A Supplement on the Paper "Totally Ordered Linear Space Structures and Extension Theorems"

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	作成者: 岩田, 一男
	メールアドレス:
	所属:
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A Supplement on the Paper "Totally Ordered Linear Space Structures and Extension Theorems"

Kazuo Iwata

Abstract

In this supplement, after modifying 24), the author pursues a relationship between [24),Theorem 4] and [22), sandwich Theorem 6.3].

Introduction. In the preceding paper 24), from the viewpoint of the *totally ordered linear space structures* of the product linear space $E \times \boldsymbol{R}$, the author improved Cotlar-Cignoli [20, III, § 1.2] type extension theorems in the direction of Anger-Lembcke [21), § 1, § 2], but qua strict cone. On the other hand, he recently found that these materials were as well studied, qua preordering, by Anger-Lembcke 22), in '74 but from different angle. At this place, sure enough, what is interesting is that there is found a close relation between the two, although they are unlike in appearance. In this supplement we slightly enlarge our [24), Ths. 1–4] by adding also new conditions (1–4.4), and therewith show that how both above are related with. This consideration, in the interests of the question, seems to be somewhat significant.

For the present article, the author was benefited by Anger-Lembcke 22), he is deeply grateful to them.

Here the author also wishes to express his sincere gratitude to Prof. S. Koshi (Hokkaido Univ.) for his constant inspection.

Preliminaries. In this paper let $E\ (\neq \{0\})$ denote a linear space over the real field $\boldsymbol{R}$. Let L denote a product linear space $E \times \boldsymbol{R}$ or topological product $E \times \boldsymbol{R}$ ($\boldsymbol{R}$ being endowed with usual topology). We put

DEFINITION 1. If on E there is defined a binary relation "$\leqslant$" satisfying all postulates in [17), Def. 1, c)] excepting perhaps 2) and 4), E is called a *preordered linear space* with respect to "$\leqslant$". Convex cone $C=\{c \geqslant 0\}$ is called the *associated cone* with "$\leqslant$". *Preordered linear topological space* my be analogized.

DEFINITION 2. a) Let h be a hypolinear functional on a pointed convex cone $K \subset E$. By $\tilde{h}$ is meant a hypolinear functional on a pointed convex cone $-K$ with $\tilde{h}(y) = h(-y)\ (y \in -K)$.

b) Let f be a linear functional on a pointed convex cone $X \subset E$. By $\bar{f}$ is meant a (unique) linear extension of f to (X), where (X) is the linear hull of X.

The set $\{(y, \eta) : y \in K,\ h(y) < \eta\} \subset L$, where h is a hypolinear functional on K, is termed the "epigraph" of h (the author owes the term "epigraph" to V. Klee). With this

c) $B_{\bar{f}}$ stands for the epigraph of linear form $\bar{f}$. In this case we have $B_{\bar{f}}=\{(x_1-x_2, \xi):f(x_1)-f(x_2)<\xi, x_1, x_2 \in X\}$.

d) C_h is the epigraph of hypolinear functional h.

e) C_{hC} is the quasi-epigraph of hypolinear functional h with respect to C: $C_{hC}=\{(y, \eta):$ there exists $c \in C$ such that $y+c \in K$ with $h(y+c)<\eta\}$.

f) $C_{\tilde{h}C}$ is the quasi-epigraph of hypolinear functional $\tilde{h}$ with respect to C: $C_{\tilde{h}C}=\{(y, \eta):$ there exists $c \in C$ such that $y+c \in -K$ with $\tilde{h}(y+c)<\eta\}$. In this case, in other words $C_{\tilde{h}C}=\{(-y, \eta):$ there exists $c \in C$ such that $y-c \in K$ with $h(y-c)<\eta\}$.

Besides, for convenience we let the notations and terminology employed in 17), 19), 23), and 24) be available.

Statement of the results. The following theorems are respectively special cases of Theorems 3 and 4 *infra*.

