

On one Method for Solving Three-Dimensional Stress Problems by Means of Finite Fourier Transforms

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On one Method for Solving Three-Dimensional Stress Problems by Means of Finite Fourier Transforms

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Abstract

The purpose of this paper is to present the general solutions of the three-dimensional stress problems concerning a rectangular parallelepiped acted by any forces on its boundaries. The six components of the stress and the three components of the displacement are by virtue of the finite Fourier transforms written in the form of the tripple trigonometrical series with respect to x , y , and z .

Introduction

By means of the three functions approach, J. C. Maxwell¹⁾ had first asserted and H. Neuber²⁾ had lately extended, or by means of Galerkin's vector which Galerkin³⁾ derived from the idea of Love's strain function⁴⁾, the general solutions were found out of the three-dimensional stress problems. It is, however, too difficult to determine these solutions to satisfy the boundary conditions. By that reason, the three-dimensional stress problem so far treated may mostly be connected with elastic medium which is infinite, semiinfinite, or bodies with axially symmetrical stress. Only a few investigators⁵⁾, the author thinks, have been dealing with the three-dimensional stress problems in regard to the finite bodies which have six side planes.

The procedure herein proposed is a kind of operator calculus in which Green's formulas, relating to the equations of equilibrium of forces, are

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used in finding out the components of the stress and the components of the displacement in the finite Fourier transforms.

Equations of Equilibrium of Forces and Green's Formulas

The stresses acting on six sides of a cubic element of an elastic medium are expressed by three normal stresses σ_x , σ_y , and σ_z ; and three shearing stresses τ_{xy} , τ_{zx} , and τ_{yz} . From the equilibrium condition applied to a cubical element of the body, the equations of equilibrium of forces take the well-known forms

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = X, \quad (1)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = Y, \quad (2)$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = Z, \quad (3)$$

where X , Y , Z denote the components of the body force per unit volume in x , y , z directions respectively.

The stresses are related to the displacements u , v , and w

$$\left. \begin{aligned} \sigma_x &= (2\mu + \lambda) \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} + \lambda \frac{\partial w}{\partial z}, \\ \sigma_y &= \lambda \frac{\partial u}{\partial x} + (2\mu + \lambda) \frac{\partial v}{\partial y} + \lambda \frac{\partial w}{\partial z}, \\ \sigma_z &= \lambda \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} + (2\mu + \lambda) \frac{\partial w}{\partial z}, \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} \tau_{xy} &= \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \\ \tau_{yz} &= \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right), \\ \tau_{zx} &= \mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right), \end{aligned} \right\} \quad (5)$$

where u , v , w , are the components of displacement in x , y , z directions respectively; and μ , λ are Lamé's constants.

If we multiply the left side of Eq. (1) by L_1 that is a function differentiable two times with respect to x , y , and z ; and integrate by parts, then with the following abbreviations

$$\left. \begin{aligned} \int_0^a \int_0^b \int_0^c f(x, y, z) dx dy dz &= \int f(x, y, z) dV, \\ \int_0^b \int_0^c f(x, y, z) dy dz &= \int f(x, y, z) dA_x, \\ \int_0^c \int_0^a f(x, y, z) dz dx &= \int f(x, y, z) dA_y, \\ \int_0^a \int_0^b f(x, y, z) dx dy &= \int f(x, y, z) dA_z, \end{aligned} \right\} \quad (6)$$

we find that

$$\begin{aligned} & \int [\sigma_x L_1]_0^a dA_x + \int [\tau_{xy} L_1]_0^b dA_y + \int [\tau_{xz} L_1]_0^c dA_z \\ & - \int \left\{ \frac{\partial L_1}{\partial x} \sigma_x + \frac{\partial L_1}{\partial y} \tau_{xy} + \frac{\partial L_1}{\partial z} \tau_{xz} \right\} dV = \int X L_1 dV \end{aligned}$$

Putting hereinto the relations (4) and again integrating by parts, we finally have

$$\left. \begin{aligned} & \int [\sigma_x L_1]_0^a dA_x + \int [\tau_{xy} L_1]_0^b dA_y + \int [\tau_{xz} L_1]_0^c dA_z - \int \left[(2\mu + \lambda) u \frac{\partial L_1}{\partial x} \right. \\ & \quad \left. + \mu v \frac{\partial L_1}{\partial y} + \mu w \frac{\partial L_1}{\partial z} \right]_0^a dA_x - \int \left[\mu u \frac{\partial L_1}{\partial y} + \lambda v \frac{\partial L_1}{\partial x} \right]_0^b dA_y \\ & \quad - \int \left[\mu u \frac{\partial L_1}{\partial z} + \lambda w \frac{\partial L_1}{\partial x} \right]_0^c dA_z + \int \left\{ \mu v \Delta L_1 + (\mu + \lambda) u \frac{\partial^2 L_1}{\partial x^2} \right\} dV \\ & \quad + (\mu + \lambda) \int \left\{ v \frac{\partial^2 L_1}{\partial x \partial y} + w \frac{\partial^2 L_1}{\partial z \partial x} \right\} dV = \int X L_1 dV, \end{aligned} \right\} \quad (7)$$

where

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Similary, Eqs. (2) and (3), yield the following Green's formulas:

$$\left. \begin{aligned} & \int [\tau_{xy} L_2]_0^a dA_x + \int [\sigma_y L_2]_0^b dA_y + \int [\tau_{yz} L_2]_0^c dA_z - \int \left[\mu u \frac{\partial L_2}{\partial x} \right. \\ & \quad \left. + (2\mu + \lambda) v \frac{\partial L_2}{\partial y} + \mu w \frac{\partial L_2}{\partial z} \right]_0^b dA_y - \int \left[\mu v \frac{\partial L_2}{\partial x} + \lambda u \frac{\partial L_2}{\partial y} \right]_0^a dA_x \\ & \quad - \int \left[\mu v \frac{\partial L_2}{\partial z} + \lambda w \frac{\partial L_2}{\partial y} \right]_0^c dA_z + \int \left\{ \mu v \Delta L_2 + (\mu + \lambda) v \frac{\partial^2 L_2}{\partial y^2} \right\} dV \\ & \quad + (\mu + \lambda) \int \left\{ w \frac{\partial^2 L_2}{\partial y \partial z} + u \frac{\partial^2 L_2}{\partial x \partial y} \right\} dV = \int Y L_2 dV, \end{aligned} \right\} \quad (8)$$

$$\left. \begin{aligned} & \int [\tau_{xz} L_3]_0^a dA_x + \int [\tau_{yz} L_3]_0^b dA_y + \int [\sigma_z L_3]_0^c dA_z - \int \left[\mu u \frac{\partial L_3}{\partial x} \right. \\ & \quad \left. + v \frac{\partial L_3}{\partial y} + (2\mu + \lambda) w \frac{\partial L_3}{\partial z} \right]_0^c dA_z - \int \left[\mu w \frac{\partial L_3}{\partial x} + \lambda u \frac{\partial L_3}{\partial z} \right]_0^a dA_x \end{aligned} \right\} \quad (9)$$

$$\left. \begin{aligned} & - \int \left[\mu w \frac{\partial L_3}{\partial y} + \lambda v \frac{\partial L_3}{\partial z} \right]_0^b dA_y + \int \left\{ \mu w \cdot \Delta L_3 + (\mu + \lambda) w \frac{\partial^2 L_3}{\partial z^2} \right\} dV \\ & + (\mu + \lambda) \int \left\{ u \frac{\partial^2 L_3}{\partial z \partial x} + v \frac{\partial^2 L_3}{\partial y \partial z} \right\} dV = \int Z L_3 dV. \end{aligned} \right\}$$

where L_2 and L_3 are each a function which can be differentiated two times with respect to x , y , and z .

Nwe Symbolic Notations

If $f(x)$ satisfies Dirichlet's conditions in the interval $(0, a)$ and if for that range its finite sine and cosine transform are defined to be

$$S_m[f(x)] = \int_0^a f(x) \sin \frac{m\pi}{a} x dx, \quad (10)$$

$$C_m[f(x)] = \int_0^a f(x) \cos \frac{m\pi}{a} x dx, \quad (11)$$

then, at each point of $(0, a)$ at which $f(x)$ is continuous,

$$f(x) = \frac{2}{a} \sum_m S_m[f(x)] \cdot \sin \frac{m\pi}{a} x, \quad (12)$$

$$f(x) = \frac{1}{a} \int_0^a f(x) dx + \frac{2}{a} \sum_m C_m[f(x)] \cdot \cos \frac{m\pi}{a} x, \quad (13)$$

which can be extended to functions of three variables. Suppose, for instance, that $f(x, y, z)$ is a function of the three independent variables, x , y , and z , and satisfies Dirichlet's conditions in the spacial domain $(0 < x < a, 0 < y < b, 0 < z < c)$; then the finite tripple Fourier transform of $f(x, y, z)$ with regard to $\sin m\pi x/a$, $\sin n\pi y/b$, and $\sin r\pi z/c$, may be written in

$$S_m S_n S_r[f(x, y, z)] = \int_0^a \int_0^b \int_0^c f(x, y, z) \cdot \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \cdot \sin \frac{r\pi}{c} z dx dy dz, \quad (14)$$

where

$$m, n, r = 1, 2, 3, 4, \dots,$$

then, at each point of that space at which $f(x, y, z)$ is continuous.

$$f(x, y, z) = \frac{8}{abc} \sum_m \sum_n \sum_r S_m S_n S_r[f(x, y, z)] \cdot \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \sin \frac{r\pi}{c} z. \quad (15)$$

In a similar way, the finite tripple Fourier transforms with respect to $\sin m\pi x/a \cos n\pi y/b \cos r\pi z/c$ and $\cos m\pi x/a \sin n\pi y/b \cos r\pi z/c$, are related with $f(x, y, z)$ by the two inversion formulas:

$$\left. \begin{aligned}
 f(x, y, z) = & \frac{2}{abc} \sum_m \sin \frac{m\pi}{a} x \left\{ \int S_m [f(x, y, z)] dA_x \right. \\
 & + 2 \sum_n \int_0^c S_m C_n [f(x, y, z)] dz \cdot \cos \frac{n\pi}{b} y + 2 \sum_r \int_0^b S_m C_r [f(x, y, z)] dy \\
 & \times \cos \frac{r\pi}{c} z + 4 \sum_n \sum_r S_m C_n C_r [f(x, y, z)] \cdot \cos \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z \Big\},
 \end{aligned} \right\} \quad (16)$$

$$\left. \begin{aligned}
 f(x, y, z) = & \frac{2}{abc} \sum_n \sin \frac{n\pi}{b} y \left\{ \int S_n [f(x, y, z)] dA_y \right. \\
 & + 2 \sum_m \int_0^c C_m S_n [f(x, y, z)] dz \times \cos \frac{m\pi}{a} x + 2 \sum_r \int_0^a S_n C_r [f(x, y, z)] dx \\
 & \times \cos \frac{r\pi}{c} z + 4 \sum_m \sum_r C_m S_n C_r [f(x, y, z)] \cdot \cos \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z \Big\}.
 \end{aligned} \right\} \quad (17)$$

Solutions of Rectangular Parallelopiped

If we locate the rectangular Cartesian coordinate as shown in Fig. 1.

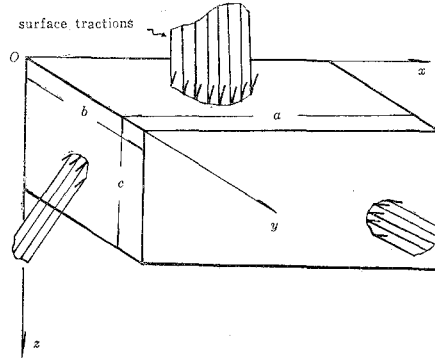


Fig. 1.

and let

$$\left. \begin{aligned}
 L_1 &= \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z, \\
 L_2 &= \sin \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z, \\
 L_3 &= \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \cdot \sin \frac{r\pi}{c} z,
 \end{aligned} \right\} \quad (18)$$

then, with the notations

$$M = \frac{m\pi}{a}, \quad N = \frac{n\pi}{b}, \quad R = \frac{r\pi}{c}.$$

Eq. (7) becomes

(171)

for $n \neq m = r = 0$,

$$\left. \begin{aligned} & \int_0^c S_n [(\sigma_x)_{x=a}] dz - \int_0^c S_n [(\sigma_x)_{x=0}] dz + \int_0^a S_n [(\tau_{zx})_{z=c}] dx \\ & - \int_0^a S_n [(\tau_{zx})_{z=0}] dx - \mu N \left\{ \int_0^c C_n [v_{x=a}] dz - \int_0^c C_n [v_{x=0}] dz \right\} \\ & - \mu N \left\{ (-1)^n \int_0^c (u_{y=b}) dA_y - \int_0^c (u_{y=0}) dA_y \right\} - \mu N^2 \int S_n [u] dA_y = \int S_n [X] dA_y \end{aligned} \right\} \quad (19)$$

for $m, n \neq 0, r = 0$,

$$\left. \begin{aligned} & (-1)^m \int_0^c S_n [(\sigma_x)_{x=a}] dz - \int_0^c S_n [(\sigma_x)_{x=0}] dz + C_m S_n [(\tau_{zx})_{z=c}] \\ & - C_m S_n [(\tau_{zx})_{z=0}] - \mu N \left\{ (-1)^m \int_0^c C_n [v_{x=a}] dz - \int_0^c C_n [v_{x=0}] dz \right\} \\ & - \mu N \left\{ (-1)^n \int_0^c C_m [u_{y=b}] dz - \int_0^c C_m [u_{y=0}] dz \right\} - \left\{ M^2 (2\mu + \lambda) + N^2 \mu \right\} \\ & \times \int_0^c C_m S_n [u] dz - MN(\mu + \lambda) \int_0^c S_m C_n [v] dz = \int_0^c C_m S_n [X] dz \end{aligned} \right\} \quad (20)$$

for $n, r \neq 0, m = 0$,

$$\left. \begin{aligned} & S_n C_r [(\sigma_x)_{x=a}] - S_n C_r [(\sigma_x)_{x=0}] + (-1)^r \int_0^a S_n [(\tau_{zx})_{z=c}] dx \\ & - \int_0^a S_n [(\tau_{zx})_{z=0}] dx - \mu N \left\{ C_n C_r [v_{x=a}] - C_n C_r [v_{x=0}] \right\} \\ & - \mu N \left\{ (-1)^n \int_0^a C_r [u_{y=b}] dx - \int_0^a C_r [u_{y=0}] dx \right\} + \mu R \left\{ S_n S_r [w_{x=a}] \right. \\ & \left. - S_n S_r [w_{x=0}] \right\} - \mu (N^2 + R^2) \int_0^a S_n C_r [u] dx = \int_0^a S_n C_r [X] dx, \end{aligned} \right\} \quad (21)$$

for $m, n, r \neq 0$,

$$\left. \begin{aligned} & (-1)^m S_n C_r [(\sigma_x)_{x=a}] - S_n C_r [(\sigma_x)_{x=0}] + (-1)^r C_m S_n [(\tau_{zx})_{z=c}] \\ & - C_m S_n [(\tau_{zx})_{z=0}] - \mu N \left\{ (-1)^m C_n C_r [v_{x=a}] - C_n C_r [v_{x=0}] \right\} \\ & - \mu N \left\{ (-1)^n C_m C_r [u_{y=b}] - C_m C_r [u_{y=0}] \right\} + \mu R \left\{ (-1)^m S_n S_r [w_{x=a}] \right. \\ & \left. - S_n S_r [w_{x=0}] \right\} + \lambda M \left\{ (-1)^r S_m S_n [w_{z=c}] - S_m S_n [w_{z=0}] \right\} \\ & - \left\{ (2\mu + \lambda) M^2 + \mu N^2 + \mu R^2 \right\} C_m S_n C_r [v] - (\mu + \lambda) MN \cdot S_m C_n C_r [v] \\ & + (\mu + \lambda) MR \cdot S_m S_n S_r [w] = C_m S_n C_r [X]. \end{aligned} \right\} \quad (22)$$

