Foundations of Objectivist Theory of Events

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Abstract

The author aims to develop some naive construction of a theory of events on the footing that a universe of primitive objects is fixed and axiomatics is taken to fit to the research of events occurring in this universe. The whole system is possibly considered to be revised historically. Logic thereupon is claimed to work on beyond the intuitionism.

1. Introduction

In this study of logical consistency of an analysis, in providing axiomatics we make much of the epistemological correlations to be claimed on its objects. This will be a way which interests us rather in material axiomatics than formal one. A universe of primitive objects might primarily be concerned with technical operations allowed by axiomatics provided for it, so that, if any paradox was found, the universe itself might possibly be taken as a sourse of that paradox. But, such may not be the case when a universe U is taken in itself to make the basic ground of events and axioms are thereupon postulated to admit some naive theory of events occurring in U. In fact some universes seem to have been from the outset admitted before axiomatics for them. For instance, the real axis has been used in geometry before any theory of irrational numbers was completed. Moreover, the euclidean geometry was perhaps essentially thought to be unchangeably absolute even when some additional axioms were by D. Hilbert discovered to be needed.[1] Thus, if a unverse is esteemed to be original to be unchangeably fixed, it is called an *objectivist universe*, and if we then intend to build up a theory of events based on that universe, the theory is called an *objectivist theory* or an *objectivism.*

In material axiomatics, axioms are postulad as the facts which are mentally convinced to hold except that there are found no rigorous means to demonstrate them. They are the results of human observations and so, if any contradiction is factually ascribed to some axioms, they must then be eliminated from the system of axioms and if needed altered or supplemented. Such is the common manner destined to all objectivist theories. Because of this manner an objectivism may be said to develop its course of analysis naturally and historically, perhaps sometimes with revisions of axiomatics.

2. Consistency

As a cogent logical complement for objectivist devices, we may adopt the *empiricist pragmatism*. This is the dogmatism which epistemologically scrutenizes over all places

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fundamental to axiomatic researches and claims to renounce what cannot have any authenticity beyond mere fictive presentation, as meaningless.

If axiomatics is based only on observational trials over an objectivist universe under the empiricist pramgmatism, it is called *objectivist axiomatics*. In such a system it is possibly promised to have some axioms altered or supplemented if any essential contradiction is revealed. Thus the axioms to date possessed cannot all be considered as absolute. In this meaning of historical subsumption, we particularly call the *proto-system* of axioms the system of axioms to date possessed, and the *proto-construction* the collection of all axioms, definitions and theorems to date obtained.

It should be so constituted that any real contradiction may be retraced and ascribed to the corresponding contrarieties amog the axioms. Thus revisions of the proto-construction are to be applied wholly to the proto-system of axioms. After any revision, the thereafter newly obtained construction (of axioms, defintions and theorems) gives again a proto-construction. By grace of such a historical renovation, an objectivism may be regarded to be free from real contradiction and be aimed only its perfect consistency, though, despite of the human desire, the present appearance of consistency may not be convinced in itself toward the future beyond the realm of the proto-construction, In fact, this apparent consistency is a sort of consistency which has been called *empirical*.[2] When we necessarily try some renovation for the proto-construction, there may possibly be found an opportunity to look upon it with some connotation, that is, epistemological insight therewith to be connected.

3. Law of Excluded Middle

The law of excluded middle in respect of (lst-order) predicates has been prevalently used when limited its application to a finite set of objects, but when applied to an infinite set of them it was sometimes overtly objected, particularly by the intuitionist school.[3] Incidentally the conception itself of infinite sets has also been exposed to several hot discussions.

An enumerable set may be defined as a set of elements which can be enumerated along the natural numbers in order, but whether it reaches the infinity or not must be abstained from discussion. By the way, the structure of an enumerable set can be considered to correspond to the procedure of mathematical induction.

Principle of Elementwise** (Mathematical)* ***Induction. *If* $N=1, 2, \cdots$ *and if the following two conditions are admitted to be satisfied, it is stated that* $(\forall n \in N)$ *(F(n) is true)* : (i) $F(1)$ *is true*; (ii) *if* $n \in N$ *and* $F(n)$ *is true, then so is* $F(n+1)$.