THEOREM $\begin{Bmatrix}1\\2\end{Bmatrix}$ *Let E be a preordered* $\begin{Bmatrix}\textit{linear}\\ \textit{linear topological}\end{Bmatrix}$ *space with associated cone C. Let K be a pointed convex cone in E, h a hypolinear functional on K. Let X be a pointed convex cone in E, f a linear functional on X. In order that there exists an* $\begin{Bmatrix}F \in E^*\\ F \in E'\end{Bmatrix}$ *extending f and satisfying $F(y) \leqslant h(y+c)$ for all $y+c \in K$ with $c \in C$, one of the following four conditions is necessary and sufficient*:

$\begin{Bmatrix}(1.1)\\(2.1)\end{Bmatrix}$ *There exist t.o.l.s $(L, \mathfrak{R})$ and* $\begin{Bmatrix}\Phi \in L^*\\ \Phi \in L'\end{Bmatrix}$ *such that*

$$B_{\bar{f}} \cup C_{hC} \subset (L, \Phi(\mathfrak{R}))^+, \quad \Phi(0,1)=1.$$

$\begin{Bmatrix}(1.2)\\(2.2)\end{Bmatrix}$ *There exists a t.o.l.s. $(L, \mathfrak{R})$ such that*

(i) $B_{\bar{f}} \cup C_{hC} \subset (L, \mathfrak{R})^+$,

(ii) $(L, \mathfrak{R})^+$ *is* $\begin{Bmatrix}\textit{absorbing}\\ \textit{a neighbourhood}\end{Bmatrix}$ *at $(0, 1)$ for L.*

$\begin{Bmatrix}(1.3)\\(2.3)\end{Bmatrix}$ *There exists a convex* $\begin{Bmatrix}\textit{absorbing set}\\ \textit{0-neighbourhood}\end{Bmatrix}$ *U in E such that $B_{\bar{f}} \cup C_{hC} \cup (U \times \{1\})$ is positively independent in L.*

$\begin{Bmatrix}(1.4)\\(2.4)\end{Bmatrix}$ *There exists a convex* $\begin{Bmatrix}\textit{absorbing set}\\ \textit{0-neighbourhood}\end{Bmatrix}$ *U in E such that $\xi+\eta+1>0$ whenever $x_1-x_2+y+u=0$ for $(x_1-x_2, \xi) \in B_{\bar{f}}$, $(y, \eta) \in C_{hC}$, $u \in U$.*

REMARK 1. Needless to say, our conditions (1.2), (2.2) are somewhat stronger than those of the former [24), Ths. 1, 2] respectively. So will be the conditions (3.2), (4.3), *infra* in comparison with the cases of [24), Ths. 3, 4]. Theorem 1 (using (1.3)) generalizes [20), III, § 1.2.3 Theorem] (cf. [24), Cor. 1(4)]).

On the other hand, next is a special case of Supplement 2 *infra.*

SUPPLEMENT 1. *Our* (1. 4) (*resp.* (2. 4)) *is equivalent to the condition due to* (*resp.* (9) *of*) Anger-Lembcke [22), *Theorem* 3. 4]:

There exists a convex absorbing set (*resp.* 0-*neighbourhood*) U *in* E *such that the set* $\{f(x_1)-f(x_2)+h(y): x_1, x_2 \in X, y \in K, x_1-x_2+y \in U+C\}$ *is bounded below.*

This means that

REMARK 2. Our Theorem 2 and its proof present another view and an alternative (self-contained) proof to [22), Theorem 3. 4]. (Almost *vice versa.*)

In this connection, in practice, conditions (1. 2−4), (2. 2−4) also seem to be useful (not to mention, Anger-Lembcke condition above quoted is well applicable). For instance, if we are concerned with the "if" part of [22), Th. 3. 5 (Bauer)], (2. 4) serves as follows: Let $x+y+u=0$ for $(x, \xi) \in B_f, (y, \eta) \in C_{hC}, u \in U$, where $U=V$ and $h=p$ with $p(y+c)<\eta$ for some $c \in C$. Then, since $-(y+c+u) \in (\eta+1)U$ where $\eta+1>0$, in view of $x=-(y+c+u)+c$, estimate $f(x) \geqslant -(\eta+1)$ follows. This implies $\xi+\eta+1>f(x)+\eta+1 \geqslant 0$ which proves the assertion.

