# Numerical Simulation of Stepwise Deployment of Membrane Structure with Booms Using Multi-Particle Approximation Method 

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#### Abstract

Deployable membrane structures are hopeful to develop future lightweight large space structures. In order to predict the dynamic behaviors of the structures in the preliminary design phase, simple and fast numerical simulation is necessary. For this purpose, multi-particle approximation method has been studied which models membranes with spring-mass-damper systems. In this study, polygonal membrane structures integrated with extendible booms are investigated. The membranes are deployed by the elasticity of the booms in a stepwise manner. A multi-particle model for one-dimensional elastic body is introduced as the boom model to the multi-particle method. Each boom is released from the tip end by several particles and the dynamic behaviors of the stepwise deployment can be obtained. The behaviors are compared with deployment experiments of booms without a membrane. Numerical simulation of the deployment of a hexagonal membrane with booms is also demonstrated. Finally, the effect of the stepwise pattern on the vibration motion of the central body due to deployment is studied.


Key Words: Membrane, Boom, Stepwise Deployment, Simulation, Multi-Particle Method

## 1. Introduction

Deployable membrane structures are important to develop high efficient large space structures in the present and near future. Membrane structures are essentially lightweight and flexible compared with conventional deployment structures which use panels and hinges. Since predicting the dynamic behaviors is important to design membrane structures, some simple and fast numerical simulation method is necessary to find appropriate various design parameters in the preliminary design phase.

In order to study basic dynamics of membrane structures, a multi-particle approximation method has been developed which simulates thin membrane with a spring-mass-damper system. The method has been validated by performing numerical simulations of centrifugal deployments of folded membranes and comparing with on-ground small-scale experiments in vacuum ${ }^{1,2)}$ as well as on-orbit behaviors of the solar power sail of "IKAROS". ${ }^{3)}$ The centrifugal deployment dynamics of the square solar sail with 14 meters in width was mainly investigated by multi-particle method and the membrane was successfully deployed in space.

On the other hand, the developments of lightweight membrane structures mounted on spacecrafts and deployed with booms, such as solar sails, ${ }^{4}$ de-orbit sails, ${ }^{5}$ ) sun shields, thin film solar arrays, and so on, is an important subject at present. Incorporating elastic elements in the multi-particle method is necessary to study their deployment dynamics.

In this study, deployable polygonal membranes stowed with modified spiral folding and integrated with extendible booms
are studied. ${ }^{5)}$ The membrane structures are deployed by the elasticity of the booms. The booms are rolled up around a central body and released from the tip ends in a stepwise manner. The boom used here is a braided bi-convex boom (BCON boom). ${ }^{6,7)}$ A multi-particle model for onedimensional elastic continuum ${ }^{8)}$ is introduced as the boom model. Numerical simulations of the deployment of booms are performed for two cases: (i) central body is fixed on the ground, (ii) central body is rotatable around its central axis. The numerical results are compared with the experimental results. ${ }^{99}$ Then the numerical simulation of the deployment of a hexagonal membrane with rolled-up booms is demonstrated. Finally, the effect of stepwise deployment pattern on the vibration of central body due to deployment is studied.

## 2. Stepwise Deployment of Membrane Structure with Rolled-Up Booms

Fig. 1(a) shows a hexagonal membrane with regular spiral folding pattern. Solid and dashed lines represent mountain folding lines and valley folding lines, respectively. Folding lines are slightly curved to deal with membrane thickness effects and the membrane can be folded up to be a cylindrical shape. The lines become straight if the membrane thickness is almost zero. In the regular spiral folding, diagonal lines are folded in a zigzag manner and rolled up around a central hub. Fig. 1(b) shows a modified spiral folding pattern. Peripheral parts are cut from the hexagonal membrane with regular folding pattern. In this case, diagonal lines are just rolled up in stowed configuration and elastic booms can be arranged along


Fig. 1. Hexagonal membrane with spiral folding pattern.


Fig. 2. Three stepwise deployment of boom.
the diagonal lines and the deployment can proceed through the release of the stowed elastic energy of the booms.

