ハノイ市における繊維質材料混合流動化処理土の埋戻し地盤への適用に関する研究

Appendix C
Simulation of moving train load with velocity of 60 km/h by Newmark numerical method

I. Inputs

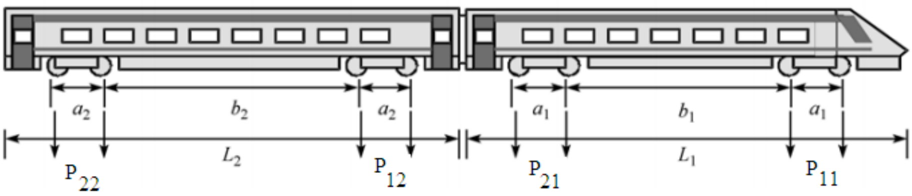
1. Parameters of metro train

Distance from analysis point to the first axial of train: $\mathbf{Lo} := 3\mathbf{m}$

Speed of train: $\mathbf{v_t} := 60\mathbf{kph}$

Total number of cars: $\mathbf{Ncar} := 6$

Geometry parameters of train



Car length:

$\mathbf{L_{car_1}} := 24\mathbf{m}$ $\mathbf{L_{car_2}} := 24\mathbf{m}$ $\mathbf{L_{car_3}} := 24\mathbf{m}$ $\mathbf{L_{car_4}} := 24\mathbf{m}$ $\mathbf{L_{car_5}} := 24\mathbf{m}$ $\mathbf{L_{car_6}} := 24\mathbf{m}$

Fixed axle spacing:

$\mathbf{a_{car_1}} := 2\mathbf{m}$ $\mathbf{a_{car_2}} := 2\mathbf{m}$ $\mathbf{a_{car_3}} := 2\mathbf{m}$ $\mathbf{a_{car_4}} := 2\mathbf{m}$ $\mathbf{a_{car_5}} := 2\mathbf{m}$ $\mathbf{a_{car_6}} := 2\mathbf{m}$

Axle spacing between 2 bogie:

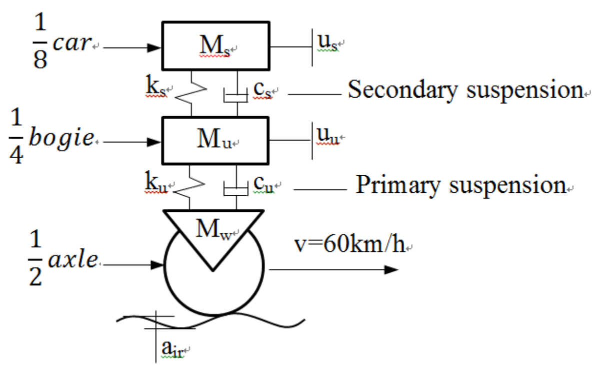
$\mathbf{b_{car_1}} := 16\mathbf{m}$ $\mathbf{b_{car_2}} := 16\mathbf{m}$ $\mathbf{b_{car_3}} := 16\mathbf{m}$ $\mathbf{b_{car_4}} := 16\mathbf{m}$ $\mathbf{b_{car_5}} := 16\mathbf{m}$ $\mathbf{b_{car_6}} := 16\mathbf{m}$

2. Parameters of cars in the analysis model

Car body mass (1/8 car): $\mathbf{m_s} := \frac{32000}{8}\mathbf{kg}$

Bogie mass (1/4 bogie): $\mathbf{m_u} := \frac{5540}{4}\mathbf{kg}$

Wheel mass (1/2 axle): $\mathbf{m_w} := \frac{1503}{2}\mathbf{kg}$



Car mass in total:

$$\mathbf{M_{car}} := 8\cdot(\mathbf{m_s} + \mathbf{m_u} + \mathbf{m_w}) = 49092\,\mathbf{kg}$$