Proceeding as before, we write Eq. (8) as follows:

for $m \neq n = r = 0$,

$$\left. \begin{aligned} & \int_0^c S_m [(\sigma_y)_{y=b}] dz - \int_0^c S_m [(\sigma_y)_{y=0}] dz + \int_0^b S_m [(\tau_{yz})_{z=c}] dy \\ & - \int_0^b S_m [(\tau_{yz})_{z=0}] dy - \mu M \left\{ \int_0^c C_m [u_{y=b}] dz - \int_0^c C_m [u_{y=0}] dz \right\} \\ & - \mu M \left\{ (-1)^m \int_0^c (v_{x=a}) dA_x - \int_0^c (v_{x=0}) dA_x \right\} - \mu \int S_m [v] dA_x = \int S_m [Y] dA_x, \end{aligned} \right\} \quad (23)$$

for $m, n \neq 0, r = 0$,

$$\left. \begin{aligned} & (-1)_n \int_0^c S_n [(\sigma_y)_{y=b}] dz - \int_0^c S_m [(\sigma_y)_{y=0}] dz + S_m C_n [(\tau_{yz})_{z=c}] \\ & - S_n C_n [(\tau_{yz})_{z=0}] - \mu M \left\{ (-1)^n \int_0^c C_m [u_{y=b}] dz - \int_0^c C_m [u_{y=0}] dz \right\} \\ & - \mu M \left\{ (-1)^m \int_0^c C_n [v_{x=a}] dz - \int_0^c C_n [v_{x=0}] dz \right\} - \left\{ \mu M^2 + (2\mu + \lambda) N^2 \right\} \\ & \times \int_0^c S_m C_n [v] dz - (\mu + \lambda) MN \int_0^c C_m S_n [u] dz = \int_0^c S_m C_n [Y] dz, \end{aligned} \right\} \quad (24)$$

for $m, r \neq 0, n = 0$,

$$\left. \begin{aligned} & S_m C_r [(\sigma_y)_{y=b}] - S_m C_r [(\sigma_y)_{y=0}] + (-1)^r \int_0^b S_m [(\tau_{yz})_{z=c}] dy \\ & - \int_0^b S_m [(\tau_{yz})_{z=0}] dy - \mu M \left\{ C_m C_r [u_{y=b}] - C_m C_r [u_{y=0}] \right\} \\ & - \mu M \left\{ (-1)^n \int_0^b C_r [v_{x=a}] dy - \int_0^b C_r [v_{x=0}] dy \right\} + \mu R \left\{ S_m S_r [w_{x=a}] \right. \\ & \left. - S_m S_n [w_{x=0}] \right\} - \mu (M^2 + R^2) \int_0^b S_m C_r [v] dy = \int_0^b S_m C_r [Y] dy, \end{aligned} \right\} \quad (25)$$

for $m, n, r \neq 0$,

$$\left. \begin{aligned} & (-1)^n S_m C_r [(\sigma_y)_{y=b}] - S_m C_r [(\sigma_y)_{y=0}] + (-1)^r S_m C_n [(\tau_{yz})_{z=c}] \\ & - S_m C_n [(\tau_{yz})_{z=0}] - \mu M \left\{ (-1)^n C_m C_r [u_{y=b}] - C_m C_r [u_{y=0}] \right\} \\ & - \mu M \left\{ (-1)^m C_n C_r [v_{x=a}] - C_n C_r [v_{x=0}] \right\} + \mu R \left\{ (-1)^n S_m S_r [w_{y=b}] \right. \\ & \left. - S_m S_r [w_{y=0}] \right\} + \lambda N \left\{ (-1)^r S_m S_n [w_{z=c}] - S_m S_n [w_{z=0}] \right\} \\ & - \left\{ \mu M^2 + (2\mu + \lambda) N^2 + \mu R^2 \right\} \cdot S_m C_n C_r [v] - (\mu + \lambda) MN \cdot C_m S_n C_r [u] \\ & + (\mu + \lambda) NR \cdot S_m S_n S_r [w] = S_m C_n C_r [Y]. \end{aligned} \right\} \quad (26)$$

Similar, Eq. (9) yields

$$\left. \begin{aligned} & -(2\mu + \lambda) R \left\{ (-1)_r S_m S_n [w_{z=c}] - S_m S_n [w_{z=0}] \right\} - \mu M \left\{ (-1)^m S_n S_r [w_{x=a}] \right. \\ & \left. - S_n S_r [w_{x=0}] \right\} - \mu N \left\{ (-1)^n S_m S_r [w_{y=b}] - S_m S_r [w_{y=0}] \right\} \\ & - \left\{ \mu M^2 + \mu N^2 + (2\mu + \lambda) R^2 \right\} \cdot S_m S_n S_r [w] + (\mu + \lambda) RM \cdot C_m S_n C_r [u] \\ & + (\mu + \lambda) NR \cdot S_m C_n C_r [v] = S_m S_n S_r [Z]. \end{aligned} \right\} \quad (27)$$

Each of the three equations (22), (26) and (27) include the finite Fourier transforms of u , v , and w . Hence, the finite Fourier transforms of u , v , and w :

$$C_m S_n C_r [u], \quad S_m C_n C_r [u], \quad S_m S_n S_r [w]$$

can, with solving Eqs. (22), (26) and (27) simultaneously, be expressed by

the double finite Fourier transforms connecting with the stresses and the displacements on the six side planes.

These double finite Fourier transforms are the boundary values to be determined as satisfying the given boundary conditions, and apparently the numbers of the double finite Fourier transforms are same as those of the boundary conditions.

To avoid the complexities of further evaluations, let us introduce the following notations:

$$\left. \begin{matrix} \pi A'_{0n} \\ \pi A_{n0} \end{matrix} \right\} = \int_0^c S_n [(\sigma_x)_{x=a}] dz \mp \int_0^c S_n [(\sigma_x)_{x=0}] dz, \quad (28)$$

$$\left. \begin{matrix} \pi B'_{m0} \\ \pi B_{m0} \end{matrix} \right\} = \int_0^c S_m [(\sigma_y)_{y=b}] dz \mp \int_0^c S_m [(\sigma_y)_{y=0}] dz, \quad (29)$$

$$\left. \begin{matrix} \pi A'_{nr} \\ \pi A_{nr} \end{matrix} \right\} = S_n C_r [(\sigma_x)_{x=a}] \mp S_n C_r [(\sigma_x)_{x=0}], \quad (30)$$

$$\left. \begin{matrix} \pi B'_{mr} \\ \pi B_{mr} \end{matrix} \right\} = S_m C_r [(\sigma_y)_{y=b}] \mp S_m C_r [(\sigma_y)_{y=0}], \quad (31)$$

$$\left. \begin{matrix} \pi D'_{0n} \\ \pi D_{0n} \end{matrix} \right\} = \int_0^a S_n [(\tau_{zx})_{z=c}] dx \mp \int_0^a S_n [(\tau_{zx})_{z=0}] dx, \quad (32)$$

$$\left. \begin{matrix} \pi F'_{m0} \\ \pi F_{m0} \end{matrix} \right\} = \int_0^b S_m [(\tau_{yz})_{z=c}] dy \mp \int_0^b S_m [(\tau_{yz})_{z=0}] dy, \quad (33)$$

$$\left. \begin{matrix} \pi D'_{mn} \\ \pi D_{mn} \end{matrix} \right\} = C_m S_n [(\tau_{zx})_{z=c}] \mp C_m S_n [(\tau_{zx})_{z=0}], \quad (34)$$

$$\left. \begin{matrix} \pi F'_{mn} \\ \pi F_{mn} \end{matrix} \right\} = S_m C_n [(\tau_{yz})_{z=c}] \mp S_m C_n [(\tau_{yz})_{z=0}], \quad (35)$$

$$\left. \begin{matrix} H'_{00} \\ H_{00} \end{matrix} \right\} = \int (v_{x=a}) dA_x \mp \int (v_{x=0}) dA_x, \quad (36)$$

$$\left. \begin{matrix} J'_{00} \\ J_{00} \end{matrix} \right\} = \int (u_{y=b}) dA_y \mp \int (u_{y=0}) dA_y, \quad (37)$$

$$\left. \begin{matrix} H'_{m0} \\ H_{m0} \end{matrix} \right\} = \int_0^c C_m [u_{y=b}] dz \mp \int_0^c C_m [u_{y=0}] dz, \quad (38)$$

$$\left. \begin{matrix} J'_{n0} \\ J_{n0} \end{matrix} \right\} = \int_0^c C_n [v_{x=a}] dz \mp \int_0^c C_n [v_{x=0}] dz, \quad (39)$$

$$\left. \begin{matrix} H'_{0r} \\ H_{0r} \end{matrix} \right\} = \int_0^a C_r [u_{y=b}] dx \mp \int_0^a C_r [u_{y=0}] dx, \quad (40)$$

$$\left. \begin{matrix} J'_{0r} \\ J_{0r} \end{matrix} \right\} = \int_0^b C_r [v_{x=a}] dy \mp \int_0^b C_r [v_{x=0}] dy, \quad (41)$$

$$\left. \begin{matrix} H'_{mr} \\ H_{mr} \end{matrix} \right\} = C_m C_r [u_{y=b}] \mp C_m C_r [u_{y=0}], \quad (42)$$

$$\left. \begin{matrix} J'_{nr} \\ J_{nr} \end{matrix} \right\} = C_n C_r [v_{x=a}] \mp C_n C_r [v_{x=0}], \quad (43)$$

$$\left. \begin{matrix} E'_{nr} \\ E_{nr} \end{matrix} \right\} = S_n S_r [w_{x=a}] \mp S_n S_r [w_{x=0}], \quad (44)$$

$$\left. \begin{matrix} G'_{mr} \\ G_{mr} \end{matrix} \right\} = S_m S_r [w_{y=b}] \mp S_m S_r [w_{y=0}], \quad (45)$$

$$\left. \begin{matrix} K'_{mn} \\ K_{mn} \end{matrix} \right\} = S_m S_n [w_{z=c}] \mp S_m S_n [w_{z=0}]. \quad (46)$$

Now, for Eqs. (19) and (23), we have

$$\begin{aligned} A_{n0} + D_{n0} - \mu N J_{n0} - \mu N \left\{ (1 + (-1)^n) H_{00} + (1 - (-1)^n) H'_{00} \right\} \\ - \mu N^2 \int S_n [u] dA_y - \mu N^2 \int S_n [u] dA_y = \int S_n [X] dA_y, \\ B_{m0} + F_{m0} - \mu M H_{m0} - \mu M \left\{ (1 + (-1)^m) J_{00} + (1 - (-1)^m) J'_{00} \right\} \\ - \mu M^2 \int S_m [v] dA_x - \mu M^2 \int S_m [v] dA_x = \int S_m [Y] dA_x, \end{aligned} \quad (47)$$

for Eqs. (21) and (25),

$$\begin{aligned} A_{nr} + D_{0n} - \mu N J_{nr} - \mu N \left\{ (1 + (-1)^n) H_{0r} + (1 - (-1)^n) H'_{0r} \right\} \\ + \mu R E_{nr} - \int_0^a S_n C_r [X] dx = \mu (N^2 + R^2) \int_0^a S_n C_r [u] dx, \\ B_{mr} + F_{m0} - \mu M H_{mr} - \mu M \left\{ (1 + (-1)^n) J_{0r} + (1 - (-1)^n) J'_{0r} \right\} \\ + \mu R G_{mr} - \int_0^b S_m C_r [Y] dy = \mu (M^2 + R^2) \int_0^b S_m C_r [v] dy, \end{aligned} \quad (48)$$

for Eqs. (20) and (24),

$$\begin{aligned} (1 + (-1)^m) A_{n0} + (1 - (-1)^m) A'_{0n} + D_{mn} - \mu N \left\{ (1 + (-1)^m) J_{n0} \right. \\ \left. + (1 - (-1)^m) J'_{n0} \right\} - \mu N \left\{ (1 + (-1)^n) H_{m0} + (1 - (-1)^n) H'_{mc} \right\} \\ - \int_0^c C_m S_n [X] dz = \left\{ (2\mu + \lambda) M^2 + \mu N^2 \right\} \int_0^c C_m S_n [u] dz \\ + MN(\mu + \lambda) \int_0^c S_m C_n [v] dz, \\ (1 + (-1)^n) B_{m0} + (1 - (-1)^n) B'_{m0} + F_{mn} - \mu M \left\{ (1 + (-1)^n) H_{m0} \right. \\ \left. + (1 - (-1)^n) H'_{m0} \right\} - \mu M \left\{ (1 + (-1)^m) J_{n0} + (1 - (-1)^m) J'_{n0} \right\} \\ - \int_0^c S_m C_n [Y] dz = \left\{ (2\mu + \lambda) N^2 + \mu M^2 \right\} \int_0^c S_m C_n [v] dz \\ + MN(\mu + \lambda) \int_0^c C_m S_n [u] dz, \end{aligned} \quad (49)$$

from which

$$\left. \begin{aligned} \int_0^c C_m S_n [u] dz &= \frac{1}{\mu} \left\{ \frac{1}{M^2 + N^2} - \frac{\mu + \lambda}{2\mu + \lambda} \frac{M^2}{(M^2 + N^2)^2} \right\} \left\{ (1 + (-1)^m) A_{n0} \right. \\ &\quad \left. + (1 - (-1)^m) A'_{n0} - \int_0^c C_m S_n [X] dz \right\} - \frac{1}{\mu} \frac{\mu + \lambda}{2\mu + \lambda} \frac{MN}{(M^2 + N^2)^2} \\ &\quad \times \left\{ (1 + (-1)^n) B_{m0} + (1 - (-1)^n) B'_{m0} - \int_0^c S_m C_n [Y] dz \right\}, \end{aligned} \right\} \quad (50)$$