Fig. 2 illustrates a stepwise deployment of a boom. To realize simple deployment, the booms and membranes are fixed to the spacecraft using several fixing mechanisms, and they are released in a stepwise manner from the tip ends.

## 3. Multi-Particle Model for Booms and Membranes

### 3.1. Boom

An elastic boom model is incorporated in multi-particle model in this study. A spring-mass model for one-dimensional elastic continuum was proposed based on Cosserat theory and soliton theory and was simplified by applying Euler-Bernoulli theory. ${ }^{7}$ ) The model expresses elastic continuum such as strings and beams by a one-dimensional spring-mass system and can describe elongation and contraction, bending and torsion of the continuum. In this study, out-of-plane bending and torsion are ignored because they are not dominant compared with in-plane bending motion.
Fig. 3 shows the spring-mass model. $\boldsymbol{N}_{n}$ represents an axial force vector due to elongation and contraction and $\boldsymbol{Q}_{n}$ represents a shear force vector due to bending, which act on mass $n$. The mechanics of the system is described by using local coordinate system $\boldsymbol{x}_{n}-\boldsymbol{y}_{n}-\boldsymbol{z}_{n}$ put along a spring element as shown in Fig. 4. The forces are described as follows:


Fig. 3. Spring-mass model for one-dimensional continuum.


Fig. 4. Local coordinate system.


Fig. 5. Bending moment and angle.

$$
\begin{gather*}
\boldsymbol{N}_{n}=E A \frac{l_{n}-l_{0}}{l_{0}} \boldsymbol{x}_{n}  \tag{1}\\
\boldsymbol{Q}_{n}=\frac{\left(\boldsymbol{M}_{n+1}-\boldsymbol{M}_{n}\right) \times \boldsymbol{x}_{n}}{l_{n}}  \tag{2}\\
\boldsymbol{M}_{n}=E I \frac{\phi_{n}-\phi_{0}}{l_{0}} \frac{\boldsymbol{x}_{n-1} \times \boldsymbol{x}_{n}}{\left|\boldsymbol{x}_{n-1} \times \boldsymbol{x}_{n}\right|}  \tag{3}\\
\boldsymbol{F}_{n}=\boldsymbol{N}_{n}-\boldsymbol{N}_{n-1}-\boldsymbol{Q}_{n}+\boldsymbol{Q}_{n-1} \tag{4}
\end{gather*}
$$

where $\boldsymbol{M}_{n}$ denotes a bending moment vector around mass $n$ when the angle between two adjacent springs around $z$ axis is $\phi_{n}$ as shown in Fig. 5. $\boldsymbol{F}_{n}$ denotes a total force vector on mass n. $E, A, l_{n}, l_{0}, I, \phi_{0}$ denote Young's modulus, section area, and current and natural lengths of the spring, second moment of area of the boom and natural angle between two springs, respectively.

In order to apply damping to the boom motion, dashpots are arranged between two masses whose forces are proportional to their relative velocity. Velocity proportional dampers are also introduced to the lateral motion of the mass. The equations of motion of the spring-mass-damper system of the boom can be described in a straightforward way.

### 3.2. Membrane

A multi-particle model for thin membrane is summarized here. A triangular membrane element shown in Fig. 6 is used. ${ }^{11)} E, \rho, h$ and $S$ represent Young's modulus, density, thickness and area of the element, respectively and $L_{i}(i=1,2,3)$ denote lengths of the sides of the triangle. The mass of the element $\rho S$ is equally distributed to the lumped masses $m_{1}, m_{2}$ and $m_{3}$. The spring constants $k_{i}$ are determined so that strain energies of the membrane and the spring-mass system coincide when the element is in one-axis stress states parallel to three sides and they are obtained by Eq. (5).

$$
\left(\begin{array}{l}
k_{1}  \tag{5}\\
k_{2} \\
k_{3}
\end{array}\right)=B_{i j}^{-1}\left(\begin{array}{l}
1 \\
1 \\
1
\end{array}\right), \quad B_{i j}=\frac{p_{i j}^{2} L_{j}^{2}}{E h S}, \quad p_{i j}=1-\frac{4(1+v) S^{2}}{L_{i}^{2} L_{j}^{2}}\left(1-\delta_{i j}\right),
$$

where $v$ and $\delta_{i j}$ denote Poisson's ratio and Kronecker delta, respectively.