Wheel spring stiffness: $\mathbf{k_u} := 2360\frac{\mathbf{kN}}{\mathbf{m}}$

Bogie spring stiffness: $\mathbf{k_s} := 530\frac{\mathbf{kN}}{\mathbf{m}}$

Wheel damping coefficient: $\mathbf{c_u} := 78.4\frac{\mathbf{kN\cdot s}}{\mathbf{m}}$

Bogie damping coefficient: $\mathbf{c_s} := 90.2\frac{\mathbf{kN\cdot s}}{\mathbf{m}}$

Wheel diameter: $\mathbf{D_w} := 0.85\mathbf{m}$

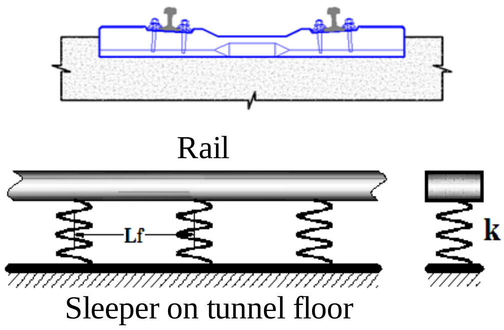
3. Parameters of track:

Sleeper spacing: $L_f := 0.65\,m$

Rail Pad stiffness: $K_{railpad} := 120\cdot 10^3\,\dfrac{kN}{m}$

Rail modulus: $E_r := 2.07\cdot 10^{11}\,\dfrac{N}{m^2}$

Inertia moment of one rail: $I_r := 30.55\cdot 10^{-6}\,m^4$



4. Parameters of wheel-rail irregularity

Types of irregularity that can be model including corrugation of rail, arbitrary wheel surface profile and wheel flat:

-) Corrugation rail surface, wavelength from 30mm to 300mm and 300mm to 1000mm, depth of irregularity from 0.01mm to 0.4mm:

$$L_{ir_1} := 200\,mm \quad a_{ir_1} := \begin{cases} 0\,mm & \text{if } 0\,mm < L_{ir_1} \le 30\,mm \\ 0.01\,mm & \text{if } 30\,mm < L_{ir_1} \le 100\,mm \\ 0.25\,mm & \text{if } 100\,mm < L_{ir_1} \le 300\,mm \\ 0.4\,mm & \text{if } 300\,mm < L_{ir_1} \le 1000\,mm \\ 0 & \text{otherwise} \end{cases}$$

Frequency from: $\dfrac{v_t}{3000\,mm} = 5.556\cdot Hz$ to: $\dfrac{v_t}{200\,mm} = 83.333\cdot Hz$

-) Arbitrary wheel surface or wheel flat with wavelength from 0.2 to 1 time of circumference of wheel, depth of irregularity 1mm:

Wavelength from: $0.2\pi\cdot D_w = 0.534\,m$ to: $1\pi\cdot D_w = 2.67\,m$

$$L_{ir_2} := 534\,mm \quad a_{ir_2} := 1\,mm$$

Frequency from: $\dfrac{v_t}{\pi\cdot D_w} = 6.241\cdot Hz$ to: $\dfrac{v_t}{0.2\pi\cdot D_w} = 31.207\cdot Hz$

Wheel-rail irregularity equation, is vertical displacement of wheel:

$$u_{ir}(t) := \sum_{i=1}^{2}\left[\left(\frac{a_{ir_i}}{2}\right)\cdot\left(1-\cos\left(2\cdot\pi\cdot\frac{v_t}{L_{ir_i}}\cdot t\right)\right)\right]$$

Vertical vibration velocity of wheel:

$$v_{ir}(t) := \sum_{i=1}^{2}\left[\left(\frac{a_{ir_i}}{2}\right)\cdot\left(2\cdot\pi\cdot\frac{v_t}{L_{ir_i}}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{v_t}{L_{ir_i}}\cdot t\right)\right]$$