$$\left. \begin{aligned} \int_0^c S_m C_n [v] dz &= \frac{1}{\mu} \left\{ \frac{1}{M^2 + N^2} - \frac{\mu + \lambda}{2\mu + \lambda} \frac{N^2}{(M^2 + N^2)^2} \right\} \left\{ (1 + (-1)^n) B_{m0} \right. \\ &\quad \left. + (1 - (-1)^n) B'_{m0} - \int_0^c S_m C_n [Y] dz \right\} - \frac{1}{\mu} \frac{\mu + \lambda}{2\mu + \lambda} \frac{MN}{(M^2 + N^2)^2} \\ &\quad \times \left\{ (1 + (-1)^m) A_{n0} + (1 - (-1)^m) A'_{n0} - \int_0^c C_m S_n [X] dz \right\}, \end{aligned} \right\} \quad (51)$$

for Eqs. (22), (26) and (27), we have

$$\left. \begin{aligned} &(1 + (-1)^m) A_{nr} + (1 - (-1)^m) A'_{nr} + (1 + (-1)^r) D_{mn} + (1 - (-1)^r) D'_{mn} \\ &- \mu N \{ (1 + (-1)^m) J_{nr} + (1 - (-1)^m) J'_{nr} \} + \mu R \{ (1 + (-1)^m) E_{nr} \\ &+ (1 - (-1)^m) E'_{nr} \} - \mu N \{ (1 + (-1)^n) H_{mr} + (1 - (-1)^n) H'_{mr} \} \\ &+ \lambda M \{ (1 + (-1)^r) K_{mn} + (1 - (-1)^r) K'_{mn} \} - C_m S_n C_r [X] \\ &= \{ (2\mu + \lambda) M^2 + \mu N^2 + \mu R^2 \} \times C_m S_n C_r [u] + (\mu + \lambda) MN \cdot S_m C_n C_r [v] \\ &- (\mu + \lambda) MR \cdot S_m S_n S_r [w], \end{aligned} \right\} \quad (52)$$

$$\left. \begin{aligned} &(1 + (-1)^n) B_{nr} + (1 - (-1)^n) B'_{nr} + (1 + (-1)^r) F_{mn} - (1 - (-1)^r) F'_{mn} \\ &- \mu M \{ (1 + (-1)^n) H_{mr} + (1 - (-1)^n) H'_{mr} \} + \mu R \{ (1 + (-1)^n) G_{mr} \\ &+ (1 - (-1)^n) G'_{mr} \} - \mu M \{ (1 + (-1)^m) J_{nr} + (1 - (-1)^m) J'_{nr} \} \\ &+ \lambda N \{ (1 + (-1)^r) K_{mn} + (1 - (-1)^r) K'_{mn} \} - S_m C_n C_r [Y] \\ &= \{ (2\mu + \lambda) N^2 + \mu M^2 + \mu R^2 \} \times S_m C_n C_r [v] + (\mu + \lambda) MN \times C_m S_n C_r [u] \\ &- (\mu + \lambda) NR \cdot S_m S_n S_r [w], \end{aligned} \right\} \quad (53)$$

$$\left. \begin{aligned} &-(2\mu + \lambda) R \{ (1 + (-1)^r) K_{mn} - (1 - (-1)^r) K'_{mn} \} - \mu M \{ (1 + (-1)^m) E_{nr} \\ &+ (1 - (-1)^m) E'_{nr} \} - \mu M \{ (1 + (-1)^n) G_{mr} + (1 - (-1)^n) G'_{mr} \} \\ &- S_m S_n S_r [Z] = \{ (2\mu + \lambda) R^2 + \mu M^2 + \mu N^2 \} \times S_m S_n S_r [w] \\ &- (\mu + \lambda) R \cdot M \times C_m S_n C_r [u] - (\mu + \lambda) NR \times S_m C_n C_r [v], \end{aligned} \right\} \quad (54)$$

from which we find that

$$\left. \begin{aligned} C_m S_n C_r [u] &= \left\{ \frac{\lambda}{2\mu + \lambda} \frac{M}{T} - \frac{2(\mu + \lambda)}{2\mu + \lambda} \frac{MR^2}{T^2} \right\} \left\{ (1 + (-1)^r) K_{mn} \right. \\ &\quad \left. + (1 - (-1)^r) K'_{mn} \right\} + \frac{2(\mu + \lambda)}{2\mu + \lambda} \frac{MNR}{T^2} \left\{ (1 + (-1)^n) G_{mr} + (1 - (-1)^n) G'_{mr} \right\} \\ &- \left\{ \frac{R}{T} - \frac{2(\mu + \lambda)}{2\mu + \lambda} \frac{M^2 R}{T^2} \right\} \left\{ (1 + (-1)^m) E_{nr} + (1 - (-1)^m) E'_{nr} \right\} \end{aligned} \right\}$$

$$\begin{aligned}
& -\frac{\mu+\lambda}{\mu(2\mu+\lambda)} \frac{MN}{T^2} \times \left\{ (1+(-1)^n) B_{mr} + (1-(-1)^n) B'_{mr} + (1+(-1)^r) F'_{mn} \right. \\
& + (1-(-1)^r) F'_{mn} - S_m C_n C_r [Y] \left. \right\} + \left\{ \frac{1}{\mu T} - \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \frac{M^2}{T^2} \right\} \left\{ (1+(-1)^m) \right. \\
& \times A_{nr} + (1-(-1)^m) A'_{nr} + (1+(-1)^r) D_{mn} + (1-(-1)^r) D'_{mn} - C_m S_n C_r [X] \left. \right\} \\
& - \left\{ \frac{N}{T} - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{M^2 N}{T^2} \right\} \left\{ (1+(-1)^n) H_{mr} + (1-(-1)^n) H'_{mr} \right. \\
& + (1+(-1)^m) J_{nr} + (1-(-1)^m) J'_{nr} \left. \right\} + \frac{\lambda}{2\mu+\lambda} \frac{MR}{T^2} S_m S_n S_c [Z], \quad (55)
\end{aligned}$$

$$\begin{aligned}
S_m C_n C_r [v] = & \left\{ \frac{\lambda}{2\mu+\lambda} \frac{N}{T} - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{NR^2}{T^2} \right\} \left\{ (1+(-1)^r) K_{mn} \right. \\
& + (1-(-1)^r) K'_{mn} \left. \right\} + \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{MNR}{T^2} \left\{ (1+(-1)^m) E_{nr} + (1-(-1)^m) E'_{nr} \right\} \\
& - \left\{ \frac{R}{T} - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{N^2 R}{T^2} \right\} \times \left\{ (1+(-1)^n) G_{mr} + (1-(-1)^n) G'_{mr} \right\} \\
& - \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \frac{MN}{T^2} \left\{ (1+(-1)^m) A_{nr} + (1-(-1)^m) A'_{nr} + (1+(-1)^r) D_{mn} \right. \\
& + (1-(-1)^r) D'_{mn} - C_m S_n C_r [X] \left. \right\} + \left\{ \frac{1}{\mu T} - \frac{2(\mu+\lambda)}{\mu(2\mu+\lambda)} \frac{N^2}{T^2} \right\} \\
& \times \left\{ (1+(-1)^n) B_{mr} + (1-(-1)^n) B'_{mr} + (1+(-1)^r) F'_{mn} + (1+(-1)^r) F'_{mn} \right. \\
& - S_m C_n C_r [Y] \left. \right\} - \left\{ \frac{M}{T} - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{MN^2}{T^2} \right\} \left\{ (1+(-1)^n) H_{mr} \right. \\
& + (1-(-1)^n) H'_{mr} + (1+(-1)^m) J_{nr} + (1-(-1)^m) J'_{nr} \left. \right\} \\
& + \frac{\lambda}{2\mu+\lambda} \frac{NR}{T^2} S_m S_n S_c [Z], \quad (56)
\end{aligned}$$

$$\begin{aligned}
S_m S_n S_r [w] = & - \left\{ \frac{R}{T} + \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{R(M^2+N^2)}{T^2} \right\} \left\{ (1+(-1)^r) K_{mn} \right. \\
& + (1-(-1)^r) K'_{mn} \left. \right\} + \left\{ \frac{M}{T} - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{MR^2}{T^2} \right\} \left\{ (1+(-1)^m) E_{nr} \right. \\
& + (1-(-1)^m) E'_{nr} \left. \right\} + \left\{ \frac{N}{T} - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{NR^2}{T^2} \right\} \left\{ (1+(-1)^n) G_{mr} \right. \\
& + (1-(-1)^n) G'_{mr} \left. \right\} + \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \frac{RM}{T^2} \left[(1+(-1)^m) A_{nr} + (1-(-1)^m) A'_{nr} \right. \\
& + (1+(-1)^r) D_{mn} + (1-(-1)^r) D'_{mn} - 2\mu N \left\{ (1+(-1)^m) J_{nr} \right. \\
& + (1-(-1)^m) J'_{nr} \left. \right\} - C_m S_n C_r [X] \left. \right] + \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \frac{NR}{T^2} \left[(1+(-1)^n) B_{mr} \right. \\
& \left. \left. + (1-(-1)^n) B'_{mr} \right] \right\} \quad (57)
\end{aligned}$$

$$\begin{aligned}
& + (1 - (-1)^n) B'_{mr} + (1 + (-1)^r) F_{mn} + (1 - (-1)^r) F'_{mn} - 2\mu M \\
& \times \left\{ (1 + (-1)^n) H_{mr} + (1 - (-1)^n) H'_{mr} \right\} - S_m C_n C_r [Y] \\
& - \left\{ \frac{1}{T} - \frac{\mu + \lambda}{\mu (2\mu + \lambda)} \frac{R^2}{T^2} \right\} \times S_m S^n S_r [Z]
\end{aligned}$$

where

$$T = M^2 + N^2 + R^2 = \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 + \left(\frac{r\pi}{c} \right)^2.$$

With the aid of the following formulas:

$$\begin{aligned}
& \sum_m (1 - (-1)^m) \frac{M}{T} \sin Mx = \frac{a}{2} \left\{ \begin{array}{l} Q^{(1)}(\alpha_{nr}\xi), \\ Q^{(2)}(\alpha_{nr}\xi), \end{array} \right. \\
& \sum_m (1 + (-1)^m) \frac{M}{T^2} \sin Mx = \frac{a^3}{4\pi^2 \alpha_{nr}^2} \left\{ \begin{array}{l} P^{(1)}(\alpha_{nr}\xi), \\ P^{(2)}(\alpha_{nr}\xi), \end{array} \right. \\
& Q^{(1)}(\alpha_{nr}\xi) = \frac{ch\pi\alpha_{nr}(1-\xi) \pm ch\pi\alpha_{nr}\xi}{ch\pi\alpha_{nr} \pm 1}, \\
& Q^{(2)}(\alpha_{nr}\xi) = \frac{\pi\alpha_{nr} \{ \xi sh\pi\alpha_{nr}(1-\xi) \pm (1-\xi) sh\pi\alpha_{nr} \}}{ch\pi\alpha_{nr} \pm 1},
\end{aligned} \quad (58)$$

$$M = \frac{m\pi}{a}, \quad T = \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 + \left(\frac{r\pi}{c} \right)^2, \quad \alpha_{nr}^2 = \frac{a^2}{b^2} n^2 + \frac{a^2}{c^2} r^2, \quad \xi = \frac{x}{a},$$

$$m = 1, 2, 3, 4, \dots,$$

$$\begin{aligned}
& \sum_n (1 - (-1)^n) \frac{1}{T} \cos Nx = \frac{a^2}{2\pi\alpha_{nr}} \left\{ \begin{array}{l} \phi^{(1)}(\alpha_{nr}\xi), \\ \phi^{(2)}(\alpha_{nr}\xi) - \frac{2}{\pi\alpha_{nr}}, \end{array} \right. \\
& \sum_n (1 + (-1)^n) \frac{M^2}{T^2} \cos Nx = \frac{a^2}{4\pi\alpha_{nr}} \left\{ \begin{array}{l} \phi^{(1)}(\alpha_{nr}\xi) - \psi^{(1)}(\alpha_{nr}\xi), \\ \phi^{(2)}(\alpha_{nr}\xi) - \psi^{(2)}(\alpha_{nr}\xi), \end{array} \right. \\
& \sum_n (1 - (-1)^n) \frac{1}{T^2} \cos Nx = \frac{a^4}{4\pi^3 \alpha_{nr}^3} \left\{ \begin{array}{l} \phi^{(1)}(\alpha_{nr}\xi) + \psi^{(1)}(\alpha_{nr}\xi), \\ \phi^{(2)}(\alpha_{nr}\xi) + \psi^{(2)}(\alpha_{nr}\xi) - \frac{2}{\pi\alpha_{nr}}, \end{array} \right. \\
& \phi^{(1)}(\alpha_{nr}\xi) = \frac{sh\pi\alpha_{nr}(1-\xi) \mp sh\pi\alpha_{nr}\xi}{ch\pi\alpha_{nr} \pm 1}, \\
& \phi^{(2)}(\alpha_{nr}\xi) = \frac{\pi\alpha_{nr} \{ \xi ch\pi\alpha_{nr}(1-\xi) \mp (1-\xi) ch\pi\alpha_{nr}\xi \}}{ch\pi\alpha_{nr} \pm 1}, \\
& \psi^{(1)}(\alpha_{nr}\xi) = \frac{\pi\alpha_{nr} \{ \xi ch\pi\alpha_{nr}(1-\xi) \mp (1-\xi) ch\pi\alpha_{nr}\xi \}}{ch\pi\alpha_{nr} \pm 1}, \\
& \psi^{(2)}(\alpha_{nr}\xi) = \frac{\pi\alpha_{nr} \{ \xi ch\pi\alpha_{nr}(1-\xi) \mp (1-\xi) ch\pi\alpha_{nr}\xi \}}{ch\pi\alpha_{nr} \pm 1},
\end{aligned} \quad (59)$$

$$m = 1, 2, 3, 4, \dots,$$

$$\begin{aligned}
& \sum_n (1 - (-1)^n) \frac{N}{T} \sin Ny = \frac{b}{2} \left\{ \begin{array}{l} Q^{(1)}(\beta_{mr}\eta), \\ Q^{(2)}(\beta_{mr}\eta), \end{array} \right. \\
& \sum_n (1 + (-1)^n) \frac{N}{T^2} \sin Ny = \frac{b^3}{4\pi^2 \beta_{mr}} \left\{ \begin{array}{l} P^{(1)}(\beta_{mr}\eta), \\ P^{(2)}(\beta_{mr}\eta), \end{array} \right.
\end{aligned} \quad (60)$$