This may be thought as a definition which also involvelves the generation of an enumtrable set itself, that is : (i) $a_1 \in A$; (ii) $a_n \in A. \Rightarrow. a_{n+1} \in A$; then A is determined to contain an enumerable set. Moreover, the law of excluded middle in respect of a predicate $\boldsymbol{p}$ may be warranted for an enumerable set $A=\{a_1, a_2, \cdots\}$ if (1) $\boldsymbol{p}a_1.\vee.\sim\boldsymbol{p}a_1$, and (ii) $\boldsymbol{p}a_n.\vee.\sim\boldsymbol{p}a_n:\Rightarrow:\boldsymbol{p}a_{n+1}.\vee.\sim\boldsymbol{p}a_{n+1}$. In this case we render it

$$(\forall n)(\boldsymbol{p}a_n.\vee.\sim\boldsymbol{p}a_n).$$

On accepting the conception of an enumerable set, it is certainly concluded that an enumerable set must be an infinite set. However, it is to be noted that, if $A = \{a_1, a_2, \cdots\}$ and $A_n = \{a_1, a_2, \cdots, a_n\}$, then the remainder

$$R_n = A - A_n = \{a_{n+1}, a_{n+2}, \cdots\}$$

is always an enumerable infinite set too, and can moreover be put into $1-1$ correspondence to A, that is, the enumerability is of unfinishing type.[4] This may be thought to be an inherent property of the set of natural numbers.

In case of a general infinite set A, the primary meaning of the sentence

$$(\forall a \in A)(\boldsymbol{p}a. \vee . \sim \boldsymbol{p}a) \tag{3.1}$$

inevitably in itself comes into question. However, if there is no a really to fail in the condition

$$a \in A. \ \&: \boldsymbol{p}a. \vee . \sim \boldsymbol{p}a,$$

then (3.1) may eventually be rendered "For each element a of the set A it is warranted that

$$\boldsymbol{p}a. \vee . \sim \boldsymbol{p}a."$$

So, let us inversely define such that, if

$$\sim (\exists a \in A)(\sim (\boldsymbol{p}a. \vee . \sim \boldsymbol{p}a)),$$

the property $\boldsymbol{p}$ is said to be *"elementwise well-defined"* in the set A. Thus the law of excluded middle in respect of a predicate $\boldsymbol{p}$ is warranted for a set A if $\boldsymbol{p}$ is considered to be elementwise well-defined in A.

In a previous paper[5] we defined a predicate $\boldsymbol{p}$ to be called *descriptive* in a set A if $\boldsymbol{p}$ fulfils the condition (3.1) there. If $R(\boldsymbol{p})$ indicates the total set of the elements a for which $\boldsymbol{P}a$ is true, $R(\boldsymbol{p})$ is the *range of* $\boldsymbol{p}$. "$R(\boldsymbol{p}) \subset R(\boldsymbol{q})$" is thought to be equivalent to the implication "$\boldsymbol{p} \Rightarrow \boldsymbol{q}$".

4. Objectivist Propositions

Let the language for an objectivism being developed on a given objectivist universe U be denoted by $\boldsymbol{L}_U$. $\boldsymbol{L}_U$ may primarily be introduced in terms of any word language, say English, French, etc., and possibly be helped by that language whenever needed.

Suppose that there is a class $\boldsymbol{Fs}$ of sets in U and is given a relation f which assigns an object $f(A)$ to each set A of $\boldsymbol{Fs}$ and a certain property $\boldsymbol{p}$ is to be examined if $f(A)$ fulfils it or not. Then $f(A)$ is called a *configuration* on A (with respect to f). We assume $\boldsymbol{Fs}$ to be the largest class of wich every set A may be assigned a unique configuration by means of f, and call it the *class of f-configuration*.

Denoting as

$$\rho(f, A) = (f(A) \text{ fulfils } \boldsymbol{p})$$

we call an event defined by

$$\boldsymbol{a} = \rho(f, A) \tag{4.1}$$

an *elementary objectivist event* or elliptically an (*objectivist*) *event* if there is no fear to be complicated. If we define a proposition $\boldsymbol{S}$ by one of the following three sorts of statements:

(I) $\boldsymbol{S}: A \in \boldsymbol{Fs}. \Rightarrow. \rho(f, A)$ is true (or valid);
(II) $\boldsymbol{S}: A \in \boldsymbol{Fs}. \Rightarrow. \rho(f. A)$ is possible;
(III) $\boldsymbol{S}: A \in \boldsymbol{Fs}. \Rightarrow. \rho(f, A)$ is impossible;

then we call $\boldsymbol{S}$ an *elementary objectivist proposition* or elliptically an *objectivist proposition* or a *proposition* if there is no fear of complication. When a proposition is in a certain way explicated out, we shall use $S(A)$ instead of $f(A)$ and instead of (4.1) we write as

$$\boldsymbol{a} = \rho(S, A) \tag{4.2}$$

Since we (for the present) convince the proto-system of axioms to be consistent, we may expect any event to be true if it is derived from the proto-construction. Thus, in this view, to say an event to be true is of the same meaning as to say it to be valid.