Vertical vibration acceleration of wheel:

$$w_{ir}(t) := \sum_{i=1}^{2}\left[\left(\frac{a_{ir_i}}{2}\right)\cdot\left(2\cdot\pi\cdot\frac{v_t}{L_{ir_i}}\right)^2\cdot\cos\left(2\cdot\pi\cdot\frac{v_t}{L_{ir_i}}\cdot t\right)\right]$$

5. Data export parameters:

Sampling frequency per 1 sec: $\mathbf{SRate} := 1000$

Sampling time: $\mathbf{Ttotal} := 9\mathrm{s}$ Choose: $\frac{1}{v_t}\cdot\left(2Lo + \sum_{i=1}^{Ncar} L_{car_i}\right) = 9\,\mathrm{s}$

Integral time step: $\Delta t := \frac{1\mathrm{sec}}{\mathrm{SRate}}$

Integral time step total: $n := \frac{\mathrm{Ttotal}}{\Delta t}$

Calculation index: $i := 0, 1 .. n$

$t := 0, \Delta t .. \mathrm{Ttotal}$

II. Calculation:

Newmark direct integration method:

1. Matrices of differential equation:

$$\mathrm{Msys} := \begin{pmatrix} m_s & 0 \\ 0 & m_u \end{pmatrix} \qquad \mathrm{Csys} := \begin{pmatrix} c_s & -c_s \\ -c_s & c_s + c_u \end{pmatrix} \qquad \mathrm{Ksys} := \begin{pmatrix} k_s & -k_s \\ -k_s & k_s + k_u \end{pmatrix}$$

$$R_i := \begin{pmatrix} 0 \\ c_u \cdot v_{ir}(i\cdot\Delta t) + k_u \cdot u_{ir}(i\cdot\Delta t) \end{pmatrix}$$

2. Condition at t=0, no vibration

$$u_i := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \mathrm{m} \qquad v_i := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \frac{\mathrm{m}}{\mathrm{s}} \qquad w_i := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \frac{\mathrm{m}}{\mathrm{s}^2}$$

3. Time step Δt, integral parameters Newmark and factors ai:

Choose: $\delta := 0.5$ $\alpha := 0.25$

$$a0 := \frac{1}{\alpha\cdot\Delta t^2} \qquad a2 := \frac{1}{\alpha\cdot\Delta t} \qquad a4 := \frac{\delta}{\alpha} - 1 \qquad a6 := \Delta t\cdot(1 - \delta)$$

$$a1 := \frac{\delta}{\alpha\cdot\Delta t} \qquad a3 := \frac{1}{2\alpha} - 1 \qquad a5 := \frac{\Delta t}{2}\cdot\left(\frac{\delta}{\alpha} - 2\right) \qquad a7 := \delta\cdot\Delta t$$

4. Effect stiffness matrix:

$$\mathrm{Keff} := \mathrm{Ksys} + a0\cdot\mathrm{Msys} + a1\cdot\mathrm{Csys}$$

5. Iterative calculation for time integral step to have u, v, w:

$i := 0, 1 .. n - 1$

$$\mathrm{Reff}_{i+1} := R_{i+1} + \mathrm{Msys}\cdot\left(a0\cdot u_i + a2\cdot v_i + a3\cdot w_i\right) + \mathrm{Csys}\cdot\left(a1\cdot u_i + a4\cdot v_i + a5\cdot w_i\right)$$

$$u_{i+1} := \mathrm{Keff}^{-1}\cdot\mathrm{Reff}_{i+1}$$

$$w_{i+1} := a0\cdot\left(u_{i+1} - u_i\right) - a2\cdot v_i - a3\cdot w_i$$

$$v_{i+1} := v_i + a6\cdot w_i + a7\cdot w_{i+1}$$

6. Displacement of Bogie and Car body:

$(u_i)_{(1,0)} =$

0
$7.019\cdot10^{-7}$
$1.296\cdot10^{-6}$
$1.683\cdot10^{-6}$
...