$$\left. \begin{aligned}
 Q^{(1)}(\beta_{mr}\eta) \} &= \frac{ch\pi\beta_{mr}(1-\eta) \pm ch\pi\beta_{mr}\eta}{ch\pi\beta_{mr} \pm 1}, \\
 Q^{(2)}(\beta_{mr}\eta) \} &= \frac{\pi\beta_{mr} \{ \eta sh\pi\beta_{mr}(1-\eta) \pm (1-\eta) sh\pi\beta_{mr}\eta \}}{ch\pi\beta_{mr} \pm 1}, \\
 P^{(1)}(\beta_{mr}\eta) \} &= \frac{b^2}{a^2} m^2 + \frac{b^2}{c^2} r^2, \quad \eta = \frac{y}{b}, \quad N = \frac{n\pi}{b}, \\
 P^{(2)}(\beta_{mr}\eta) \} &= \frac{b^2}{a^2} m^2 + \frac{b^2}{c^2} r^2, \quad \eta = \frac{y}{b}, \quad N = \frac{n\pi}{b}, \\
 n &= 1, 2, 3, 4, \dots,
 \end{aligned} \right\}$$

$$\left. \begin{aligned}
 \sum_n \frac{(1-(-1)^n)}{(1+(-1)^n)} \frac{1}{T} \cos Ny &= \frac{a^2}{2\pi\beta_{mr}} \left\{ \frac{\phi^{(1)}(\beta_{mr}\eta)}{\phi^{(2)}(\beta_{mr}\eta)} - \frac{2}{\pi\beta_{mr}} \right\}, \\
 \sum_n \frac{(1-(-1)^n)}{(1+(-1)^n)} \frac{N^2}{T^2} \cos Ny &= \frac{a^2}{4\pi\beta_{mr}} \left\{ \frac{\phi^{(1)}(\beta_{mr}\eta) - \Psi^{(1)}(\beta_{mr}\eta)}{\phi^{(2)}(\beta_{mr}\eta) - \Psi^{(2)}(\beta_{mr}\eta)} \right\}, \\
 \sum_n \frac{(1-(-1)^n)}{(1+(-1)^n)} \frac{1}{T^2} \cos Ny &= \frac{a^4}{4\pi^3\beta_{mr}^3} \left\{ \frac{\phi^{(1)}(\beta_{mr}\eta) + \Psi^{(1)}(\beta_{mr}\eta)}{\phi^{(2)}(\beta_{mr}\eta) + \Psi^{(2)}(\beta_{mr}\eta)} - \frac{2}{\pi\beta_{mr}} \right\}, \\
 \phi^{(1)}(\beta_{mr}\eta) \} &= \frac{sh\pi\beta_{mr}(1-\eta) \mp sh\pi\beta_{mr}\eta}{ch\pi\beta_{mr} \pm 1}, \\
 \phi^{(2)}(\beta_{mr}\eta) \} &= \frac{\pi\beta_{mr} \{ (1-\eta)ch\pi\beta_{mr}\eta \mp \eta ch\pi\beta_{mr}(1-\eta) \}}{ch\pi\beta_{mr} \pm 1}, \\
 \Psi^{(1)}(\beta_{mr}\eta) \} &= \frac{\pi\beta_{mr} \{ (1-\eta)ch\pi\beta_{mr}\eta \mp \eta ch\pi\beta_{mr}(1-\eta) \}}{ch\pi\beta_{mr} \pm 1}, \\
 \Psi^{(2)}(\beta_{mr}\eta) \} &= \frac{\pi\beta_{mr} \{ (1-\eta)ch\pi\beta_{mr}\eta \mp \eta ch\pi\beta_{mr}(1-\eta) \}}{ch\pi\beta_{mr} \pm 1}, \\
 n &= 1, 2, 3, 4, \dots,
 \end{aligned} \right\} \quad (61)$$

$$\left. \begin{aligned}
 \sum_r \frac{(1-(-1)^r)}{(1+(-1)^r)} \frac{R}{T} \sin Rz &= \frac{c}{2} \left\{ \frac{Q^{(1)}(\gamma_{mn}\zeta)}{Q^{(2)}(\gamma_{mn}\zeta)} \right\}, \\
 \sum_r \frac{(1-(-1)^r)}{(1+(-1)^r)} \frac{R}{T^2} \sin Rz &= \frac{c^3}{4\pi^2\gamma_{mn}^2} \left\{ \frac{P^{(1)}(\gamma_{mn}\zeta)}{P^{(2)}(\gamma_{mn}\zeta)} \right\}, \\
 Q^{(1)}(\gamma_{mn}\zeta) \} &= \frac{ch\pi\gamma_{mn}(1-\zeta) \mp ch\pi\gamma_{mn}\zeta}{ch\pi\gamma_{mn} \pm 1}, \\
 Q^{(2)}(\gamma_{mn}\zeta) \} &= \frac{ch\pi\gamma_{mn}(1-\zeta) \mp ch\pi\gamma_{mn}\zeta}{ch\pi\gamma_{mn} \pm 1}, \\
 P^{(1)}(\gamma_{mn}\zeta) \} &= \frac{\pi\gamma_{mn} \{ (1-\zeta)sh\pi\gamma_{mn}\zeta \pm \zeta sh\pi\gamma_{mn}(1-\zeta) \}}{ch\pi\gamma_{mn} \pm 1}, \\
 P^{(2)}(\gamma_{mn}\zeta) \} &= \frac{\pi\gamma_{mn} \{ (1-\zeta)sh\pi\gamma_{mn}\zeta \pm \zeta sh\pi\gamma_{mn}(1-\zeta) \}}{ch\pi\gamma_{mn} \pm 1}, \\
 \gamma_{mn}^2 &= \frac{c^2}{a^2} m^2 + \frac{c^2}{b^2} n^2, \quad \zeta = \frac{z}{c}, \quad R = \frac{r\pi}{c}, \\
 r &= 1, 2, 3, 4, \dots,
 \end{aligned} \right\} \quad (62)$$

$$\left. \begin{aligned}
 \sum_r \frac{(1-(-1)^r)}{(1+(-1)^r)} \frac{1}{T} \cos Rz &= \frac{c^2}{2\pi\gamma_{mn}} \left\{ \frac{\phi^{(1)}(\gamma_{mn}\zeta)}{\phi^{(2)}(\gamma_{mn}\zeta)} - \frac{2}{\pi\gamma_{mn}} \right\}, \\
 \sum_r \frac{(1-(-1)^r)}{(1+(-1)^r)} \frac{R^2}{T^2} \cos Rz &= \frac{c^2}{4\pi\gamma_{mn}} \left\{ \frac{\phi^{(1)}(\gamma_{mn}\zeta) - \Psi^{(1)}(\gamma_{mn}\zeta)}{\phi^{(2)}(\gamma_{mn}\zeta) - \Psi^{(2)}(\gamma_{mn}\zeta)} \right\}, \\
 \sum_r \frac{(1-(-1)^r)}{(1+(-1)^r)} \frac{1}{T^2} \cos Rz &= \frac{c^3}{4\pi^3\gamma_{mn}^3} \left\{ \frac{\phi^{(1)}(\gamma_{mn}\zeta) + \Psi^{(1)}(\gamma_{mn}\zeta)}{\phi^{(2)}(\gamma_{mn}\zeta) + \Psi^{(2)}(\gamma_{mn}\zeta)} - \frac{2}{\pi\gamma_{mn}} \right\}, \\
 \end{aligned} \right\} \quad (63)$$

$$\left. \begin{aligned} \phi^{(1)}(\gamma_{mn}\xi) &= \frac{sh\pi\gamma_{mn}(1-\xi) \mp sh\pi\gamma_{mn}\xi}{ch\pi\gamma_{mn} \pm 1}, \\ \phi^{(2)}(\gamma_{mn}\xi) &= \frac{\pi\gamma_{mn} \{ \xi ch\pi\gamma_{mn}(1-\xi) \mp (1-\xi) ch\pi\gamma_{mn}\xi \}}{ch\pi\gamma_{mn} \pm 1}, \\ r &= 1, 2, 3, 4, \dots, \end{aligned} \right\}$$

the inversion theorem stated by Eqs. (17), (16) and (15), yield the components of the displacement without the body forces X , Y , and Z , as follows:

$$\left. \begin{aligned} u = & \sum_m \sum_n \frac{c^2 m}{2a\gamma_{mn}} \left[K_{mn} \left\{ -\frac{\mu}{2\mu+\lambda} \phi^{(1)}(\gamma_{mn}\xi) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\gamma_{mn}\xi) \right\} \right. \\ & + K'_{mn} \left\{ -\frac{\mu}{2\mu+\lambda} \phi^{(2)}(\gamma_{mn}\xi) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\gamma_{mn}\xi) \right\} \Big] \times \cos \frac{m\pi}{a} x \\ & \times \sin \frac{n\pi}{b} y + \sum_m \sum_n \frac{b^3 m r}{2ca\beta_{mr}^2} \frac{\mu+\lambda}{2\mu+\lambda} \left\{ G_{mr} P^{(1)}(\beta_{mr}\eta) + G'_{mr} P^{(2)}(\beta_{mr}\eta) \right\} \\ & \times \cos \frac{m\pi}{a} x \times \cos \frac{r\pi}{c} z - \sum_n \sum_r \frac{ra^2}{2ca_{nr}} \left[E_{nr} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(1)}(\alpha_{nr}\xi) \right. \right. \\ & + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\alpha_{nr}\xi) \Big\} + E'_{nr} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\alpha_{nr}\xi) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\alpha_{nr}\xi) \right\} \Big] \cdot \\ & \sin \frac{n\pi}{b} y \cos \frac{r\pi}{c} z + \sum_m \sum_r \frac{b}{2} \left[H_{mr} \left\{ Q^{(1)}(\beta_{mr}\eta) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{1}{\beta_{mr}^2} \left(\frac{mb}{a} \right)^2 \right. \right. \\ & \times P^{(1)}(\beta_{mr}\eta) \Big\} + H'_{mr} \left\{ Q^{(2)}(\beta_{mr}\eta) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{1}{\beta_{mr}^2} \left(\frac{mb}{a} \right)^2 \cdot P^{(2)}(\beta_{mr}\eta) \right\} \Big] \cdot \\ & \cos \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_n \sum_r \frac{a^2 n}{2ba_{nr}} \left[J_{nr} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(1)}(\alpha_{nr}\xi) + \frac{\mu+\lambda}{2\mu+\lambda} \right. \right. \\ & \psi^{(1)}(\alpha_{nr}\xi) \Big\} + J'_{nr} \left\{ \frac{\mu}{2\mu+\lambda} P^{(2)}(\alpha_{nr}\xi) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\alpha_{nr}\xi) \right\} \Big] \cdot \sin \frac{n\pi}{b} y \cdot \\ & \cos \frac{r\pi}{c} z - \sum_m \sum_r \frac{b^3 m}{4a\beta_{mr}^2} \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \left\{ B_{mr} P^{(1)}(\beta_{mr}\eta) + B'_{mr} P^{(2)}(\beta_{mr}\eta) \right\} \\ & \times \cos \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_m \sum_n \frac{c^4 mn}{4ab(\gamma_{mn})^3} \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \left[F_{mn} \left\{ \phi^{(1)}(\gamma_{mn}\xi) \right. \right. \\ & + \psi^{(1)}(\gamma_{mn}\xi) \Big\} + F'_{mn} \left\{ \phi^{(2)}(\gamma_{mn}\xi) + \psi^{(2)}(\gamma_{mn}\xi) \right\} \Big] \cdot \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \\ & + \sum_n \sum_r \frac{a^2}{4a_{nr}} \left[A_{nr} \left\{ \frac{3\mu+\lambda}{\mu(2\mu+\lambda)} \phi^{(1)}(\alpha_{nr}\xi) + \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \psi^{(1)}(\alpha_{nr}\xi) \right\} \right. \\ & + A'_{nr} \left\{ \frac{3\mu+\lambda}{\mu(2\mu+\lambda)} \phi^{(2)}(\alpha_{nr}\xi) + \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \psi^{(2)}(\alpha_{nr}\xi) \right\} \Big] \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z \\ & + \sum_m \sum_n \frac{c^2}{4\gamma_{mn}\mu} \left[D_{mn} \left\{ \left(2 - \frac{\mu+\lambda}{2\mu+\lambda} \frac{c^2 m^2}{a^2 \gamma_{mn}^2} \right) \times \phi^{(1)}(\gamma_{mn}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} \right. \right. \\ & \times \frac{c^2 m^2}{a^2 \gamma_{mn}^2} \psi^{(1)}(\gamma_{mn}\xi) \Big\} + D'_{mn} \left\{ \left(2 - \frac{\mu+\lambda}{2\mu+\lambda} \frac{c^2 m^2}{a^2 \gamma_{mn}^2} \right) \cdot \phi^{(2)}(\gamma_{mn}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} \right. \end{aligned} \right\} \quad (64)$$

$$\begin{aligned}
& \times \frac{c^2 m^2}{a^2 \gamma_{mn}^2} \psi^{(2)}(\gamma_{mn} \xi) \left\} \right] \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y + \frac{a^2}{8\mu} \sum_n \frac{b}{an} \left[A_{n0} \left\{ \frac{3\mu+\lambda}{2\mu+\lambda} \right. \right. \\
& \times \phi^{(1)}(\alpha_{n0} \xi) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\alpha_{n0} \xi) \left\} + A'_{n0} \left\{ \frac{3\mu+\lambda}{2\mu+\lambda} \times \phi^{(2)}(\alpha_{n0} \xi) + \frac{\mu+\lambda}{2\mu+\lambda} \right. \right. \\
& \times \psi^{(2)}(\alpha_{n0} \xi) \left\} \right] \sin \frac{n\pi}{b} y + \frac{b}{4} \sum_m \left[H_{m0} \left\{ Q^{(1)}(\beta_{m0} \eta) - \frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\beta_{m0} \eta) \right\} \right. \\
& + H'_{m0} \left\{ Q^{(2)}(\beta_{m0} \eta) - \frac{\mu+\lambda}{2\mu+\lambda} P^{(2)}(\beta_{m0} \eta) \right\} \left] \cos \frac{m\pi}{a} x + \frac{a}{4} \sum_n \left[J_{n0} \left\{ \frac{\mu}{2\mu+\lambda} \right. \right. \\
& \times \phi^{(1)}(\alpha_{n0} \xi) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\alpha_{n0} \xi) \left\} + J'_{n0} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\alpha_{n0} \xi) + \frac{\mu+\lambda}{2\mu+\lambda} \right. \right. \\
& \times \psi^{(2)}(\alpha_{n0} \xi) \left\} \right] \sin \frac{n\pi}{b} y - \sum_m \frac{ab}{8m} \frac{1}{2\mu+\lambda} \left\{ B_{m0} \times P^{(1)}(\beta_{m0} \eta) + B'_{m0} \right. \\
& \times P^{(2)}(\beta_{m0} \eta) \left\} \cos \frac{m\pi}{a} x + \sum_n \frac{bc}{4n} \left\{ D_{0n} \phi^{(1)}(\gamma_{0n} \xi) + D'_{0n} \phi^{(2)}(\gamma_{0n} \xi) \right\} \sin \frac{n\pi}{b} y \\
& - \frac{b}{4} \sum_r \left\{ J_{0r} Q^{(1)}(\beta_{0r} \eta) + J'_{0r} Q^{(2)}(\beta_{0r} \eta) \right\} \cos \frac{r\pi}{c} z - \frac{b}{8} \left\{ J_{00}(1-2\gamma) + J_{00} \right\} \\
& m, n, r = 1, 2, 3, 4, \dots,
\end{aligned}$$

