + B\tau_{ni})^{1+\frac{1}{A}}} \tag{16}$$

When $t \to \infty$, $P(s) \sim s^{-\gamma}$ where $\gamma = 1 + \frac{1}{A}$, $A = \alpha \cdot \frac{2\delta+1}{2\delta+2}$. We can conclude that the node strength follows the power-law distribution whose scaling exponent varies from 2 to 3 as $\frac{2}{3} \leq \alpha \leq 1(\delta = 1)$.

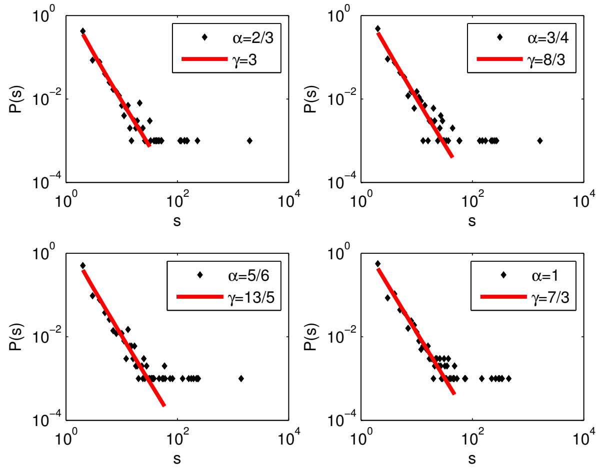

Fig. 1. Node strength distribution $P(s)$ of the network with parameter $\frac{2}{3} \leq \alpha \leq 1$.

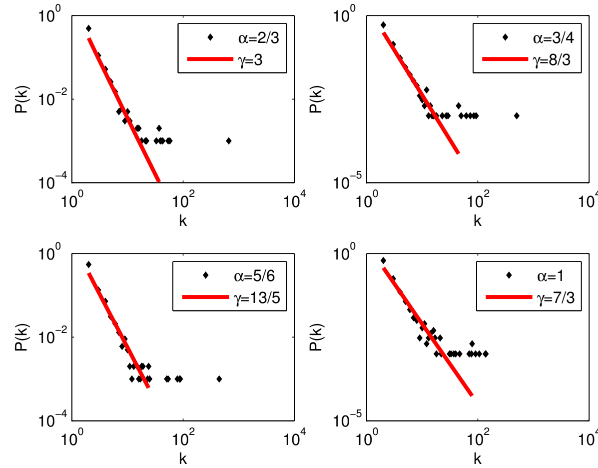

Fig. 2. Degree distribution $P(k)$ of the network with parameter $\frac{2}{3} \leq \alpha \leq 1$.

Likewise, $P(k) \sim k^{-\gamma}$ where $\gamma = 1 + \frac{1}{A}$, $A = \alpha \cdot \frac{2\delta+1}{2\delta+2}$. We can conclude that the node degree follows the power-law distribution whose scaling exponent varies from 2 to 3 as $\frac{2}{3} \leq \alpha \leq 1 (\delta = 1)$.

As we can see from the above theoretic analysis, node strength and degree of the network both follow the power-law distribution with an exponent $\gamma \in [2, 3]$ as $\frac{2}{3} \leq \alpha \leq 1 (\delta = 1)$. In particular, when $\alpha = 1$, the network evolving condition is exactly same as the BBV model [18]. This kind of scale-free property has been discovered in many real-world networks, so our analytic results here have demonstrated the practicality of our proposed model.

V. Numerical simulation

In order to verify the validity of the obtained analytical predictions, we performed extensive numerical simulations of networks generated by proposed model with a different value of parameter α. In the simulation, we fix $\delta_i = \delta = 1$, $w_0 = 1$, $m_0 = 5$, $m = 2$, $N = 1000$ and $\beta \in U(0, 1)$.

A. Node degree distribution and strength distribution

In the simulation, we fix $\alpha = \frac{2}{3}$, $\alpha = \frac{3}{4}$, $\alpha = \frac{5}{6}$ and $\alpha = 1$ respectively.

As shown in Fig. 1 and Fig. 2, both node strength and degree follow the power-law distribution with an exponent $\gamma \in [2, 3]$ when $\frac{2}{3} \leq \alpha \leq 1 (\delta = 1)$, the red lines in figures represent the scaling exponent of the power-law distribution. The numerical simulations are coherent with the results from theoretical analysis in section 3 and consistent with the statistical results of many real-world networks [30], [31].

B. Linear correlation between node degree and strength

In the simulation, parameter α is assign to $\frac{2}{3}$, $\delta = 0.5$, $\delta = 1$, $\delta = 2$ and $\delta = 5$ are fixed respectively to experiment the linear relationship between node degree and strength.

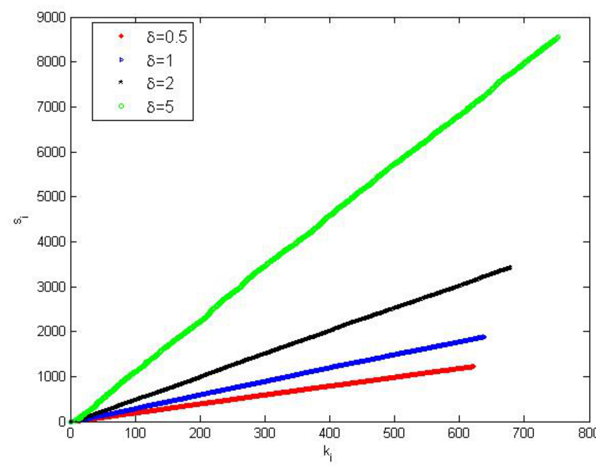

Fig. 3. Strength s_i versus k_i with different values of δ.