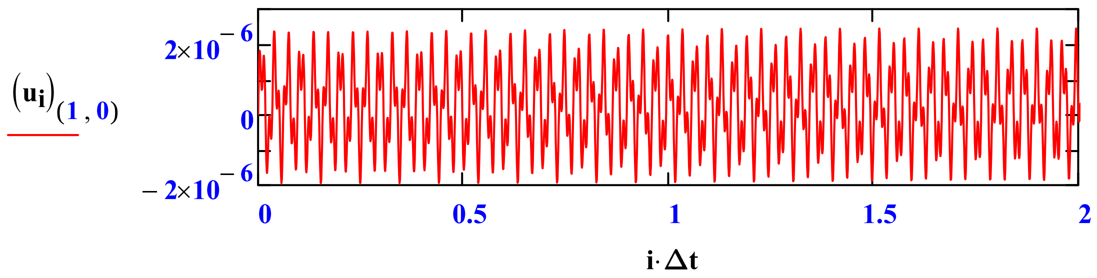
m

$(u_i)_{(0,0)} =$

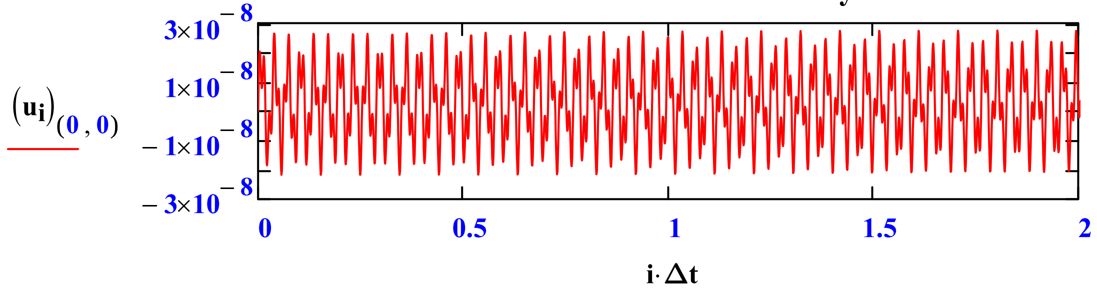
0
$7.848\cdot10^{-9}$
$1.449\cdot10^{-8}$
$1.881\cdot10^{-8}$
...

m

Vertical motion of bogie



Vertical motion of car body



7. Dynamic force on rail:

Car body acceleration $Wcar_i := (w_i)_{(0,0)}$

Bogie acceleration: $Wbogie_i := (w_i)_{(1,0)}$

Wheel acceleration: $Wwheel_i := w_{ir}(i\cdot\Delta t)$

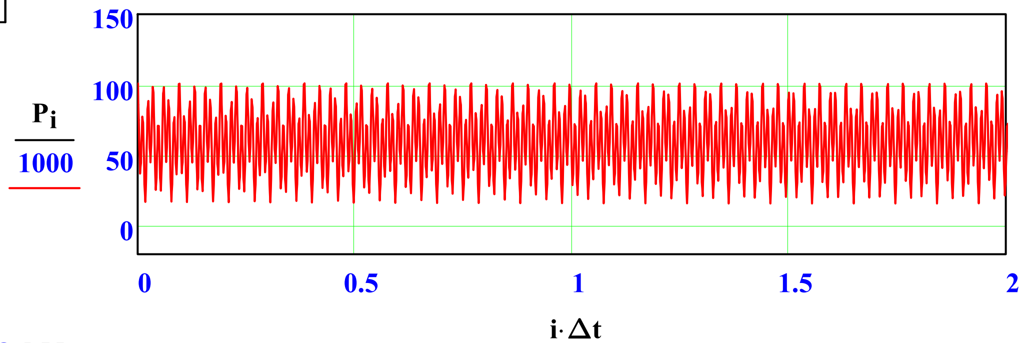
Dynamic force on rail: $P_i := (m_w + m_u + m_s)\cdot g + (m_w\cdot Wwheel_i + m_u\cdot Wbogie_i + m_s\cdot Wcar_i)$

$P =$

	0
0	100.382
1	100.669
2	85.793
3	75.024
4	...