$$\begin{aligned}
v = & - \sum_m \sum_n \frac{c^2 n}{2b\gamma_{mn}} \left[K_{mn} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(1)}(\gamma_{mn} \xi) - \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\gamma_{mn} \xi) \right\} \right. \\
& + K'_{mn} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\gamma_{mn} \xi) - \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\gamma_{mn} \xi) \right\} \left] \sin \frac{m\pi}{a} x \cos \frac{n\pi}{b} y \\
& + \sum_n \sum_r \frac{a^3 nr}{2cb a_{nr}^2} \frac{\mu+\lambda}{2\mu+\lambda} \left\{ E_{nr} P^{(1)}(\alpha_{nr} \xi) + E'_{nr} P^{(2)}(\alpha_{nr} \xi) \right\} \cos \frac{n\pi}{b} y \cos \frac{r\pi}{c} z \\
& - \sum_n \sum_r \frac{a^3 n}{4ba_{nr}^2} \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \left\{ A_{nr} P^{(1)}(\alpha_{nr} \xi) + A'_{nr} P^{(2)}(\alpha_{nr} \xi) \right\} \cos \frac{n\pi}{b} y \cdot \cos \\
& \times \frac{r\pi}{c} z - \sum_m \sum_r \frac{rb^2}{2c\beta_{mr}} \left[G_{mr} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(1)}(\beta_{mr} \eta) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\beta_{mr} \eta) \right\} \right. \\
& + G'_{mr} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\beta_{mr} \eta) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\beta_{mr} \eta) \right\} \left] \sin \frac{m\pi}{a} x \cos \frac{r\pi}{c} z \\
& + \sum_n \sum_r \frac{a}{2} \left[J_{nr} \left\{ Q^{(1)}(\alpha_{nr} \xi) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{n^2 a^2}{b^2 a_{nr}^2} P^{(1)}(\alpha_{nr} \xi) \right\} + J'_{nr} \left\{ Q^{(2)}(\alpha_{nr} \xi) \right. \right. \\
& - \frac{\mu+\lambda}{2\mu+\lambda} \frac{n^2 a^2}{b^2 a_{nr}^2} P^{(2)}(\alpha_{nr} \xi) \left. \right\} \left] \cos \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_m \sum_n \frac{b^2 m}{4a\beta_{mr}} \\
& \times \left[H_{mr} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(1)}(\beta_{mr} \eta) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\beta_{mr} \eta) \right\} + H'_{mr} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\beta_{mr} \eta) \right. \right. \\
& + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\beta_{mr} \eta) \left. \right\} \left] \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z - \sum_m \sum_n \frac{c^4 mn}{4ab(\gamma_{mn}^3)} \\
& \times \frac{\mu+\lambda}{2\mu+\lambda} \left[D_{mn} \left\{ \phi^{(1)}(\gamma_{mn} \xi) + \psi^{(1)}(\gamma_{mn} \xi) \right\} + D'_{mn} \left\{ \phi^{(1)}(\gamma_{mn} \xi) + \psi^{(1)}(\gamma_{mn} \xi) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
& \times \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y + \sum_m \sum_r \frac{b^2}{4\beta_{mr}} \left[B_{mr} \left\{ \frac{3\mu+\lambda}{\mu(2\mu+\lambda)} \phi^{(1)}(\beta_{mr}\eta) \right. \right. \\
& + \left. \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \psi^{(1)}(\beta_{mr}\eta) \right\} + B'_{mr} \left\{ \frac{3\mu+\lambda}{\mu(2\mu+\lambda)} \phi^{(2)}(\beta_{mr}\eta) + \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \right. \\
& \times \left. \psi^{(2)}(\beta_{mr}\eta) \right\} \left] \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z + \sum_m \sum_n \frac{c^2}{4\gamma_{mn}\mu} \left[F_{mn} \left\{ \left(2 - \frac{\mu+\lambda}{2\mu+\lambda} \right. \right. \right. \\
& \times \left. \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \right\} \phi^{(1)}(\gamma_{mn}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \psi^{(1)}(\gamma_{mn}\xi) \left. \right\} + F'_{mn} \left\{ \left(2 - \frac{\mu+\lambda}{2\mu+\lambda} \right. \right. \\
& \times \left. \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \right\} \phi^{(2)}(\gamma_{mn}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \psi^{(2)}(\gamma_{mn}\xi) \left. \right\} \left] \sin \frac{m\pi}{a} x \cos \frac{n\pi}{b} y \right. \\
& - \frac{b^2}{8\mu} \sum_m \frac{a}{bm} \left[B_{m0} \left\{ \frac{3\mu+\lambda}{2\mu+\lambda} \phi^{(1)}(\beta_{m0}\eta) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\beta_{m0}\eta) \right\} + B'_{m0} \right. \\
& \times \left. \left\{ \frac{3\mu+\lambda}{2\mu+\lambda} \phi^{(2)}(\beta_{m0}\eta) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\beta_{m0}\eta) \right\} \right] \sin \frac{m\pi}{a} x + \frac{a}{4} \sum_n \left[J_{n0} \right. \\
& \times \left. \left\{ Q^{(1)}(\alpha_{n0}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\alpha_{n0}\xi) \right\} + J'_{n0} \left\{ Q^{(2)}(\alpha_{n0}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} P^{(2)}(\alpha_{n0}\xi) \right\} \right] \\
& \times \cos \frac{n\pi}{b} y + \frac{b}{4} \sum_m \left[H_{m0} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(1)}(\beta_{m0}\eta) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\beta_{m0}\eta) \right\} \right. \\
& + H'_{m0} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\beta_{m0}\eta) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\beta_{m0}\eta) \right\} \left] \sin \frac{m\pi}{a} x - \sum_n \frac{ba}{8n} \right. \\
& \times \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \left\{ A_{n0} \times P^{(1)}(\alpha_{n0}\xi) + A'_{n0} \times P^{(2)}(\alpha_{n0}\xi) \right\} \cos \frac{n\pi}{b} y + \sum_n \frac{ac}{4m} \\
& \times \left\{ F_{m0} \phi^{(1)}(\gamma_{m0}\xi) + F'_{m0} \phi^{(2)}(\gamma_{m0}\xi) \right\} \sin \frac{m\pi}{a} x - \frac{a}{4} \sum_r \left\{ H_{0r} Q^{(1)}(\beta_{0r}\eta) \right. \\
& + H'_{0r} Q^{(2)}(\beta_{0r}\eta) \left. \right\} \cos \frac{r\pi}{c} z - \frac{a}{8} \left\{ H_{00}(1-2\xi) + H_{00} \right\} \\
& m, n, r = 1, 2, 3, 4, \dots,
\end{aligned} \tag{65}$$

$$\begin{aligned}
w = & - \sum_m \sum_n \frac{c}{2} \left[K_{mn} \left\{ Q^{(1)}(\gamma_{mn}\xi) + \frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\gamma_{mn}\xi) \right\} + K'_{mn} \left\{ Q^{(2)}(\gamma_{mn}\xi) \right. \right. \\
& + \left. \frac{\mu+\lambda}{2\mu+\lambda} P^{(2)}(\gamma_{mn}\xi) \right\} \left] \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y + \sum_n \sum_r \frac{a}{2} \left[E_{nr} \left\{ Q^{(1)}(\alpha_{nr}\xi) \right. \right. \\
& - \left. \frac{\mu+\lambda}{2\mu+\lambda} \frac{r^2 a^2}{c^2 \alpha_{nr}^2} P^{(1)}(\alpha_{nr}\xi) \right\} + E'_{nr} \left\{ Q^{(2)}(\alpha_{nr}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{r^2 a^2}{c^2 \alpha_{nr}^2} P^{(2)}(\alpha_{nr}\xi) \right\} \left] \right. \\
& \times \sin \frac{n\pi}{b} y \cdot \sin \frac{r\pi}{c} z + \sum_m \sum_r \frac{b}{2} \left[G_{mr} \left\{ Q^{(1)}(\beta_{mr}\eta) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{r^2 b^2}{c^2 \beta_{mr}^2} \right. \right. \\
& \times \left. P^{(1)}(\beta_{mr}\eta) \right\} + G'_{mr} \left\{ Q^{(2)}(\beta_{mr}\eta) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{r^2 b^2}{c^2 \beta_{mr}^2} P^{(2)}(\beta_{mr}\eta) \right\} \left] \sin \frac{m\pi}{a} x \right. \\
& \times \sin \frac{r\pi}{c} z + \sum_n \sum_r \frac{a^3 r}{4c \alpha_{nr}^2} \frac{\mu+\lambda}{\mu(2\mu+\lambda)} \left\{ A_{nr} P^{(1)}(\alpha_{nr}\xi) + A'_{nr} P^{(2)}(\alpha_{nr}\xi) \right\}
\end{aligned}$$

$$\begin{aligned}
& \times \sin \frac{n\pi}{b} y \sin \frac{r\pi}{c} z + \sum_m \sum_n \frac{c^3 m}{4a\gamma_{mn}^2} \frac{\mu + \lambda}{2\mu + \lambda} \left\{ D_{mn} P^{(1)}(\gamma_{mn}\xi) \right. \\
& + D'_{mn} P^{(2)}(\gamma_{mn}\xi) \left. \right\} \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y - \sum_n \sum_r \frac{a^3 nr}{2bca_{nr}^2} \frac{\mu + \lambda}{2\mu + \lambda} \\
& \times \left\{ J_{nr} P^{(1)}(\alpha_{nr}\xi) + J'_{nr} P^{(2)}(\alpha_{nr}\xi) \right\} \sin \frac{n\pi}{b} y \cdot \sin \frac{r\pi}{c} z \\
& + \sum_m \sum_n \frac{b^3 r}{4c\beta_{mr}^2} \frac{\mu + \lambda}{\mu(2\mu + \lambda)} \left\{ B_{mr} P^{(1)}(\beta_{mr}\eta) + B'_{mr} P^{(2)}(\beta_{mr}\eta) \right\} \sin \frac{m\pi}{a} x \cdot \sin \\
& \times \frac{r\pi}{c} z + \sum_m \sum_n \frac{c^3 n}{4b\gamma_{mn}^2} \frac{\eta + \lambda}{2\mu + \lambda} \left\{ F_{mn} P^{(1)}(\gamma_{mn}\xi) + F'_{mn} P^{(2)}(\gamma_{mn}\xi) \right\} \sin \\
& \times \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y + \sum_m \sum_r \frac{b^3 mr}{2ac\beta_{mr}^2} \frac{\mu + \lambda}{2\mu + \lambda} \left\{ H_{mr} P^{(1)}(\beta_{mr}\eta) \right. \\
& + H'_{mr} P^{(2)}(\beta_{mr}\eta) \left. \right\} \sin \frac{m\pi}{a} x \cdot \sin \frac{r\pi}{c} z, \\
& m, n, r = 1, 2, 3, 4, \dots,
\end{aligned} \tag{66}$$

The dilatation is, therefore, written in the form:

$$\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = e = \sum_m \sum_n \frac{\mu}{2\mu + \lambda} \gamma_{mn} \left\{ K_{mn} \phi^{(1)}(\gamma_{mn}\xi) \right. \\
+ K'_{mn} \phi^{(2)}(\gamma_{mn}\xi) \left. \right\} \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y + \sum_n \sum_r \frac{\mu}{2\mu + \lambda} \frac{ar}{c} \left\{ E_{nr} Q^{(1)}(\alpha_{nr}\xi) \right. \\
+ E'_{nr} Q^{(2)}(\alpha_{nr}\xi) \left. \right\} \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z + \sum_m \sum_r \frac{\mu}{2\mu + \lambda} \frac{br}{c} \left\{ G_{mr} Q^{(1)}(\beta_{mr}\eta) \right. \\
+ G'_{mr} Q^{(2)}(\beta_{mr}\eta) \left. \right\} \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_m \sum_r \frac{bm}{a} \frac{\mu}{2\mu + \lambda} \left\{ H_{mr} Q^{(1)}(\beta_{mr}\eta) \right. \\
+ H'_{mr} Q^{(2)}(\beta_{mr}\eta) \left. \right\} \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_n \sum_r \frac{an}{b} \frac{\mu}{2\mu + \lambda} \left\{ J_{nr} Q^{(1)}(\alpha_{nr}\xi) \right. \\
+ J'_{nr} Q^{(2)}(\alpha_{nr}\xi) \left. \right\} \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z - \sum_m \sum_r \frac{b}{2\mu + \lambda} \left\{ B_{mr} Q^{(1)}(\beta_{mr}\eta) \right. \\
+ B'_{mr} Q^{(2)}(\beta_{mr}\eta) \left. \right\} \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_n \sum_r \frac{a}{2\mu + \lambda} \left\{ A_{nr} Q^{(1)}(\alpha_{nr}\xi) \right. \\
+ A'_{nr} Q^{(2)}(\alpha_{nr}\xi) \left. \right\} \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z - \sum_n \sum_r \frac{c^2 n}{2b\gamma_{mn}^2} \frac{1}{2\mu + \lambda} \\
\times \left\{ F_{mn} \phi^{(1)}(\gamma_{mn}\xi) + F'_{mn} \phi^{(2)}(\gamma_{mn}\xi) \right\} \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y - \sum_m \sum_n \frac{c^2 m}{2a\gamma_{mn}^2} \\
\times \frac{1}{2\mu + \lambda} \left\{ D_{mn} \phi^{(1)}(\gamma_{mn}\xi) + D'_{mn} \phi^{(2)}(\gamma_{mn}\xi) \right\} \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z \\
- \sum_n \frac{a}{4} \frac{1}{2\mu + \lambda} \left\{ A_{n0} Q^{(1)}(\alpha_{n0}\xi) + A'_{n0} Q^{(2)}(\alpha_{n0}\xi) \right\} \sin \frac{n\pi}{b} y \\
- \sum_m \frac{b}{4} \frac{1}{2\mu + \lambda} \left\{ B_{m0} Q^{(1)}(\beta_{m0}\eta) + B'_{m0} Q^{(2)}(\beta_{m0}\eta) \right\} \sin \frac{m\pi}{a} x
\end{aligned} \tag{67}$$

$$\left. \begin{aligned} & - \sum_m \frac{bm}{2a} \frac{\mu}{2\mu+\lambda} \left\{ H_{m0} Q^{(1)}(\beta_{m0}\eta) + H'_{m0} Q^{(2)}(\beta_{m0}\eta) \right\} \sin \frac{m\pi}{a} x \\ & - \sum_n \frac{an}{2b} \frac{\mu}{2\mu+\lambda} \left\{ J_{n0} Q^{(1)}(\alpha_{n0}\xi) + J'_{n0} Q^{(2)}(\alpha_{n0}\xi) \right\} \sin \frac{n\pi}{b} y. \end{aligned} \right\}$$