The restoring forces of the spring is assumed to become constant when the length of the spring is less than a critical value $l_{\text {cr }}$ to simulate buckling of thin membrane as shown in Fig. 7. The critical value $l_{\text {cr }}$ is expressed using Euler buckling


Fig. 6. Triangular element.



Table 2. Step time and length.

| Step | Start <br> time | Deployment <br> Length |
| :---: | ---: | ---: |
| 1 | 0 sec | 215 mm |
| 2 | 0.6 sec | 340 mm |
| 3 | 2.3 sec | 271 mm |

Fig. 8. Experiment model of hexagonal body.

Fig. 7. Buckling property.
Table 1. Properties of experiment model.

| Central <br> body | Radius | 65 mm |
| :--- | :--- | :---: |
|  | Mass | 1.2 kg |
|  | Moment of inertia | $2.05 \mathrm{E}-3 \mathrm{~kg} \mathrm{~m}$ |
| Boom | Length | 826 mm |
|  | Width | 16 mm |
|  | Thickness | 5.5 mm |
|  | Density | $3178 \mathrm{~kg} / \mathrm{m}^{3}$ |
|  | Section area | $7.52 \mathrm{E}-6 \mathrm{~m}^{2}$ |
|  | Second moment of area | $2.15 \mathrm{E}-12 \mathrm{~m}^{4}$ |



Fig. 9. Braided bi-convex boom (BCON boom).
strength of slender column as:

$$
\begin{equation*}
l_{c r}=L_{i}-\alpha \frac{\pi^{2} h^{2}}{12 L_{i}} \tag{6}
\end{equation*}
$$

where $\alpha$ is a parameter to tune buckling strength to actual membranes.

Velocity proportional dampers parallel to the springs between two masses are also introduced.

### 3.3. Central body

In the case that central body on which the deployable structure is attached can rotate, the rotation is coupled with the deployment dynamics. In this study, the central body is assumed to be a rigid body and have only a rotational degree of freedom around its central axis. The moment of inertia of boom masses not yet released is added to that of the central body. The motion is computed taking account of the conservation of total angular momentum. The friction torque of the bearing is considered. The contact between particles and the central body is also taken into account by applying penalty springs in radial direction when the radius of a particle becomes smaller than its initial radius.

## 4. Experiments and Corresponding Simulation Model

### 4.1. Deployment experiments

In order to observe deployment behavior of the deployable structure and to create an appropriate simulation model, deployment experiments has been carried out. ${ }^{10)}$ Experiments of a six boom model without a membrane were first conducted. Fig. 8 shows the pictures of the model. The properties are summarized in Table 1. Six extendible booms are rolled up


Fig. 10. Bending stiffness.
around a central body which is equipped with a stepwise deployment mechanism. The central body can rotate on a bearing around its central axis. The overview of the boom is shown in Fig. 9. The boom consists of two uni-convex tapes covered by a braid sleeve and has a lenticular section, which is referred to as "BCON boom". ${ }^{\text {. }}$ When the boom is stowed, convex tapes can be flattened and slide each other inside the braid and can be easily rolled up around the central body.

In order to measure the bending property of the boom, three-point bending test was conducted using a specimen boom with 100 mm in length. The blue line in Fig. 10 shows the bending stiffness with respect to bending angle obtained by the measurement.

The deployment experiments were conducted for two cases: (i) central body was fixed and (ii) central body was rotatable. The number of steps of stepwise deployment was three. The start times and deployment lengths of three step are given in Table 2.