As we can see from Fig. 3, no matter what value of parameter δ is, node degree k_i and strength s_i are always linear, which is same as the result from theoretical analysis.

C. Clustering coefficient and average shortest path length

In the simulations, we fix $\alpha = \frac{2}{3}$, $\alpha = \frac{5}{6}$ and $\alpha = 1$ respectively. The values of average clustering coefficient and average shortest path length of the network are calculated by averaging over 20 independent runs.

Clustering coefficient C is often used for the characterization on the correlation degree between nodes in the network. The clustering coefficient of one given node i is defined as the ratio between existing and potential numbers of neighboring connections of node i. The formula for calculating the clustering coefficient of node i in the network can be defined as:

$$C_i = \frac{2E_i}{k_i(k_i - 1)} \tag{17}$$

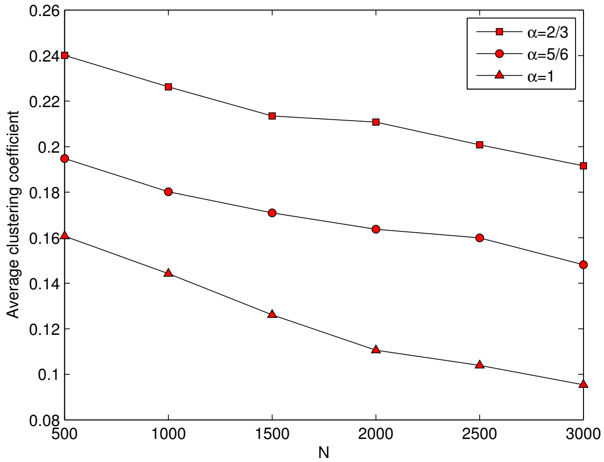

Fig. 4. Average clustering coefficient of the network as parameter $\frac{2}{3} \leq \alpha \leq 1$.

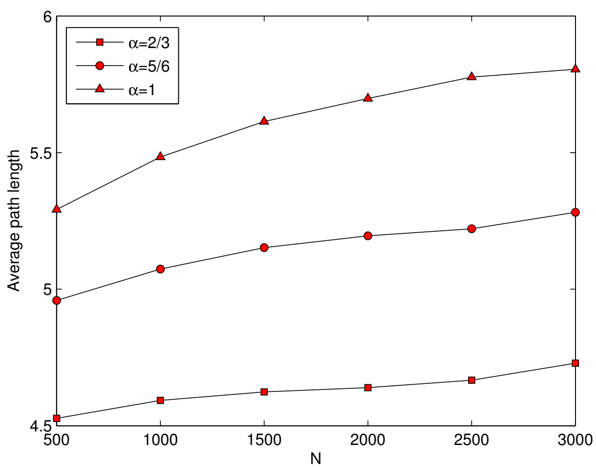

Fig. 5. Average path length of the network as parameter $\frac{2}{3} \leq \alpha \leq 1$.

where parameter k_i represents the number of node i's neighbors, and E_i represents the number of existing connections between node i and its neighbors. The average clustering coefficient C is the mean value of all the nodes' clustering coefficient in the network.

Average shortest path length is another significant property for evaluating the distance between nodes. The distance d_{ij} denotes the length of shortest path between node i and node j. The average shortest path length L is defined as the average length of the shortest paths between any pair of two nodes in the network.

$$L = \frac{1}{\frac{1}{2}N(N+1)} \sum_{i \geq j} d_{ij} \tag{18}$$

where parameter N is the total nodes number of the network.

From Fig. 4 and Fig. 5, we can see that the values of average clustering coefficient C and average shortest path length L both change as a function of network size. While the number of nodes grows, the former values decrease and the latter ones increase, which just shows the small-world property of the network. The average clustering coefficient C decreases as the value of parameter α increases, that is, the smaller parameter α is, the easier nodes will cluster together. As we see from the topological growth rule of our model, when the parameter α becomes smaller, the weight of the correlation between nodes in the evolutionary process becomes greater. When the maximum value (the value is 1) is fixed to α, node strength is the only factor to be considered in the network evolution rule, and the network changes into the BBV network. In other words, the dual assessment of both node strength and correlation makes nodes be more likely to get clustered together, which leads to a higher probability to cluster, a better connectivity of the whole network. On the contrary, with the increase of parameter α, average path length increase distinctly because of the lower clustering degree between the network nodes.

From the above simulation results, we conclude that node strength distribution $P(s)$ and degree distribution $P(k)$ of the network both exhibit a power-law behavior as $\frac{2}{3} \leq \alpha \leq 1$. By adjusting parameters α, we can adjust the weights of node strength and correlation in evaluating rules. As demonstrated by the numerical simulations, the smaller parameter α is, and the easier nodes can get clustered together, the higher propagation velocity network has.

VI. Conclusion

In this paper, we propose a weighted network model based on the correlations between nodes which takes the node strength and node correlation into consideration during the network evolution, and the weights of these two evolving assessment factors are adjusted by parameter α. As we can see from the both theoretical analysis and numerical simulations, the network model shows the scale-free property and the small-world property that are observed in many real-world networks. Moreover, we have demonstrated that the dual assessment of node strength and node correlation leads to a higher propagation velocity of the whole network. In a word, the weighted network model we introduced in this paper better reveals the evolving mechanisms behind various real-world networks than many existing models.

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