Then, the relations (4) and (5) yield the component of stresses as follows:

$$\begin{aligned} \sigma_x &= 2\mu \frac{\partial u}{\partial x} + \lambda e \\ &= \sum_n \sum_r \left[K_{mn} \left\{ \left(\frac{\mu^2}{2\mu+\lambda} \frac{c^2 m^2}{a^2 r_{mn}^2} + \frac{\lambda \mu}{2\mu+\lambda} r_{mn} \right) \phi^{(1)}(r_{mn}\zeta) \right. \right. \\ &\quad \left. \left. - \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \frac{c^2 m^2}{a^2 r_{mn}^2} \psi^{(1)}(r_{mn}\zeta) \right\} + K'_{mn} \left\{ \left(\frac{\mu^2}{2\mu+\lambda} \frac{c^2 m^2}{a^2 r_{mn}^2} + \frac{\lambda \mu}{2\mu+\lambda} r_{mn} \right) \right. \right. \\ &\quad \left. \left. \times \phi^{(2)}(r_{mn}\zeta) - \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \frac{c^2 m^2}{a^2 r_{mn}^2} \psi^{(2)}(r_{mn}\zeta) \right\} \right] \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \\ &\quad + \sum_m \sum_r \frac{br}{c} \left[G_{mr} \left\{ \frac{\lambda \mu}{2\mu+\lambda} Q^{(1)}(\beta_{mr}\eta) - \frac{b^2 m^2}{a^2 \beta_{mr}^2} \frac{\mu(\lambda+\mu)}{2\mu+\lambda} P^{(1)}(\beta_{mr}\eta) \right\} \right. \\ &\quad \left. + G'_{mr} \left\{ \frac{\lambda \mu}{2\mu+\lambda} Q^{(2)}(\beta_{mr}\eta) - \frac{b^2 m^2}{a^2 \beta_{mr}^2} \frac{\mu(\lambda+\mu)}{2\mu+\lambda} P^{(2)}(\beta_{mr}\eta) \right\} \right] \sin \frac{m\pi}{a} x \\ &\quad \times \cos \frac{r\pi}{c} z + \sum_n \sum_r \frac{ra}{c} \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \left\{ E_{nr} P^{(1)}(\alpha_{nr}\xi) + E'_{nr} P^{(2)}(\alpha_{nr}\xi) \right\} \\ &\quad \times \sin \frac{n\pi}{b} y \cos \frac{r\pi}{c} z - \sum_m \sum_r \frac{bm}{a} \left[H_{mr} \left\{ \frac{\mu(3\mu+\lambda)}{2\mu+\lambda} Q^{(1)}(\beta_{mr}\eta) \right. \right. \\ &\quad \left. \left. + \frac{\lambda(\mu+\lambda)}{2\mu+\lambda} \frac{b^2 m^2}{a^2 \beta_{mr}^2} P^{(1)}(\beta_{mr}\eta) \right\} + H'_{mr} \left\{ \frac{\mu(3\mu+\lambda)}{2\mu+\lambda} Q^{(2)}(\beta_{mr}\eta) \right. \right. \\ &\quad \left. \left. + \frac{\lambda(\mu+\lambda)}{2\mu+\lambda} \frac{b^2 m^2}{a^2 \beta_{mr}^2} P^{(2)}(\beta_{mr}\eta) \right\} \right] \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_m \sum_r \frac{an}{b} \\ &\quad \times \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \left\{ J_{nr} P^{(1)}(\alpha_{nr}\xi) + J'_{nr} P^{(2)}(\alpha_{nr}\xi) \right\} \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z - \sum_m \sum_r \\ &\quad \times \frac{b}{2} \left[B_{mr} \left\{ \frac{\lambda}{2\mu+\lambda} Q^{(1)}(\beta_{mr}\eta) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{b^2 m^2}{a^2 \beta_{mr}^2} P^{(1)}(\beta_{mr}\eta) \right\} + B'_{mr} \right. \\ &\quad \left. \times \left\{ \frac{\lambda}{2\mu+\lambda} Q^{(2)}(\beta_{mr}\eta) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{b^2 m^2}{a^2 \beta_{mr}^2} P^{(2)}(\beta_{mr}\eta) \right\} \right] \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z \\ &\quad - \sum_m \sum_n \frac{c^2 n}{2br_{mn}} \left[F'_{mn} \left\{ \left(\frac{\lambda}{2\mu+\lambda} + \frac{\lambda(\mu+\lambda)}{\mu(2\mu+\lambda)} \frac{c^2 m^2}{a^2 r_{mn}^2} \right) \phi^{(1)}(r_{mn}\zeta) \right. \right. \\ &\quad \left. \left. - \frac{c^2 m^2}{a^2 r_{mn}^2} \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(r_{mn}\zeta) \right\} + F'_{mn} \left\{ \left(\frac{\lambda}{2\mu+\lambda} + \frac{\lambda(\mu+\lambda)}{\mu(2\mu+\lambda)} \frac{c^2 m^2}{a^2 r_{mn}^2} \right) \right. \right. \\ &\quad \left. \left. \times \phi^{(2)}(r_{mn}\zeta) - \frac{c^2 m^2}{a^2 r_{mn}^2} \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(r_{mn}\zeta) \right\} \right] \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \\ &\quad - \sum_n \sum_r \frac{a}{2} \left[A_{nr} \left\{ Q^{(1)}(\alpha_{nr}\xi) + \frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\alpha_{nr}\xi) \right\} + A'_{nr} \left\{ Q^{(2)}(\alpha_{nr}\xi) \right. \right. \end{aligned} \quad \left. \right\} \quad (68)$$

$$\begin{aligned}
& + \frac{\mu + \lambda}{2\mu + \lambda} P^{(2)}(\alpha_{nr}\xi) \Big\} \Big] \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z - \sum_n \sum_r \frac{c^2 m}{2a r_{mn}} \left[D_{mn} \right. \\
& \times \left\{ \left(\frac{4\mu + 3\lambda}{2\mu + \lambda} - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 m^2}{a^2 r_{mn}^2} \right) \times \phi^{(1)}(r_{mn}\xi) - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 m^2}{a^2 r_{mn}^2} \psi^{(1)}(r_{mn}\xi) \right\} \\
& - D_{mn} \left\{ \left(\frac{4\mu + 3\lambda}{2\mu + \lambda} - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 m^2}{a^2 r_{mn}^2} \right) \phi^{(2)}(r_{mn}\xi) - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 m^2}{a^2 r_{mn}^2} \right. \\
& \times \psi^{(2)}(r_{mn}\xi) \Big\} \Big] \sin \frac{m\pi}{a} x \times \sin \frac{n\pi}{b} y - \sum_n \frac{a}{4} \left[A_{n0} \left\{ Q^{(1)}(\alpha_{n0}\xi) \right. \right. \\
& + \frac{\mu + \lambda}{2\mu + \lambda} P^{(1)}(\alpha_{n0}\xi) \Big\} + A_{n0}' \left\{ Q^{(2)}(\alpha_{n0}\xi) + \frac{\mu + \lambda}{2\mu + \lambda} P^{(2)}(\alpha_{n0}\xi) \right\} \Big] \sin \frac{n\pi}{b} y \\
& - \sum_n \frac{bm}{2a} \left[H_{m0} \left\{ \frac{\mu(3\mu + \lambda)}{2\mu + \lambda} Q^{(1)}(\beta_{m0}\eta) + \frac{\lambda(\mu + \lambda)}{2\mu + \lambda} \times P^{(1)}(\beta_{m0}\eta) \right\} \right. \\
& + H_{m0}' \left\{ \frac{\mu(3\mu + \lambda)}{2\mu + \lambda} Q^{(2)}(\beta_{m0}\eta) + \frac{\lambda(\mu + \lambda)}{2\mu + \lambda} P^{(2)}(\beta_{m0}\eta) \right\} \Big] \sin \frac{m\pi}{a} x \\
& - \sum_n \frac{an}{2b} \frac{\mu(\mu + \lambda)}{2\mu + \lambda} \left\{ J_{n0} P^{(1)}(\alpha_{n0}\xi) + J_{n0}' P^{(2)}(\alpha_{n0}\xi) \right\} \sin \frac{n\pi}{b} y \\
& - \sum_n \frac{b}{4} \left[B_{m0} \left\{ \frac{\lambda}{2\mu + \lambda} Q^{(1)}(\beta_{m0}\eta) - \frac{\mu + \lambda}{2\mu + \lambda} P^{(1)}(\beta_{m0}\eta) \right\} \right. \\
& + B_{m0}' \left\{ \frac{\lambda}{2\mu + \lambda} Q^{(2)}(\beta_{m0}\eta) - \frac{\mu + \lambda}{2\mu + \lambda} P^{(2)}(\beta_{m0}\eta) \right\} \Big] \sin \frac{m\pi}{a} x, \\
\sigma_y = 2\mu \frac{\partial v}{\partial y} + \lambda e = & \sum_n \sum_r \left[K_{mn} \left\{ \left(\frac{\mu^2}{2\mu + \lambda} \frac{c^2 n^2}{b^2 r_{mn}} + \frac{\mu\lambda}{2\mu + \lambda} r_{mn} \right) \phi^{(1)}(r_{mn}\xi) \right. \right. \\
& - \frac{\mu(\mu + \lambda)}{2\mu + \lambda} \frac{c^2 n^2}{b^2 r_{mn}} \psi^{(1)}(r_{mn}\xi) \Big\} + K_{mn}' \left\{ \left(\frac{\mu^2}{2\mu + \lambda} \frac{c^2 n^2}{b^2 r_{mn}} + \frac{\mu\lambda}{2\mu + \lambda} r_{mn} \right) \right. \\
& \times \phi^{(2)}(r_{mn}\xi) - \frac{\mu(\mu + \lambda)}{2\mu + \lambda} \frac{c^2 n^2}{b^2 r_{mn}} \psi^{(2)}(r_{mn}\xi) \Big\} \Big] \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \\
& - \sum_n \sum_r \frac{ar}{c} \left[E_{nr} \left\{ \frac{a^2 n^2}{b^2 a_{nr}^2} \frac{\mu(\mu + \lambda)}{2\mu + \lambda} P^{(1)}(\alpha_{nr}\xi) - \frac{\mu\lambda}{2\mu + \lambda} Q^{(1)}(\alpha_{nr}\xi) \right\} \right. \\
& + E_{nr}' \left\{ \frac{a^2 n^2}{b^2 a_{nr}^2} \frac{\mu(\mu + \lambda)}{2\mu + \lambda} P^{(2)}(\alpha_{nr}\xi) - \frac{\mu\lambda}{2\mu + \lambda} Q^{(2)}(\alpha_{nr}\xi) \right\} \Big] \sin \frac{n\pi}{b} y \\
& \times \cos \frac{r\pi}{c} z + \sum_n \sum_r \frac{rb}{c} \frac{\mu(\mu + \lambda)}{2\mu + \lambda} \times \left\{ G_{mr} P^{(1)}(\beta_{mr}\eta) + G_{mr}' P^{(2)}(\beta_{mr}\eta) \right\} \\
& \times \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z + \sum_n \sum_r \frac{an}{b} \left[J_{nr} \left\{ \frac{\mu(3\mu + \lambda)}{2\mu + \lambda} Q^{(1)}(\alpha_{nr}\xi) \right. \right. \\
& + \frac{\lambda(\mu + \lambda)}{2\mu + \lambda} \frac{a^2 n^2}{b^2 a_{nr}^2} P^{(1)}(\alpha_{nr}\xi) \Big\} + J_{nr}' \left\{ \frac{\mu(3\mu + \lambda)}{2\mu + \lambda} Q^{(2)}(\alpha_{nr}\xi) \right. \\
& + \frac{\lambda(\mu + \lambda)}{2\mu + \lambda} \frac{a^2 n^2}{b^2 a_{nr}^2} P^{(2)}(\alpha_{nr}\xi) \Big\} \Big] \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z - \sum_n \sum_r \frac{am}{b} \frac{\mu(\mu + \lambda)}{2\mu + \lambda} \\
& \times \left\{ H_{mr} P^{(1)}(\beta_{mr}\eta) + H_{mr}' P^{(2)}(\beta_{mr}\eta) \right\} \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_n \sum_r \frac{a}{2}
\end{aligned}$$