### 4.2. Simulation model

The multi-particle model for deployment simulation is illustrated in Fig. 11. The booms are rolled up around the hexagonal body taking account of thickness in stowed configuration. The number of particles of each boom is 32 . The bending stiffness $E I$ of the boom is assumed to be a function of bending angle $\phi$ and is approximated from the result of bending test as follows:


Fig. 11. Multi-particle model of six boom model.

$$
E I= \begin{cases}\left\{-1.68 \times\left(|\phi|-\frac{\pi}{9}\right)+3.8\right\} \times 10^{-3} & \left(|\phi| \geq \frac{\pi}{9}\right)  \tag{7}\\ 0.307 \times\left(|\phi|-\frac{\pi}{9}\right)^{2}+3.48 \times 10^{-3} & \left(\frac{\pi}{30} \leq|\varphi|<\frac{\pi}{9}\right) . \\ 0.0879 & \left(|\phi|<\frac{\pi}{30}\right)\end{cases}
$$

The red line in Fig. 10 shows this function. In this study, the damping ratio of the vibration of the boom is 0.035 based on a vibration test of the beam in cantilever configuration. ${ }^{7}$

To perform the stepwise deployment, first all the masses are fixed on the central body then appropriate number of masses are released from the tip ends in three steps. In the case that the central body can rotate, the friction coefficient of the bearing and its effective radius assumed here are 0.001 and 10 mm , respectively.

## 5. Numerical Results

### 5.1. Comparison with experiments

Numerical results for the case that the central body is fixed is presented first. Fig. 12 shows snapshots of the stepwise deployment behavior. Figs. 12 (a)-(f), (g)-(l) and (m)-(u) show the first, second and third steps, respectively. The scales of the figures are different for three steps. Time history of the radius of tip ends of the booms is shown with the experimental result

(d) 0.3 sec

(j) 1.2 sec

(m) 2.3 sec

(p) 2.8 sec


(b) 0.1 sec

(e) 0.4 sec

(h) 0.8 sec

(k) 1.4 sec

(n) 2.4 sec

(q) 3.0 sec

(t) 3.6 sec

(c) 0.2 sec

(f) 0.5 sec

(i) 1.0 sec

(1) 1.6 sec

(o) 2.6 sec

(r) 3.2 sec

(u) 3.8 sec

Fig. 12. Snapshots of deployment simulation (central body fixed) (a)-(f): 1st step, (g)-(1): 2nd step, (m)-(u): 3rd step.
in Fig. 13. The numerical simulation can qualitatively depict the stepwise deployment of the booms by their elasticity as expected. It is observed that overshoots arise just after the start of steps and that transient vibrations follow. The deployment and transient vibrations in the simulation is faster than the experimental result.

Fig. 14 and 15 show the snapshots and the time histories of deployment radius for the case that the central body can rotate. Numerical results can explain the deployment behavior qualitatively. In this case, the booms unroll quicker and deformations during deployment become larger than the previous case. It is noted that the unrolling starts from the inside of the booms and that contacts and passing between booms are observed in Figs. 14 (h) and (n)-(o). This is because the central body rotates in the opposite direction of the unrolling to assists deployment. Time histories of rotation angle and angular velocity of the central body are shown in


Fig. 13. Time history of radius of tip ends (central body fixed).


Fig. 14. Snapshots of deployment simulation (central body rotatable) (a)-(f): 1st step, (g)-(l): 2nd step, (m)-(u): 3rd step.


Fig. 15. Time history of radius of tip ends(central body rotatable).


Fig. 16. Time history of rotation angle of central body.


Fig. 17. Time history of angular velocity of central body.
Fig. 16 and 17, respectively. Since the booms are unrolled in clockwise direction, the central hub rotates in anticlockwise direction and transient vibration occurs. In the simulation, the vibration is severer and the frequency of it is higher than the experiment. On the other hand, rotation angles converged in three steps are close to the experimental result.

In order to improve the accuracy of the simulation, further investigations of the characteristics of BCON boom will be necessary. Probably, the main reason of the difference between simulations and experiments is the damping. The velocity proportional damping used in this study may not be sufficient because the friction among the convex tapes and braid sleeve may produce other kind of damping effect. Besides, the fixed points of the booms in the first and second steps are restrained by the deployment mechanism in the experiment while they are strictly fixed in the simulation. The contact between booms also needs to be considered.