If $\rho(\boldsymbol{S}, A)$ is valid, a is called an *example* of $\boldsymbol{S}$, and if impossible (or invalid) a *counter-example* of $\boldsymbol{S}$. By $L(\boldsymbol{S})$ we mean the set of all events $\boldsymbol{a}$ defind by (4.1) (or by (4.2)) and call it the *level* of the proposition $\boldsymbol{S}$. Denoting as

$$E(\boldsymbol{S}) = \{\boldsymbol{a} \mid \boldsymbol{a} \text{ is an example of } \boldsymbol{S}\}$$

and

$$C(\boldsymbol{S}) = \{\boldsymbol{a} \mid \boldsymbol{a} \text{ is a counterexample of } \boldsymbol{S}\},$$

if we have

$$L(\boldsymbol{S}) = E(\boldsymbol{S}) \cup C(\boldsymbol{S}),$$

then we say $\boldsymbol{S}$ to be a *decidable* (or *descriptive*) proposition. If $L(\boldsymbol{S}) = \varnothing$, $\boldsymbol{S}$ is called *vacuous*.

When a proposition $\boldsymbol{S}$ is not vacuous and is decidable, if $C(\boldsymbol{S}) = \varnothing$, $\boldsymbol{S}$ is a theorem in case of type (I) or (II), and if $E(\boldsymbol{S}) = \varnothing$, $\boldsymbol{S}$ is a false proposition or a fallacy in case of type (I) or (II), but is a theorem in case of type (III).

We claim that the collection of all objectivist propositions in a given objectivism can be made enumerated. Let the enumeration indices of the propositions (with respect to an enumeration) be called *Gödel numbers*. When we actually take up a proposition $\boldsymbol{S}$, Gödel numbers precedent to that of $\boldsymbol{S}$ make up together at most only a finite set. In this meaning we say "$\boldsymbol{S}$ is set *on a finite stage*". Inversely, naive admission of the fact that any objectivist proposition, if actually taken up for analysis, must necessarily be set on a finite stage (in the proto-construction), may be assumed to induce the enumerability of the total collection of the objectivist propositions (with the aid of the notion of historical proceeding).

5. Negation of a Proposition

If $\boldsymbol{S}$ is a theorem we write '$\vdash \boldsymbol{S}$'. Then, if $\boldsymbol{S}$ is a decidable proposition, we have

$$\vdash \boldsymbol{S}. \vee. \sim \vdash \boldsymbol{S}. \tag{5.1}$$

By several authors, instead of (5.1) it has been written as

$$\boldsymbol{S}. \vee. \sim \boldsymbol{S}.$$

However, the expression '$\sim \boldsymbol{S}$' is not generally to be connected to the same content as '$\sim \vdash \boldsymbol{S}$', because the expression '$\sim \boldsymbol{S}$' must properly suggest the collection of all propositions the levels of which are really different from $L(\boldsymbol{S})$ and '$\sim \boldsymbol{S}$' thus cannot work to be on a finite stage. This being so, we shall treat the expression '$\sim \boldsymbol{S}$' as a metalogical object alien from the present implementation of an objectivism.

When $\boldsymbol{S}$ is a (non-vacuous) decidable proposition, we may constitutionally have the following stipulations:

$$\vdash \boldsymbol{S}=(C(\boldsymbol{S})=\varnothing) \quad \text{and} \quad \sim\vdash \boldsymbol{S}=(C(\boldsymbol{S})\neq\varnothing)$$

in case of type (I) and

$$\vdash \boldsymbol{S}=(E(\boldsymbol{S})\neq\varnothing) \quad \text{and} \quad \sim\vdash \boldsymbol{S}=(E(\boldsymbol{S})=\varnothing)$$

in case of type (II). Thus we shall accordingly stipulate that

$$\sim(\boldsymbol{A} \text{ is } \boldsymbol{B})=(\boldsymbol{A} \text{ is not } \boldsymbol{B}),\ \sim(\boldsymbol{A} \text{ is not } \boldsymbol{B})=(\boldsymbol{A} \text{ is } \boldsymbol{B}),$$
$$\sim(\boldsymbol{A}\Rightarrow\boldsymbol{B})=(\boldsymbol{A}\not\Rightarrow\boldsymbol{B}),\ \sim(\boldsymbol{A}\not\Rightarrow\boldsymbol{B})=(\boldsymbol{A}\Rightarrow\boldsymbol{B}),$$