$$\begin{aligned}
& \times \left[A_{nr} \left\{ \frac{\lambda}{2\mu+\lambda} Q^{(1)}(\alpha_{nr}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{\alpha^2 n^2}{b^2 a_{nr}^2} P^{(1)}(\alpha_{nr}\xi) \right\} + A'_{nr} \right. \\
& \times \left. \left\{ \frac{\lambda}{2\mu+\lambda} Q^{(2)}(\alpha_{nr}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{\alpha^2 n^2}{b^2 a_{nr}^2} P^{(2)}(\alpha_{nr}\xi) \right\} \right] \sin \frac{n\pi}{b} y \cos \frac{r\pi}{c} z \\
& - \sum_m \sum_r \frac{c^2 m}{2a\gamma_{mn}} \left[D_{mn} \left\{ \left(\frac{\lambda}{2\mu+\lambda} + \frac{\lambda(\mu+\lambda)}{\mu(2\mu+\lambda)} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \right) \phi^{(1)}(\gamma_{mn}\xi) - \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \right. \right. \\
& \times \left. \left. \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\gamma_{mn}\xi) \right\} + D'_{mn} \left\{ \left(\frac{\lambda}{2\mu+\lambda} + \frac{\lambda(\mu+\lambda)}{\mu(2\mu+\lambda)} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \right) \phi^{(2)}(\gamma_{mn}\xi) \right. \right. \\
& \times \left. \left. \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\gamma_{mn}\xi) \right\} \right] \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y - \sum_m \sum_r \frac{b}{2} \\
& \times \left[B_{mr} \left\{ Q^{(1)}(\beta_{mr}\eta) + \frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\beta_{mr}\eta) \right\} + B'_{mr} \left\{ Q^{(2)}(\beta_{mr}\eta) \right. \right. \\
& + \left. \left. \frac{\mu+\lambda}{2\mu+\lambda} P^{(2)}(\beta_{mr}\eta) \right\} \right] \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_m \sum_n \frac{c^2 m}{2b\gamma_{mn}} \left[F_{mn} \right. \\
& \times \left. \left\{ \left(\frac{4\mu+3\lambda}{2\mu+\lambda} - \frac{\mu+\lambda}{2\mu+\lambda} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \right) \phi^{(1)}(\gamma_{mn}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \psi^{(1)}(\gamma_{mn}\xi) \right\} \right. \\
& + \left. F'_{mn} \left\{ \left(\frac{4\mu+3\lambda}{2\mu+\lambda} - \frac{\mu+\lambda}{2\mu+\lambda} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \right) \phi^{(2)}(\gamma_{mn}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \psi^{(2)}(\gamma_{mn}\xi) \right\} \right] \\
& \times \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y - \sum_m \frac{b}{4} \left[B_{m0} \left\{ Q^{(1)}(\beta_{m0}\eta) + \frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\beta_{m0}\eta) \right\} \right. \\
& + \left. B'_{m0} \left\{ Q^{(2)}(\beta_{m0}\eta) + \frac{\mu+\lambda}{2\mu+\lambda} P^{(2)}(\beta_{m0}\eta) \right\} \right] \sin \frac{m\pi}{a} x - \sum_n \frac{an}{2b} \left[J_{n0} \right. \\
& \times \left. \left\{ \frac{\mu(3\mu+\lambda)}{2\mu+\lambda} Q^{(1)}(\alpha_{n0}\xi) + \frac{\lambda(\mu+\lambda)}{2\mu+\lambda} P^{(1)}(\alpha_{n0}\xi) \right\} + J'_{n0} \left\{ \frac{\mu(3\mu+\lambda)}{2\mu+\lambda} Q^{(2)}(\alpha_{n0}\xi) \right. \right. \\
& + \left. \left. \frac{\lambda(\mu+\lambda)}{2\mu+\lambda} P^{(2)}(\alpha_{n0}\xi) \right\} \right] \sin \frac{n\pi}{b} y - \sum_m \frac{bm}{2a} \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \left\{ H_{m0} P^{(1)}(\beta_{m0}\eta) \right. \\
& + \left. H'_{m0} P^{(2)}(\beta_{m0}\eta) \right\} \sin \frac{m\pi}{a} x - \sum_n \frac{a}{4} \left[A_{n0} \left\{ \frac{\lambda}{2\mu+\lambda} Q^{(1)}(\alpha_{n0}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} \right. \right. \\
& \times \left. \left. P^{(1)}(\alpha_{n0}\xi) \right\} + A'_{n0} \left\{ \frac{\lambda}{2\mu+\lambda} Q^{(2)}(\alpha_{n0}\xi) - \frac{\mu+\lambda}{2\mu+\lambda} P^{(2)}(\alpha_{n0}\xi) \right\} \right] \sin \frac{n\pi}{b} y,
\end{aligned} \tag{69}$$

$$\begin{aligned}
\sigma_z &= 2\mu \frac{\partial w}{\partial z} + \lambda e = \sum_m \sum_n \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \gamma_{mn} \left[K_{mn} \left\{ \phi^{(1)}(\gamma_{mn}\xi) + \psi^{(1)}(\gamma_{mn}\xi) \right\} \right. \\
& + \left. K'_{mn} \left\{ \phi^{(2)}(\gamma_{mn}\xi) + \psi^{(2)}(\gamma_{mn}\xi) \right\} \right] \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y + \sum_n \sum_r \frac{ar}{c} \\
& \times \left[E_{nr} \left\{ \frac{2\mu(\mu+\lambda)}{2\mu+\lambda} Q^{(1)}(\alpha_{nr}\xi) - \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \frac{r^2 a^2}{c^2 a_{nr}^2} P^{(1)}(\alpha_{nr}\xi) \right\} + \left[E'_{nr} \right. \right. \\
& \times \left. \left. \left\{ \frac{2\mu(\mu+\lambda)}{2\mu+\lambda} Q^{(2)}(\alpha_{nr}\xi) - \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \frac{r^2 a^2}{c^2 a_{nr}^2} P^{(2)}(\alpha_{nr}\xi) \right\} \right] \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z \right. \\
& + \left. \sum_m \sum_r \frac{br}{c} \left[G_{mr} \left\{ \frac{2\mu(\mu+\lambda)}{2\mu+\lambda} Q^{(1)}(\beta_{mr}\eta) - \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \times \frac{r^2 b^2}{c^2 \beta_{mr}^2} P^{(1)}(\beta_{mr} \eta) \Big\} + G'_{mr} \Big\{ \frac{2\mu(\mu+\lambda)}{2\mu+\lambda} Q^{(2)}(\beta_{mr} \eta) - \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \frac{r^2 b^2}{c^2 \beta_{mr}^2} \\
& \times P^{(1)}(\beta_{mr} \eta) \Big\} \Big] \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_n \sum_r \frac{a}{2} \Big[A_{nr} \Big\{ \frac{\lambda}{2\mu+\lambda} Q^{(1)}(\alpha_{nr} \xi) \\
& - \frac{a^2 r^2}{c^2} \frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\alpha_{nr} \xi) \Big\} + A'_{nr} \Big\{ \frac{\lambda}{2\mu+\lambda} Q^{(2)}(\alpha_{nr} \xi) - \frac{a^2 r^2}{c^2} \frac{\mu+\lambda}{2\mu+\lambda} \\
& \times P^{(2)}(\alpha_{nr} \xi) \Big\} \Big] \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z + \sum_m \sum_n \frac{c^2 m}{4a\gamma_{mn}} \Big[D_{mn} \Big\{ \frac{\mu}{2\mu+\lambda} \phi^{(1)}(\gamma_{mn} \zeta) \\
& - \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\gamma_{mn} \zeta) \Big\} + D'_{mn} \Big\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\gamma_{mn} \zeta) - \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\gamma_{mn} \zeta) \Big\} \Big] \\
& \times \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y - \sum_m \sum_n \frac{an}{b} \Big[J_{nr} \Big\{ \frac{\mu\lambda}{2\mu+\lambda} Q^{(1)}(\alpha_{nr} \xi) + \frac{r^2 a^2}{c^2} \\
& \times \frac{\mu(\mu+\lambda)}{2\mu+\lambda} P^{(1)}(\alpha_{nr} \xi) \Big\} + J'_{nr} \Big\{ \frac{\mu\lambda}{2\mu+\lambda} Q^{(2)}(\alpha_{nr} \xi) + \frac{r^2 a^2}{c^2} \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \\
& \times P^{(2)}(\alpha_{nr} \xi) \Big\} \Big] \sin \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z - \sum_m \sum_r \frac{b}{2} \Big[B_{mr} \Big\{ \frac{\lambda}{2\mu+\lambda} Q^{(1)}(\beta_{mr} \eta) \\
& + \frac{b^2 r^2}{c^2} \frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\beta_{mr} \eta) \Big\} + B'_{mr} \Big\{ \frac{\lambda}{2\mu+\lambda} Q^{(2)}(\beta_{mr} \eta) + \frac{b^2 r^2}{c^2} \frac{\mu+\lambda}{2\mu+\lambda} \\
& \times P^{(2)}(\beta_{mr} \eta) \Big\} \Big] \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z + \sum_m \sum_n \frac{c^2 n}{4b\gamma_{mn}} \Big[F_{mn} \Big\{ \frac{\mu}{2\mu+\lambda} \\
& \times \phi^{(1)}(\gamma_{mn} \zeta) - \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\gamma_{mn} \zeta) \Big\} - F'_{mn} \Big\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\gamma_{mn} \zeta) - \frac{\mu+\lambda}{2\mu+\lambda} \\
& \times \psi^{(2)}(\gamma_{mn} \zeta) \Big\} \Big] \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y - \sum_m \sum_r \frac{bm}{a} \Big[H_{mr} \Big\{ \frac{\mu\lambda}{2\mu+\lambda} \\
& \times Q^{(1)}(\beta_{mr} \eta) + \frac{r^2 b^2}{c^2 \beta_{mr}^2} \frac{\mu(\mu+\lambda)}{2\mu+\lambda} P^{(1)}(\beta_{mr} \eta) \Big\} + H'_{mr} \Big\{ \frac{\mu\lambda}{2\mu+\lambda} Q^{(2)}(\beta_{mr} \eta) \\
& + \frac{b^2 r^2}{c^2 \beta_{mr}^2} \frac{\mu(\mu+\lambda)}{2\mu+\lambda} P^{(2)}(\beta_{mr} \eta) \Big\} \Big] \sin \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z - \sum_n \frac{a}{4} \frac{\lambda}{2\mu+\lambda} \\
& \times \Big\{ A_{n0} Q^{(1)}(\alpha_{n0} \xi) + A'_{n0} Q^{(2)}(\alpha_{n0} \xi) \Big\} \sin \frac{n\pi}{b} y - \sum_n \frac{an}{2b} \frac{\lambda}{2\mu+\lambda} \Big\{ J_{n0} Q(\alpha_{n0} \xi) \\
& + J'_{n0} Q^{(2)}(\alpha_{n0} \xi) \Big\} \sin \frac{n\pi}{b} y - \sum_m \frac{bm}{2a} \frac{\mu\lambda}{2\mu+\lambda} \Big\{ H_{m0} Q^{(1)}(\beta_{m0} \eta) \\
& + H'_{m0} Q^{(2)}(\beta_{m0} \eta) \Big\} \sin \frac{m\pi}{a} x - \sum_m \frac{b}{4} \frac{\lambda}{2\mu+\lambda} \Big\{ B_{m0} Q^{(1)}(\beta_{m0} \eta) \\
& + B'_{m0} Q^{(2)}(\beta_{m0} \eta) \Big\} \sin \frac{m\pi}{a} x,
\end{aligned} \tag{70}$$

$$\begin{aligned}
\tau_{yz} = \mu \Big(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \Big) = & - \sum_m \sum_n \frac{cn}{b} \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \Big\{ K_{mn} P^{(1)}(\gamma_{mn} \zeta) \\
& + K'_{mn} P^{(2)}(\gamma_{mn} \zeta) \Big\} \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y - \sum_m \sum_r \frac{\pi \beta_{mr}}{2} \Big[G_{mr} \Big\{ \phi^{(1)}(\beta_{mr} \eta)
\end{aligned}$$

$$\begin{aligned}
& - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{b^2 r^2}{c^2 \beta_{mr}^2} \psi^{(1)}(\beta_{mr}\eta) \Big\} + G'_{mr} \Big\{ \phi^{(2)}(\beta_{mr}\eta) - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{b^2 r^2}{c^2 \beta_{mr}^2} \\
& \times \psi^{(2)}(\beta_{mr}\eta) \Big\} \Big] \sin \frac{m\pi}{a} x \cdot \sin \frac{r\pi}{c} z + \sum_n \sum_r \mu \frac{an}{2b} \Big[E_{nr} \Big\{ Q^{(1)}(\alpha_{nr}\xi) \\
& - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{a^2 r^2}{c^2 a_{nr}^2} P^{(1)}(\alpha_{nr}\xi) \Big\} + E'_{nr} \Big\{ Q^{(2)}(\alpha_{nr}\xi) - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{a^2 r^2}{c^2 a_{nr}^2} \\
& \times P^{(2)}(\alpha_{nr}\xi) \Big\} \Big] \cos \frac{n\pi}{b} y \sin \frac{r\pi}{b} z - \sum_n \sum_r \mu \frac{br}{2c} \Big[J_{nr} \Big\{ Q^{(1)}(\alpha_{nr}\xi) \\
& - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{n^2 a^2}{b^2 a_{nr}^2} P^{(1)}(\alpha_{nr}\xi) \Big\} + J'_{nr} \Big\{ Q^{(2)}(\alpha_{nr}\xi) - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{n^2 a^2}{b^2 a_{nr}^2} \\
& \times P^{(2)}(\alpha_{nr}\xi) \Big\} \Big] \cos \frac{n\pi}{b} y \cdot \sin \frac{r\pi}{c} z + \sum_m \sum_r \mu \frac{b^2 mr}{2ac \beta_{mr}} \Big[H_{mr} \Big\{ \frac{\mu}{2\mu+\lambda} \\
& \times \phi^{(1)}(\beta_{mr}\eta) + \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(2)}(\beta_{mr}\eta) \Big\} + H'_{mr} \Big\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\beta_{mr}\eta) \\
& + \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(1)}(\beta_{mr}\eta) \Big\} \Big] \sin \frac{m\pi}{a} x \cdot \sin \frac{r\pi}{c} z + \sum_n \sum_r \mu \frac{a^3 rn}{2bc} \frac{1}{\beta_{mr}^2} \\
& \times \frac{\mu+\lambda}{2\mu+\lambda} \Big\{ A_{nr} P^{(1)}(\alpha_{nr}\xi) + A'_{nr} P^{(2)}(\alpha_{nr}\xi) \Big\} \cos \frac{n\pi}{b} y \cdot \sin \frac{r\pi}{c} z + \sum_m \sum_n \\
& \times \frac{c^3 mn}{2ab r_{mn}^2} \frac{\mu+\lambda}{2\mu+\lambda} \Big\{ D_{mn} P^{(1)}(r_{mn}\zeta) + D'_{mn} P^{(2)}(r_{mn}\zeta) \Big\} \sin \frac{m\pi}{a} x \cdot \cos \\
& \times \frac{n\pi}{b} y - \sum_m \sum_r \frac{b^2 r}{4c \beta_{mr}} \Big[B_{mr} \Big\{ \frac{2\mu}{2\mu+\lambda} \phi^{(1)}(\beta_{mr}\eta) + \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(1)}(\beta_{mr}\eta) \Big\} \\
& + B'_{mr} \Big\{ \frac{2\mu}{2\mu+\lambda} \phi^{(2)}(\beta_{mr}\eta) + \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(2)}(\beta_{mr}\eta) \Big\} \Big] \sin \frac{m\pi}{a} x \cdot \sin \\
& \times \frac{r\pi}{c} z - \sum_m \sum_n \frac{c}{2} \Big[F_{mn} \Big\{ Q^{(1)}(r_{mn}\zeta) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{c^2 n^2}{b^2 r_{mn}^2} P^{(1)}(r_{mn}\zeta) \Big\} \\
& + F'_{mn} \Big\{ Q^{(2)}(r_{mn}\zeta) - \frac{\mu+\lambda}{2\mu+\lambda} \frac{c^2 n^2}{b^2 r_{mn}^2} \times P^{(2)}(r_{mn}\zeta) \Big\} \Big] \cos \frac{n\pi}{b} y \cdot \sin \frac{m\pi}{a} x \\
& - \sum_n \frac{c}{4} \Big\{ F_{m0} Q^{(1)}(r_{m0}\zeta) + F'_{m0} Q^{(2)}(r_{m0}\zeta) \Big\} \sin \frac{n\pi}{b} y + \sum_r \frac{ar}{4c} \\
& \times \Big\{ H_{0r} Q^{(1)}(\alpha_{0r}\xi) + H'_{0r} Q^{(2)}(\alpha_{0r}\xi) \Big\} \sin \frac{r\pi}{c} z,
\end{aligned} \tag{71}$$