Incidentally there holds the following which (the latter half) we need in Remark 5(2).

COROLLARY. *Let* E *be a preordered linear topological space with associated cone* C, M *a linear subspace of* E, *and* f *a linear form on* M. *Let* K *be a linear subspace of* E *with* $M \supsetneqq K$ (*resp.* $M \supset K$), h *a hypolinear fuctional on* K. *Temporarily designating by*

(M) $f(x) \leqslant h(x)$ *for all* $x \in M \cap K$,

(F) *there exists a convex* 0-*neighbourhood* U *in* E *for which* f *is bounded above on* $M \cap (U-C)$,

(H) *there exists a convex* 0-*neighbourhood* U *in* E *such that* $U-C$ *does not meet* $\{y \in K: h(y)=1\}$,

(M) *plus* (F) *is necessary for* (2. 4) *of Theorem* 2, *and if* h *is a gauge function on* K, (M) *plus* (H) *is sufficient for* (2. 4) (*resp.* (M) *plus* (F) *is necessary and sufficient for* (2. 4)).

PROOF. (Necessities) (M) is necessary is obvious. With this U of (2. 4), suppose that $f(x)>1$ for some $x \in M \cap (U-C)$. Then, since $h(0)=0$, there would exist $c \in C$, ξ, ε such that $(-x, -\xi) \in B_f, (-c, \varepsilon) \in C_{hC}, x+c \in U$ with $-\xi+\varepsilon+1=0$, contrary to hypothesis. (Sufficiencies) We begin with the first half. Letting $x+y+u=0$ for $(x, \xi) \in B_f, (y, \eta) \in C_{hC}, u \in U$, let $y+c \in K, h(y+c)<\eta$ for some $c \in C$. Then in view of $x+(y+c)+u-c=0$, it comes, by hypothesis, that $0 \leqslant f(x)+h(y+c)+h(u-c)<\xi+\eta+1$. For the latter let U be chosen so that the upper bound in question be 1. Noticing that $y+c \in K \subset M$, $u-c \in M \cap (U-C)$, proof is almost similar to the above.

On this corollary, it should be noted that

REMARK 3. On the first half, if h is once a hypolinear functional on K, (M) plus

(H) not always implies (2. 4) (for a counterexample, we can quote [21), Example (2. 12)]). However, this assertion, for reference (letting $C=\{0\}$) yields [23), Cor. 1 to Th. 2]. In the latter half, if $K=\{0\}$, condition (M) may be dropped (since $f(0)=h(0)$), yielding* the Bauer-Namioka extension theorem. In passing, Theorem 1 of course admits a simple extension of f.

Our usual argument** now gives the following which are the main theorems of this paper.

THEOREM $\begin{Bmatrix}3\\4\end{Bmatrix}$. *Let* E *be a* $\begin{Bmatrix}\textit{linear}\\\textit{linear topological}\end{Bmatrix}$ *space. Let* I, J *be disjoint index sets with* $I\cup J\neq\varnothing$. *For each* $\lambda\in I\cup J$, *let* C_λ *be a pointed convex cone in* E, *and for each* $\lambda\in I\cup J$, *let* K_λ *be a pointed convox cone in* E, h_λ *a hypolinear functional on* K_λ. *Let* X *be a pointed convex cone in* E, f *a linear functional on* X. *In order that*

$\begin{Bmatrix}(3.0)\\(4.0)\end{Bmatrix}$ *there exists an* $\begin{Bmatrix}F\in E^*\\F\in E'\end{Bmatrix}$ *extending* f *and satisfying*

(a) $-h_i(y-c)\leqslant F(y)$ *whenever* $y-c\in K_i$, $c\in C_i$ *for* $i\in I$,

(b) $F(y)\leqslant h_j(y+c)$ *whenever* $y+c\in K_j$, $c\in C_j$ *for* $j\in J$,

one of the following four conditions is necessary and sufficient:

$\begin{Bmatrix}(3.1)\\(4.1)\end{Bmatrix}$ *There exist t.o.l.s.* $(L,\mathfrak{R})$ *and* $\begin{Bmatrix}\Phi\in L^*\\\Phi\in L'\end{Bmatrix}$ *such that*

$$B_{\bar f}\cup(\bigcup_{i\in I}C_{\tilde h_i c_i})\cup(\bigcup_{j\in J}C_{h_j c_j})\subset(L,\Phi(\mathfrak{R}))^+;\quad \Phi(0,1)=1.$$

$\begin{Bmatrix}(3.2)\\(4.2)\end{Bmatrix}$ *There exists a t.o.l.s.* $(L,\mathfrak{R})$ *such that*

(i) $B_{\bar f}\cup(\bigcup_{i\in I}C_{\tilde h_i c_i})\cup(\bigcup_{j\in J}C_{h_j c_j})\subset(L,\mathfrak{R})^+$;

(ii) $(L,\mathfrak{R})^+$ *is* $\begin{Bmatrix}\textit{absorbing}\\\textit{a neighbourhood}\end{Bmatrix}$ *at* $(0,1)$ *for* L.

$\begin{Bmatrix}(3.3)\\(4.3)\end{Bmatrix}$ *There exists a convex* $\begin{Bmatrix}\textit{absorbng set}\\\textit{0-neighbourhood}\end{Bmatrix}$ U *in* E *such that*

$B_{\bar f}\cup(\bigcup_{i\in I}C_{\tilde h_i c_i})\cup(\bigcup_{j\in J}C_{h_j c_j})\cup(U\times\{1\})$ *is positively independent in* L.

$\begin{Bmatrix}(3.4)\\(4.4)\end{Bmatrix}$ *There exists a convex* $\begin{Bmatrix}\textit{absorbing set}\\\textit{0-neighbourhood}\end{Bmatrix}$ U *in* E *such that*

$\xi+\sum_{\nu\in N}\eta_\nu+1>0$ *whenever* $x_1-x_2-\sum_{\nu\in I\cap N}y_\nu+\sum_{\nu\in J\cap N}y_\nu+u=0$ *for* $(x_1-x_2,$
$\xi)\in B_{\bar f}$, $(-y_\nu,\eta_\nu)\in C_{\tilde h_\nu c_\nu}(\nu\in I\cap N)$, $(y_\nu,\eta_\nu)\in C_{h_\nu c_\nu}(\nu\in J\cap N)$, $(u,1)\in U\times\{1\}$, *where* $N\subset I\cup J$ *is finite.*

PROOF OF THEOREM 3. In the light of [14), p.56], [21), (1. 7) Remark], i.e., in view of the "convex core topology" of E, Theorem 3 is viewed as a special case of Theorem 4. (Of course, for its own sake, simple (nearly algebraic) proof can be made: paraphrase the proof of Theorem 4.)

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* This answers for the latter part of [24), p.740 footnote "*"] (and p.739 footnote "*" ibid).