and call the sign '$\sim$' in this usage the *binary negation*. In this context we may have

$$\sim\sim(\boldsymbol{A} \text{ is } \boldsymbol{B})=(\boldsymbol{A} \text{ is } \boldsymbol{B}),\ \sim\sim(\boldsymbol{A} \text{ is not } \boldsymbol{B})=(\boldsymbol{A} \text{ is not } \boldsymbol{B}),$$
$$\sim\sim(\boldsymbol{A}\Rightarrow\boldsymbol{B})=(\boldsymbol{A}\Rightarrow\boldsymbol{B}),\ \sim\sim(\boldsymbol{A}\not\Rightarrow\boldsymbol{B})=(\boldsymbol{A}\not\Rightarrow\boldsymbol{B}),$$

and subsequently

$$\sim(X(\boldsymbol{S})=\varnothing)=(X(\boldsymbol{S})\neq\varnothing),\ \sim(X(\boldsymbol{S})\neq\varnothing)=(X(\boldsymbol{S})=\varnothing),$$
$$\sim\sim(X(\boldsymbol{S})=\varnothing)=(X(\boldsymbol{S})=\varnothing),\ \sim\sim(X(\boldsymbol{S})\neq\varnothing)=(X(\boldsymbol{S})\neq\varnothing),$$

where $X(\boldsymbol{S})$ renders either $E(\boldsymbol{S})$ or $C(\boldsymbol{S})$. Thus we may have

$$\sim\sim\vdash \boldsymbol{S}=\vdash \boldsymbol{S}^{*)} \tag{5.2}$$

on condition that '$\sim$' is the binary negation, when $\boldsymbol{S}$ is an objectivist proposition.

6. Decision

If $\boldsymbol{y}$ is neither a valid event nor an invalid one and yet appears to be occurrable (in respect of some statement to define it) on an objectivist universe U, then $\boldsymbol{y}$ is said to give an *undecidable event* for the proto-construction on U. If an elementary objectivist proposition $\boldsymbol{Q}$ is neither valid nor invalid, then $\boldsymbol{Q}$ is an *undecidable proposition*. In this case we may have a partition of $L(\boldsymbol{Q})$ such that

$$L(\boldsymbol{Q})=E(\boldsymbol{Q})\cup C(\boldsymbol{Q})\cup D(\boldsymbol{Q}) \tag{5.1}$$

and $D(\boldsymbol{Q})$ consists of all undecidable events in respect to the proposition $\boldsymbol{Q}$.

In this section we restrict $\boldsymbol{Q}$ to be a proposition of type (I). Then if $C(\boldsymbol{Q})\neq\varnothing, \boldsymbol{Q}$ is decided as a fallacy. So, for a proposition $\boldsymbol{Q}$ to be undecidable it is necessary that

$$C(\boldsymbol{Q})=\varnothing.$$

If $\boldsymbol{y}$ is an undecidable event, it may not be difficult to see that $\boldsymbol{y}$ is assumable as well to be valid as to be invalid, that is, whether $\boldsymbol{y}$ is assumed to be valid or invalid cannot be affected by the proto-system of axioms. So then, if we make an axiom which claims that $\boldsymbol{y}$ is a valid event on U and add it to the proto-system of axoms, consistency of the constrction may yet hold on.

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*) This formula has not been admitted by the intuitionists when no such stipulations as above-stated have been made.

In case of (5.1), if $\boldsymbol{Q}$ is certainly undecidable, some events of $D(\boldsymbol{Q})$ may, by virtue of the above reasoning, be assumed as simultaneously valid to be added to the proto-construction, with no affection to its consistency. Thus, if $D'(\boldsymbol{Q})$ is the part of $D(\boldsymbol{Q})$ which consists of such events that are assumed simultaneously to be valid, the equation

$$L(\boldsymbol{Q}')=E(\boldsymbol{Q})\cup D'(\boldsymbol{Q})$$

will define a proposition $\boldsymbol{Q}'$ after some addition of axioms, to make a theorem of which $L(\boldsymbol{Q}')$ is the level. The additional axioms are claimed to make the events of $D'(\boldsymbol{Q})$ valid and thus the proto-construction is extended. The theorem $\boldsymbol{Q}'$ is to be provable in this extended construction.