$$\begin{aligned}
\tau_{xx} &= \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) = - \sum_m \sum_n \frac{cm}{a} \frac{\mu(\mu+\lambda)}{2\mu+\lambda} \Big\{ K_{mn} P^{(1)}(r_{mn}\zeta) + K'_{mn} \\
& \times P^{(2)}(r_{mn}\zeta) \Big\} \cdot \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y - \sum_n \sum_r \mu \frac{a_{nr}}{2} \Big[E_{nr} \Big\{ \phi^{(1)}(\alpha_{nr}\xi) \\
& - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{a^2 r^2}{c^2 a_{nr}^2} \psi^{(1)}(\alpha_{nr}\xi) \Big\} + E'_{nr} \Big\{ \phi^{(2)}(\alpha_{nr}\xi) - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{a^2 r^2}{c^2 a_{nr}^2} \\
& \times \psi^{(2)}(\alpha_{nr}\xi) \Big\} \Big] \sin \frac{n\pi}{b} y \cdot \sin \frac{r\pi}{c} z + \sum_r \sum_m \mu \frac{bm}{2a} \Big[G_{mr} \Big\{ Q^{(1)}(\beta_{mr}\eta)
\end{aligned}$$

$$\begin{aligned}
& - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{a^2 r^2}{c^2 \beta_{mr}^2} P^{(1)}(\beta_{mr} \eta) \Big\} + G'_{mr} \left\{ Q^{(2)}(\beta_{mr} \eta) - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{a^2 r^2}{c^2 \beta_{mr}^2} \right. \\
& \times P^{(2)}(\beta_{mr} \eta) \Big\} \Big] \cos \frac{m\pi}{a} x \cdot \sin \frac{r\pi}{c} z + \sum_n \sum_r \mu \frac{a^2 n r}{2 b a_{nr}} \left[J_{nr} \left\{ \frac{\mu}{2\mu+\lambda} \right. \right. \\
& \times \phi^{(1)}(\alpha_{nr} \xi) + \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(1)}(\alpha_{nr} \xi) \Big\} + J'_{nr} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\alpha_{nr} \xi) \right. \\
& + \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(2)}(\alpha_{nr} \xi) \Big\} \Big] \sin \frac{n\pi}{b} y \cdot \sin \frac{r\pi}{c} z + \sum_m \sum_n \mu \frac{a r}{2 c} \left[H_{mr} \right. \\
& \times \left\{ Q^{(1)}(\beta_{mr} \eta) - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{m^2 b^2}{a^2 \beta_{mr}^2} P^{(1)}(\beta_{mr} \eta) \right\} + H'_{mr} \left\{ Q^{(2)}(\beta_{mr} \eta) \right. \\
& - \frac{2(\mu+\lambda)}{2\mu+\lambda} \frac{m^2 b^2}{a^2 \beta_{mr}^2} P^{(2)}(\beta_{mr} \eta) \Big\} \Big] \cos \frac{m\pi}{a} x \cdot \sin \frac{r\pi}{c} z + \sum_m \sum_r \frac{b^3 r m}{2 a c \beta_{mr}^2} \\
& \times \frac{\mu+\lambda}{2\mu+\lambda} \left\{ B_{mr} P^{(1)}(\beta_{mr} \eta) + B'_{mr} P^{(2)}(\beta_{mr} \eta) \right\} \cos \frac{m\pi}{a} x \cdot \sin \frac{r\pi}{c} z \\
& + \sum_m \sum_n \frac{c^3 m n}{2 a b \gamma_{mn}^2} \frac{\mu+\lambda}{2\mu+\lambda} \left\{ F_{mn} P^{(1)}(\gamma_{mn} \xi) + F'_{mn} P^{(2)}(\gamma_{mn} \xi) \right\} \\
& \times \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y - \sum_n \sum_r \frac{a^2 r}{4 c a_{nr}} \left[A_{nr} \left\{ \frac{2\mu}{2\mu+\lambda} \phi^{(1)}(\alpha_{nr} \xi) \right. \right. \\
& + \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(1)}(\alpha_{nr} \xi) \Big\} + A'_{nr} \left\{ \frac{2\mu}{2\mu+\lambda} \phi^{(2)}(\alpha_{nr} \xi) + \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(2)}(\alpha_{nr} \xi) \right\} \Big] \\
& \times \sin \frac{n\pi}{b} y \cdot \sin \frac{r\pi}{c} z - \sum_m \sum_n \frac{c}{2} \left[D_{mn} \left\{ Q^{(1)}(\gamma_{mn} \xi) + \frac{\mu+\lambda}{2\mu+\lambda} P^{(1)}(\gamma_{mn} \xi) \right\} \right. \\
& + D'_{mn} \left\{ Q^{(2)}(\gamma_{mn} \xi) + \frac{\mu+\lambda}{2\mu+\lambda} P^{(2)}(\gamma_{mn} \xi) \right\} \Big] \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \\
& - \sum_n \frac{c}{4} \left\{ D_{0n} Q^{(1)}(\gamma_{0n} \xi) + D'_{0n} Q^{(2)}(\gamma_{0n} \xi) \right\} \sin \frac{n\pi}{b} y + \sum_r \frac{b r}{4 c} \left\{ J_{0r} Q^{(1)}(\beta_{0r} \eta) \right. \\
& + J'_{0r} Q^{(2)}(\beta_{0r} \eta) \Big\} \sin \frac{r\pi}{c} z,
\end{aligned} \tag{72}$$

$$\begin{aligned}
\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = & - \sum_m \sum_n \mu \frac{c^2}{a b} \frac{m n}{\gamma_{mn}} \left[K_{mn} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(1)}(\gamma_{mn} \xi) \right. \right. \\
& + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(1)}(\gamma_{mn} \xi) \Big\} + K'_{mn} \left\{ \frac{\mu}{2\mu+\lambda} \phi^{(2)}(\gamma_{mn} \xi) + \frac{\mu+\lambda}{2\mu+\lambda} \psi^{(2)}(\gamma_{mn} \xi) \right\} \Big] \\
& \times \cos \frac{m\pi}{a} x \times \cos \frac{n\pi}{b} y + \sum_m \sum_r \mu \frac{b^3 m r}{2 c a \beta_{mr}} \left[G_{mr} \left\{ \frac{\lambda}{2\mu+\lambda} \phi^{(1)}(\beta_{mr} \eta) \right. \right. \\
& - \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(1)}(\beta_{mr} \eta) \Big\} + G'_{mr} \left\{ \frac{\lambda}{2\mu+\lambda} \phi^{(2)}(\beta_{mr} \eta) - \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(2)}(\beta_{mr} \eta) \right\} \Big] \\
& \times \cos \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z + \sum_n \sum_r \mu \frac{a^3 n r}{2 c b a_{nr}} \left[E_{nr} \left\{ \frac{\lambda}{2\mu+\lambda} \phi^{(1)}(\alpha_{nr} \xi) \right. \right. \\
& - \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(1)}(\alpha_{nr} \xi) \Big\} + E'_{nr} \left\{ \frac{\lambda}{2\mu+\lambda} \phi^{(2)}(\alpha_{nr} \xi) - \frac{2(\mu+\lambda)}{2\mu+\lambda} \psi^{(2)}(\alpha_{nr} \xi) \right\} \Big]
\end{aligned}$$

$$\begin{aligned}
& \times \psi^{(2)}(\alpha_{nr}\xi) \Big] \cos \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z + \sum_m \sum_n \frac{c^2 m}{2a\gamma_{mn}} \left[F_{mn} \left\{ \left(1 - \frac{\mu + \lambda}{2\mu + \lambda} \right. \right. \right. \\
& \times \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \Big) \times \phi^{(1)}(\gamma_{mn}\xi) - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \psi^{(1)}(\gamma_{mn}\xi) \Big\} + F'_{mn} \\
& \times \left\{ \left(1 - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \right) \phi^{(2)}(\gamma_{mn}\xi) - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 n^2}{b^2 \gamma_{mn}^2} \psi^{(2)}(\gamma_{mn}\xi) \right\} \Big] \\
& \times \cos \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y + \sum_m \sum_n \frac{c^2 n}{2b\gamma_{mn}} \left[D_{mn} \left\{ \left(1 - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 m^2}{a^2 \gamma_{mn}^2} \right) \right. \right. \\
& \times \phi^{(1)}(\gamma_{mn}\xi) - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 m^2}{a^2 \gamma_{mn}^2} \psi^{(1)}(\gamma_{mn}\xi) \Big\} + D'_{mn} \left\{ \left(1 - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 m^2}{a^2 \gamma_{mn}^2} \right) \right. \\
& \times \phi^{(2)}(\gamma_{mn}\xi) - \frac{\mu + \lambda}{2\mu + \lambda} \frac{c^2 m^2}{a^2 \gamma_{mn}^2} \psi^{(2)}(\gamma_{mn}\xi) \Big\} \Big] \cos \frac{\pi m}{a} x \cdot \cos \frac{n\pi}{b} y \\
& + \sum_n \sum_r \frac{a^2 n}{2b\alpha_{nr}} \left[A_{nr} \left\{ \frac{\mu}{2\mu + \lambda} \phi^{(1)}(\alpha_{nr}\xi) + \frac{\mu + \lambda}{2\mu + \lambda} \psi^{(1)}(\alpha_{nr}\xi) \right\} \right. \\
& + A'_{nr} \left\{ \frac{\mu}{2\mu + \lambda} \phi^{(2)}(\alpha_{nr}\xi) + \frac{\mu + \lambda}{2\mu + \lambda} \psi^{(2)}(\alpha_{nr}\xi) \right\} \Big] \cos \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y \\
& + \sum_m \sum_r \frac{b^2 m}{2a\beta_{mr}} \left[B_{mr} \left\{ \frac{\mu}{2\mu + \lambda} \phi^{(1)}(\beta_{mr}\eta) + \frac{\mu + \lambda}{2\mu + \lambda} \psi^{(1)}(\beta_{mr}\eta) \right\} \right. \\
& + B'_{mr} \left\{ \frac{\mu}{2\mu + \lambda} \phi^{(2)}(\beta_{mr}\eta) + \frac{\mu + \lambda}{2\mu + \lambda} \psi^{(2)}(\beta_{mr}\eta) \right\} \Big] \cos \frac{m\pi}{a} x \cdot \cos \frac{r\pi}{c} z \\
& - \sum_m \sum_r \mu \frac{\beta_{mr}}{2} \left\{ H_{mr} \left(1 + \frac{b^2 m^2}{a^2 \beta_{mr}^2} \right) \phi^{(1)}(\beta_{mr}\eta) + H'_{mr} \left(1 + \frac{b^2 m^2}{a^2 \beta_{mr}^2} \right) \right. \\
& \times \phi^{(2)}(\beta_{mr}\eta) \Big\} \cos \frac{m\pi}{a} x \times \cos \frac{r\pi}{c} z - \sum_n \sum_r \mu \frac{\alpha_{nr}}{2} \left\{ J_{nr} \left(1 + \frac{a^2 n^2}{b^2 \alpha_{nr}^2} \right) \right. \\
& \times \phi^{(1)}(\alpha_{nr}\xi) + J'_{nr} \left(1 + \frac{a^2 n^2}{b^2 \alpha_{nr}^2} \right) \phi^{(2)}(\alpha_{nr}\xi) \Big\} \cos \frac{n\pi}{b} y \cdot \cos \frac{r\pi}{c} z \\
& + \sum_m \mu \frac{b}{4} \left[B_{m0} \left\{ \frac{\mu}{2\mu + \lambda} \phi^{(1)}(\beta_{m0}\eta) + \frac{\mu + \lambda}{2\mu + \lambda} \psi^{(1)}(\beta_{m0}\eta) \right\} + B'_{m0} \right. \\
& \times \left\{ \frac{\mu}{2\mu + \lambda} \phi^{(2)}(\beta_{m0}\eta) + \frac{\mu + \lambda}{2\mu + \lambda} \psi^{(2)}(\beta_{m0}\eta) \right\} \Big] \cos \frac{m\pi}{a} x + \sum_n \sum_r \frac{a}{4} \\
& \times \left[A_{n0} \left\{ \frac{\mu}{2\mu + \lambda} \phi^{(1)}(\alpha_{n0}\xi) + \frac{\mu + \lambda}{2\mu + \lambda} \psi^{(1)}(\alpha_{n0}\xi) \right\} + A'_{n0} \left\{ \frac{\mu}{2\mu + \lambda} \phi^{(2)}(\alpha_{n0}\xi) \right. \right. \\
& + \frac{\mu + \lambda}{2\mu + \lambda} \psi^{(2)}(\alpha_{n0}\xi) \Big\} \Big] \cos \frac{n\pi}{b} y - \sum_m \mu \frac{mb}{2a} \left\{ H_{m0} \phi^{(1)}(\beta_{m0}\eta) \right. \\
& + H'_{m0} \phi^{(2)}(\beta_{m0}\eta) \Big\} \cos \frac{m\pi}{a} x - \sum_n \mu \frac{na}{2b} \left\{ J_{n0} \phi^{(1)}(\alpha_{n0}\xi) + J'_{n0} \phi^{(2)}(\alpha_{n0}\xi) \right\} \\
& \times \cos \frac{n\pi}{b} y + \sum_n \mu \frac{c}{4} \left\{ D_{0n} \phi^{(1)}(\gamma_{0n}\xi) + D'_{0n} \phi^{(2)}(\gamma_{0n}\xi) \right\} \cos \frac{n\pi}{b} y \\
& - \sum_r \mu \frac{br}{4c} \left\{ J_{0r} \phi^{(1)}(\beta_{0r}\eta) + J'_{0r} \phi^{(2)}(\beta_{0r}\eta) \right\} \cos \frac{r\pi}{c} z + \sum_m \mu \frac{c}{4}
\end{aligned} \tag{73}$$

$$\times \left\{ F_{m0} \phi^{(1)}(r_{m0} \zeta) + F_{m0} \phi^{(2)}(r_{m0} \zeta) \right\} \cos \frac{m\pi}{a} x - \sum_r \mu \frac{ra}{4c} \left\{ H_{0r} \phi^{(1)}(\alpha_{0r} \xi) + H_{0r} \phi^{(2)}(\alpha_{0r} \xi) \right\} \cos \frac{r\pi}{c} x + \frac{1}{4} (H_{00} + J_{00}).$$

Eighteen unknown values A_{nr} , A'_{nr} , B_{mr} , B'_{mr} , D_{mn} , D'_{mn} , E_{nr} , E'_{nr} , F_{mn} , F'_{mn} , G_{mr} , G'_{mr} , H_{mr} , H'_{mr} , J_{nr} , J'_{nr} , K_{mn} , K'_{mn} can be all determined to satisfy the eighteen boundary conditions on the side planes. The terms involving the unknown values whose index n , m , or r are zero, indicate the behaviour in regard to the plane elasticity.

In virtue of Formulas (64)~(72), we could, for instance, investigate the differences between the simply supported plate and the hinged plate; or to find out suitability of Kirchhoff's assumption for the edge reaction of the thin plate.

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