** For this we owe much to [6), § 12], [20), III, § 1. 2] and others.

PROOF OF THEOREM 4 (expatiation). (4. 0)$\Rightarrow$(4. 3) : Noticing that $L' \ni \Psi(x, \xi) = -F(x)+\xi$ is positive (>0) on every positive cone $C_{\tilde{h_i}c_i}(i \in I)$, $C_{h_jc_j}(j \in J)$, one can take $\{x \in E: F(x)<1\}$ for U. (4. 3)$\Rightarrow$(4. 2) : By means of [23), Rem. 2], apply [17), Lemma 1]. (4. 2)$\Rightarrow$ (4. 1) : [17), Lemmas 3, 4] tell us that there exists $\Phi \in L^*$ such that $(L, \mathfrak{R})=(L, \Phi(\mathfrak{R}))$ and $\Phi(0,1)=1$. Moreover $\overline{\Phi^{-1}(0)} \neq L$. (4. 1)$\Rightarrow$(4. 0) : Comparing $\varphi(x, \xi)=-\tilde{f}(x)+\xi$ (on $(X)\times \boldsymbol{R} \ni (0,1)$) with Φ(on L), one can exactly find an $F_1 \in E'$ extending f and satisfying (a), (b). (4. 3)$\Rightarrow$(4. 4) is clear. (4. 4)$\Rightarrow$(4. 3) : Anyway $B_{\tilde{f}}$, $C_{\tilde{h_i}c_i}(i \in I)$, $C_{h_jc_j}(j \in J)$ are all convex cones without vertex zero and U is convex. With this in mind, first we can examine e.g. the following : (4. 4) guarantees the positive independence of $(\bigcup_{i\in I} C_{\tilde{h_i}c_i}) \cup (\bigcup_{j\in J} C_{h_jc_j}) \cup (U\times\{1\})$. Substance of (4. 4) pertaining to $u=0 \in U$ does (esp. characterizes) the same for $B_{\tilde{f}} \cup (\bigcup_{i\in I} C_{\tilde{h_i}c_i}) \cup (\bigcup_{j\in J} C_{h_jc_j})$ (since $\rho 0 \in U$ and $\rho c_\nu \in C_\nu(\nu \in N)$ for all $\rho>0$). Thus the final examination is rather simple from (4. 4), which establishes the assertion.

REMARK 4. Condition (3. 4) (resp. (4. 4)) still more simplifies (3. 2) (resp. (4. 2)). In practice conditions (3. 2−4), (4. 2−4) also seem to be useful.

On the other hand, there holds

SUPPLEMENT 2. *Our condition* (3. 4) (*resp.* (4. 4)) *is equivalent to*

$\left\{\begin{matrix}(3.5)\\(4.5)\end{matrix}\right\}^*$ *There exists a convex* $\left\{\begin{matrix}\textit{absorbing set}\\ \textit{0-neighbourhood}\end{matrix}\right\}$ U *in* E *such that the set* $\{f(x_1)-f(x_2)+\sum_{\nu\in N} h_\nu(y_\nu): x_1, x_2 \in X,\ N\subset I\cup J$ *finite,* $y_\nu \in K_\nu(\nu \in N),\ x_1-x_2-\sum_{\nu\in I\cap N} y_\nu+\sum_{\nu\in J\cap N} y_\nu \in U+\sum_{\nu\in N} C_\nu\}$ *is bounded below.*

PROOF. To see this, without losing generality, one can assume that U is symmetric. (Necessity) With this U of (3. 4) (resp. (4. 4)), let $u=x_1-x_2-\sum_{\nu\in I\cap N}(y_\nu+c_\nu)+\sum_{\nu\in J\cap N}(y_\nu-c_\nu) \in U$ for $x_1, x_2 \in X$, $y_\nu \in K_\nu(\nu \in N)$, and $c_\nu \in C_\nu(\nu \in N)$. Then even if $h_\nu(y_\nu)<+\infty$ for all $\nu \in N$, from the viewpoint of $u+(-u)=0, (y_\nu+c_\nu)-c_\nu=y_\nu \in K_\nu(\nu\in I\cap N)$, $(y_\nu-c_\nu)+c_\nu=y_\nu \in K_\nu(\nu \in J\cap N)$, it follows that $f(x_1)-f(x_2)+\sum_{\nu\in N} h_\nu(y_\nu) \geqslant -1$ (cf. if $\xi'+\sum_{\nu\in N}\eta'_\nu<-1$, there exist $\xi>\xi', \eta_\nu>\eta'_\nu(\nu\in N)$ such that $\xi+\sum_{\nu\in N}\eta_\nu+1=0$), which proves the assertion. For the converses, let U be chosen so that the lower bound in question be -1. Let $x_1-x_2-\sum_{\nu\in I\cap N} y_\nu+\sum_{\nu\in J\cap N} y_\nu+u=0$ for $(x_1-x_2, \xi)\in B_{\tilde{f}}$, $(-y_\nu, \eta_\nu) \in C_{\tilde{h_\nu}c_\nu}(\nu\in I\cap N)$, $(y_\nu, \eta_\nu)\in C_{h_\nu c_\nu}(\nu \in J\cap N)$, $(u,1)\in U\times\{1\}$. Then since there are respective vectors $c_\nu \in C_\nu$ $(\nu\in N)$ such that $y_\nu-c_\nu \in K_\nu$ $(\nu\in I\cap N)$ with $h_\nu(y_\nu-c_\nu)<\eta_\nu$ and that $y_\nu+c_\nu \in K_\nu(\nu\in J\cap N)$ with $h_\nu(y_\nu+c_\nu)<\eta_\nu$, from the viewpoint of $x_1-x_2-\sum_{\nu\in I\cap N}(y_\nu-c_\nu)+\sum_{\nu\in J\cap N}(y_\nu+c_\nu) \in U+\sum_{\nu\in N} C_\nu$, it follows that $0 \leqslant f(x_1)-f(x_2)+\sum_{\nu\in I\cap N} h_\nu(y_\nu-c_\nu)+\sum_{\nu\in J\cap N} h_\nu(y_\nu+c_\nu)+1<\xi+\sum_{\nu\in N}\eta_\nu+1$.