### 5.2. Deployment of booms with membrane

Numerical simulation of the stepwise deployment of the booms with a hexagonal membrane is conducted. Fig. 18 shows the multi-particle model of the hexagonal membrane with the booms. The folding lines are the modified spiral folding pattern mentioned above. The properties of the membrane model are summarized in Table 3. The membrane and booms are connected with short springs at five points per a boom as indicated by black circles in Fig. 18. Since the thickness of the boom is much larger than that of the membrane, it is difficult to strictly connect them in stowed


Table 3. Properties of membrane model.

| Outer radius | 844 mm |
| :--- | :---: |
| Inner radius | 65 mm |
| Thickness | $25 \mu \mathrm{~m}$ |
| Number of particles | 1386 |
| Number of springs | 4008 |
| Number of elements | 2622 |
| Young's modulus | 3.1 GPa |
| Poisson's ratio | 0.34 |
| Density | $1420 \mathrm{~kg} / \mathrm{m}^{3}$ |

Fig. 18. Multi-particle model of hexagonal membrane with booms.
configuration. Figs 19 and 20 illustrate snapshots of the simulation result and the time history of the deployment radius of the boom, respectively. The central body is fixed and the stepwise pattern is the same as the experiment. The stepwise deployment of the membrane by the elasticity of the booms is clearly demonstrated although the deployment behavior is asymmetric. It should be noted that the deployment radius shrinks instantaneously in the beginning of the third step. Fig. 19(n) corresponds to the shrink. This behavior was also found in the boom only model and the drop becomes severer when the membrane is attached. Verification by deployment experiments of a membrane with booms will be necessary. Besides, the membrane is not fully stretched after deployment because they are connected loosely with short springs.


Fig. 19. Snapshots of deployment simulation with membrane (central body fixed) (a)-(f): 1st step, (g)-(l): 2nd step, (m)-(u): 3rd step.


Fig. 20. Time history of radius of tip ends(with membrane, central body fixed).

### 5.3. Study on stepwise deployment pattern

Considering that the deployment structure is attached on a spacecraft, severe vibration of the central body due to the unrolling deployment is undesirable. The stepwise deployment pattern can be arranged to reduce the vibration. Fig. 21 illustrates numerical results of the angular velocities of the central body with three different patterns. The number of steps is three and the time interval of each step is one second. The release lengths of the steps of three patterns are listed in Table 4. In case $A$ where the third step is the longest, the peak amplitudes of the first and second steps are small but that of the last step becomes large. In case B where the first step is the longest, the peak amplitudes of the first step is large. In case $C$, the release lengths of three steps are arranged to




Fig. 21. Angular velocity variations by three stepwise patterns.
Table 4. Stepwise deployment patterns.

|  | Case A | Case B | Case C |
| :---: | :---: | :---: | :---: |
| 1st step | 220 mm | 430 mm | 342 mm |
| 2nd step | 209 mm | 214 mm | 326 mm |
| 3rd step | 397 mm | 183 mm | 158 mm |

equalize the peak amplitudes of three steps. From these results, it is found that the release length of the first step is the most effective on the vibration motion. Although significant difference is not observed in these results, case C seems moderate compared with other patterns including the experiment.

## 6. Conclusions

A spring-mass model for one-dimensional elastic continuum was incorporated to the multi-particle model for thin membrane and the stepwise deployment behavior of a deployable membrane structure with elastic booms was studied. Numerical simulations of deployment experiments of six braided bi-convex booms without membrane were first performed and the validity of the simulation was examined. The deployment simulation of a hexagonal membrane with six booms was also demonstrated and the effect of stepwise deployment pattern on the rotation of central body was studied. In order to further improve the numerical simulation, damping property of the boom and contacts among the booms, membrane and central body will be important subjects. The asymmetric deployment and the stability of simulation when the central body is rotatable also seems to be examined.

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