$\cdot$kN

Dynamic force on 1 rail



$\max(P) = 100.938\cdot$kN

8. Loading ditribution function on tunnel floor

Bending stiffeness of rail: $EI := E_r \cdot I_r$

Stiffness of railpad: $K_{pad} := \dfrac{K_{railpad}}{L_f}$

Characteristic length of load distribution on tunnel floor through railpad: $\alpha_{rail} := \sqrt[4]{\dfrac{4 \cdot EI}{K_{pad}}}$

Loading ditribution function on tunnel floor through railpad:

$$\Phi_{rail}(x) := \frac{1}{2 \cdot \alpha_{rail}} \cdot e^{\frac{-|x|}{\alpha_{rail}}} \cdot \left(\cos\left(\frac{|x|}{\alpha_{rail}} \right) + \sin\left(\frac{|x|}{\alpha_{rail}} \right) \right)$$

9. Dynamic loading on tunnel floor:

$$\Theta(x) := \Phi_{rail}(x)$$

$$F_{1w}(t) := \sum_{i=1}^{Ncar} \left(P_{\mathrm{round}\left(\frac{\left| v_t t - Lo - \sum_{j=0}^{i-1} L_{car_j} \right|}{v_t \cdot \Delta t} \right)} \cdot \Theta\left(v_t t - Lo - \sum_{j=0}^{i-1} L_{car_j} \right) \right)$$

$$F_{2w}(t) := \sum_{i=1}^{Ncar} \left(P_{\mathrm{round}\left(\frac{\left| v_t t - Lo - \sum_{j=0}^{i-1} L_{car_j} - a_{car_i} \right|}{v_t \cdot \Delta t} \right)} \cdot \Theta\left(v_t t - Lo - \sum_{j=0}^{i-1} L_{car_j} - a_{car_i} \right) \right)$$

$$F_{3w}(t) := \sum_{i=1}^{Ncar} \left(P_{\mathrm{round}\left(\frac{\left| v_t t - Lo - \sum_{j=0}^{i-1} L_{car_j} - a_{car_i} - b_{car_i} \right|}{v_t \cdot \Delta t} \right)} \cdot \Theta\left(v_t t - Lo - \sum_{j=0}^{i-1} L_{car_j} - a_{car_i} - b_{car_i} \right) \right)$$

$$F_{4w}(t) := \sum_{i=1}^{Ncar} \left(P_{\mathrm{round}\left(\frac{\left| v_t t - Lo - \sum_{j=0}^{i-1} L_{car_j} - 2 \cdot a_{car_i} - b_{car_i} \right|}{v_t \cdot \Delta t} \right)} \cdot \Theta\left(v_t t - Lo - \sum_{j=0}^{i-1} L_{car_j} - 2 \cdot a_{car_i} - b_{car_i} \right) \right)$$

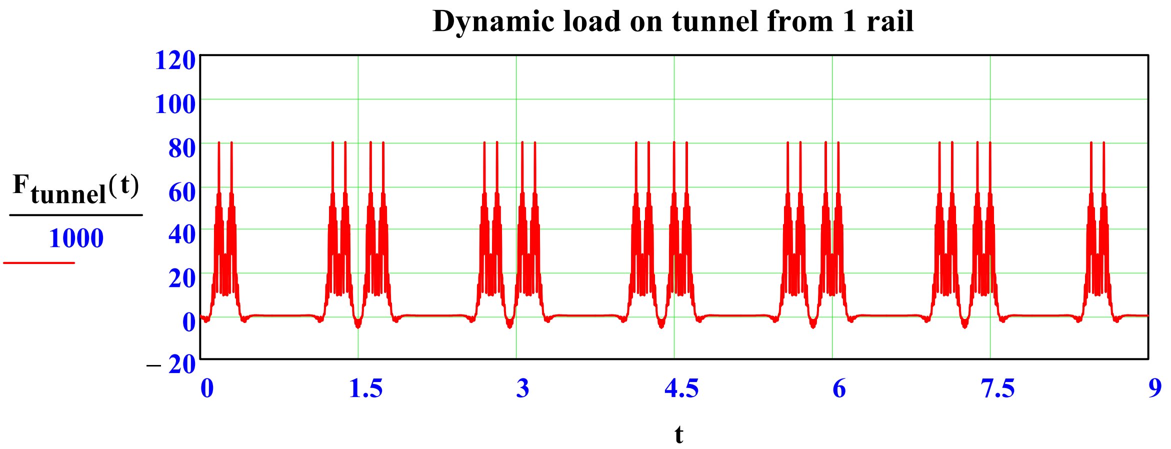
$$F_{tunnel}(t) := F_{1w}(t) + F_{2w}(t) + F_{3w}(t) + F_{4w}(t)$$