This assures Remark 2 and implies

REMARK 5. (1) Our Theorem 4 coincides with [22), Theorem 6. 3] if E is a

* These conditions were introduced after the model of 21) (and 22)).

locally convex linear topological space and $C_\lambda = C\ (\lambda \in I \cup J)$. (2) We now give substance to [24), Rem. 2] (cf. Remark 3 above).

A little computation gives the following which generalizes the latter half of Corollary of Theorem 2.

COROLLARY. *In Theorem* 4, *let in particular* X, $K_\lambda (\lambda \in I \cup J)$ *be linear subspaces of* E *with* $X \supset K_\lambda$. *Designating by*

$$\text{(M)} \quad \begin{cases} -h_i(x) \leqslant f(x) \text{ whenever } i \in I,\ x \in K_i \\ f(x) \leqslant h_j(x) \text{ whenever } j \in J,\ x \in K_j, \end{cases}$$

(F) *there exists a convex* 0-*neighbourhood* U *in* E *for which* f *is bounded above on every* $X \cap (U - \sum_{\nu \in N} C_\nu)$, *where* $N \subset I \cup J$ *is finite,*
(M) *plus* (F) *is necessary and sufficient for* (4. 4).

Finally we add the following.

REMARK 6. It is easy to see that there are three other analogues about Theorem 4 (so are also about Theorem 3). These are of the forms described in terms of the positive independence of

$$(4.3') \quad B_{\bar f} \cup (\bigcup_{i \in I} C_{\hat{h}_i(-c_i)}) \cup (\bigcup_{j \in J} C_{h_j c_j}) \cup (U \times \{1\}),$$
$$(4.3'') \quad B_{\bar f} \cup (\bigcup_{i \in I} C_{\hat{h}_i c_i}) \cup (\bigcup_{j \in J} C_{h_j(-c_j)}) \cup (U \times \{1\}),$$
$$(4.3''') \quad B_{\bar f} \cup (\bigcup_{i \in I} C_{\hat{h}_i(-c_i)}) \cup (\bigcup_{j \in J} C_{h_j(-c_j)}) \cup (U \times \{1\}),$$

respectively, where U is a suitable convex 0-neighbourhood in E. The corresponding conditions (4. 0′), (4. 0″), (4. 0‴) ; and others may be realized without difficulty ; the details are omitted.

REMARK 7. Letting in particular $h_{i_0} = -q$, $K_{i_0} = C$, $C_{i_0} = \{0\}$; $h_{j_0} = p$, $K_{j_0} = E$, $C_{j_0} = \{0\}$ (i.e., I, J both being singleton) in Theorem 3, Bonsall's result appearing in [18), p.13] a fortiori follows. To see this, for instance, it suffices to take $B_f = \{(0, \xi) : \xi > 0\}$, $U = \{y : p(y) < 1\}$ for (3. 3). In fact, under the hypothesis, positive independence of $C_{\widetilde{-q}} \cup C_p\ (\supset B_f \cup (U \times \{1\}))$ is easily verified. (3. 0) meets the conclusion.

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