In the theory of objectivism, a heuristic principle of logic shall be found in that, whatever sort of relation is found and taken up, it is expected to be descriptive, that is, the law of excluded middle is expected thereby to be conformed to. The previous definition of "elementwise well-defined" predicate (in Sect. 3) may also be recurred in connection with this principle. Thus the (proto-) construction of an objectivism is expected to be provided with a two-valued system of logic.

However, if exactly two-valuedly should all our steps of analysis be destined, the notion of undecidability might not be allowed to be raised. So then, an undecidable event should be taken as of imperfect descriptivity, or to be an event which could not be perfectly defined out.

If really no undecidable event is found, we may continue our happy investigations under two-valued logic. However, mathematics has sometimes experienced real undecidabilities. These events have surely given us opportunities to extend our construction through additional axiomatization. The imperfectness of descriptivity above pointed out, might always suggest the implication of some possible extensibility on axiomatics, with some renovation too if needed. As a matter of fact, an event may not easily be seen whether decidable or no, until any proof is discovered about its decidability. If we cannot renounce an object of being regarded as an objectivist event and if we cannot discover any proof to admit it to be a decidable one, there is undeniable possibility that it may be an undecieable one.

Presumptively, to admit a statement

$$\boldsymbol{y}=\rho(f,\ Y)$$

to give an objectivist event in our construction, is to accept it as a descriptive event, since an objectivist construction is, as previously explained, to be dominated by a bivalent system of logic. Thus, if we accept $\boldsymbol{y}$ as an event, it must be that $f(Y)$ is either valid (or true) or invalid (or false), i. e.,

$$\vdash\rho(f,\ Y).\vee.\sim\vdash\rho(f,\ Y),$$

that is, the event $\boldsymbol{y}$ must be uniquely determined either to be valid or to be invalid. On this occasion, though the alternative of two validity values might be thought certainly to be left to free selection, our decision should be laid down through deliberate contemplation, particularly on epistemological grounds. Such is the objectivist manner of decision.

Incidentally, if both $\boldsymbol{y}$ and $\sim\boldsymbol{y}$ are to be renounced, it must be that the sentence $\rho(f, Y)$ itself cannot be admitted to the language $\boldsymbol{L}_U$, that is, $\rho(f, Y)$ is a meaningless sentence and to be renounced from L_U.

7. Historical Contingency

The analysis of the level $L(\boldsymbol{P})$ of an objectivist proposition $\boldsymbol{P}$ is of course correlated to the proof of the validity of $\boldsymbol{P}$. Particularly, if $\boldsymbol{P}$ is rendered "$(\forall n=1, 2\cdots)(f(n)$ is true)", then defining as

$$\boldsymbol{a}_n=(f(n) \text{ is true})$$

and

$$N=\{1, 2, \cdots\},$$

we have

$$L(\boldsymbol{P})=\cup_{n\in N}\{\boldsymbol{a}_n\}.$$

If there to date is found no proof of $\boldsymbol{P}$ except that

$$(\forall m\in M)(\mathrm{f}(m) \text{ is valid}).$$

then, for $n\in N-M, f(n)$ is neither known to be valid nor to be invalid. However, it may not be denied out that, in the future, som k may be found such that

$$k\in N-M. \ \&\ .f(k) \text{ is invalid.} \tag{7.1}$$

If for any particular value of n we can always examine and determine the validity of $f(n)$, the set M will evidently be increased. However, the existence of such a k posited by (7.1) may not be convinced until it is really discovered. We say such a phenomenon as the existence of a k above-mentioned to be a *historical contingency*.

The proposition $\boldsymbol{P}$ above-stated seems apparently to be solvable, because it must be either true or false. However, it is also possible to be actully unsolvable only because the existence of its counterexample (i. e., the case of (7.1)) is to date a historical contingency. Thus the proposition $\boldsymbol{P}$ should not be taken as undecidable, but it might be said that the validity of $\boldsymbol{P}$ is hanged in a *historical dilemma*.

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References

1) E. g., Kleene, S. C., *Mathematical Logic* (John Wiley & Sons Inc.)198(1966) ; DeLong, H., *A Profile of Mathematical Logic* (Addison-Wesley Pub. Co.) 56–57(1970).
2) E. g., DeLong, H., *A Profile of Mathematical Logic* 216.
3) E. g., Heyting, A., *Intuitionism* (North-Holland Pub. Co.) 99–100 (1966).
4) Kinokuniya, Y., Mem. Muroran Inst. Tch. 8(1) 32(1973).
5) Kinokuniya, Y., Mem. Muroran Inst. Tch. 7(2) 602(1971).