III. Results.

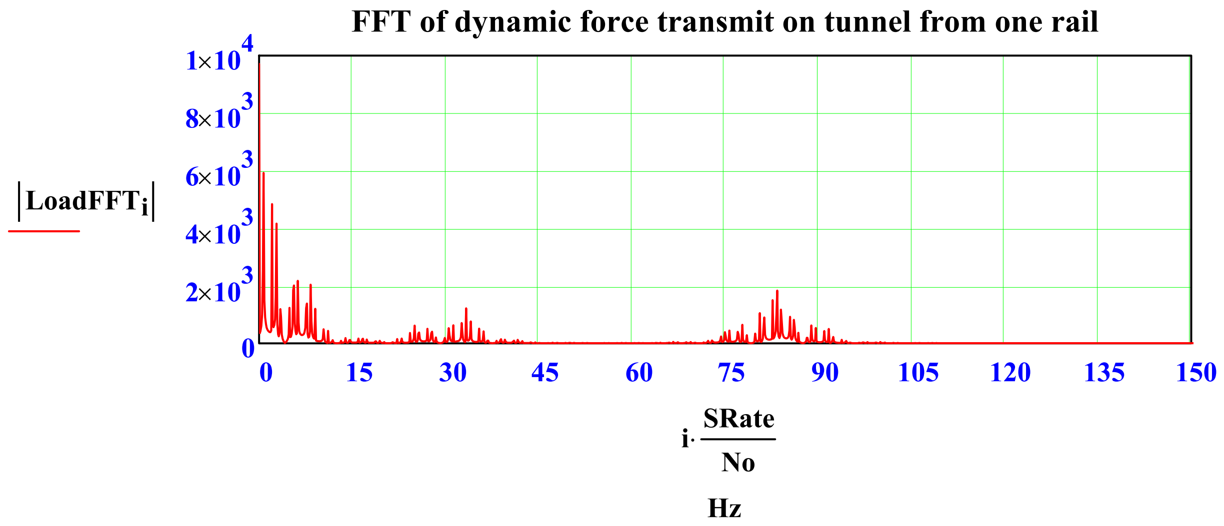
$F_{tunnel}(t) =$

-0.323
-0.34
-0.286
-0.25
-0.164
-0.141
-0.106
...

$\cdot \frac{kN}{m}$



$No := 2^{13} \quad i := 0..No - 1 \quad Time_i := \frac{i \cdot s}{SRate} \quad Val_i := F_{tunnel}(Time_i) \quad LoadFFT := FFT(Val)$



$\max(Val) = 79.885 \cdot \frac{kN}{m}$

$$f_{DynLoad_1} := \begin{vmatrix} \frac{v_t}{L_{ir_1}} & \text{if } a_{ir_1} > 0 \\ 0 & \text{otherwise} \end{vmatrix} \qquad f_{DynLoad_2} := \begin{vmatrix} \frac{v_t}{L_{ir_2}} & \text{if } a_{ir_2} > 0 \\ 0 & \text{otherwise} \end{vmatrix}$$

$f_{Load} := \max(f_{DynLoad}) = 83.333 \cdot Hz$