Some Methodological Views on Objectivism

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Abstract

Theories which are proceeded aiming to hold their consistency by means of revision of axiomatics, are *objectivisms.* Undecidabilities may be thought to emerge from the imperfect correspondence between the original constitutions and their set-theoretical interpretations. So the *annexed set theory* is taken up and is found, in effect, to yield much work in association with the intended objectivism.

0. Introduction

Given a universe U of primitive objects, if we intend to see which events on U are to be thought as true, the theory of the events there to be developed is an *objectivism.* On looking back over the history, we find some theories such as geometry and the number theory etc. had been preceded by a handsome amount of knowledges obtained in several ways before they were systematized. Devices were later on laid down in order to make these knowledges completely clarified and extensively advanced. Signs and definitions were made to build up a proper language for the intended objectivism and axiomatics was searched to raze out all ambiguousnesses from its course of proceeding.

It is the objectivist point of view that, if some disagreeable result is concluded from the construction presently assumed, revisions should be tried on its axiomatics until it ceases to reproduce such viciousness again. The whole system of the definitions, axioms and the results (or theorems) to date obtained is called the *proto-construction* (or the *proto-system*) of the intended objectivism. Axioms are particularly criticized in connection to epistemological reflections and are changed if needed. Such a treatment may be founnd as of the same stand with the 'realist' attitude which Bourbaki has adopted.[1)]

To say a proposition to be *valid* means that it is provable in the proto-construction. Though we conveniently say a proposition is true when it is valid, a true proposition may not always be valid. For example, the proposition "There are infinitely many pairs of twin prime numbers" might be true, though it has not yet been shown to be valid. Incidentally, after some renovation has been made, something formerly thought to be valied may possibly be thought to be invalid (or false) in the new construction.

In an objectivism every inquiry is put forward in the form "Whether so and so is true or not", that is, it is of bivalent prospect. However, we have really experienced several inquiries which could neither be concluded as valid nor invalid, that is, were undecidable though at the outses had been assumed to be of bivalent prospect. In so far as the proto-system is consistent, suce inquiries may occur only because their contents cannot perfectly be therein defind out. Yet, on the other side, they may truely cast new lights to promise

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some supplemental leaps to the intended objectivism, if sufficient reflections are paid. Thus, in trying analyses on various sorts of events, to reveal imperfectness of the proto-system of axioms to decompose out their contents will make a very important part of our study.

1. Annexed Set Theory

If for each member of a finite set of events $\{e_1,\cdots,e_n\} = E_n$ a property p is uniformly fulfilled, we say p is *verified* in E_n. In this case each e_k $(k=1, 2, \cdots, n)$ is considered to give an *evidence* for the property p and if n is found to be > 1 p is considered as a possible property in the system. Generally, if we are convinced that for every event contained in a set A p is fulfilled, we say also that p is verified in A. If the supremum $R(p)$ of such A's exists as a set (i.e., a determinate aggregate), and if p is surely verifiable in $R(p)$, then $R(p)$ is referred as the *range* of p. In this connection the set $R(p)$ is, as it were, the maximum extension of E_n.

As it is, such an $R(p)$ as defined in the above canot always give a clear object. For instance, the total collection of things which are not considered to be 'white' may only be an intractable divergence as by several authors has been noticed. To be exempted from such divergence of conception, there must precede a universal set of events $L(p)$ to be existent to restrict such that

$$E_n \subseteq R(p) \subseteq L(p).$$

This $L(p)$ is to be introduced as the total aggregate of events over which is properly inquired whether the property p is fulfilled or not, and in this connection is referred as the *level* of p. Thus a property p cannot be haphazardly presented, but it is thought requisite to be associated with a set $L(p)$ as its level, that is, to be *quantified* by the set $L(p)$. Quantified properties are *objectivisit properties*. However, if no confliction is feared for, we simply call them *properties*.

For two properties p_1 and p_2, if every event of $L(p_2)$ is found to be an event on $L(p_1)$ (or an event of events, or an event of events of events etc.), then we say "$L(p_2)$ is of *higher order* than $L(p_1)$" or "p_2 stands on a higher level than p_1". However, we shall simply use the term 'event' regardless of its level, if no need of showing it.

Starting from the primitive universe, on repeating definitions we may obtain various levels and for each of them we may take its subsets. An objectivism may thus have a variety of sets for observation, so that may there-upon be constituted a theory of sets peculiar to it. We call this theory the *annexed set theory* of the intended objectivism, It will be particularly notable that sets or classes from the annexed set theory may in their turn emerge in a similar way to that of information sources. In this connection it is important that the annexed set theory is found to be based on a proper extension of the set of the direct interpretations from the proto-construction.

To assume a proposition p either to be true or false, we think it necessary that P can be quantitativery interpreted in the annexed set theory, that is, there can be found a proposition P' in the annexed set theory such that

$$P' \cdot\!\Longleftrightarrow\!\cdot P$$

(read : If P' is true so is P, and inversely). Such a proposition P is called an *objectivist proposition.* But we shall henceforth mean by '*a proposition*' an objectivist one. Transference of observation from about P to about P' is considered to be an objectivist analysis.

2. Annexed Methodology

If two properties p and q have the same level L and if $R(p) \subseteq R(q)$ ($\subseteq L$), then we write as

$$p \Rightarrow q, \tag{2.1}$$

and say "*p implies q*". We shall not adopt the material implication (or the conditional sentence)

$$p \supset q. \equiv . \sim p \vee q.$$

This is found to be a property of which the range $R(p \supset q)$ is calculated as

$$R(p \supset q) = (\mathrm{L} - R(p)) \cup R(q),$$

whereas the (logical) implication given by (2.1) is not a property of which the range is found in L, but a statement which is thought to stand on a higher level than p and q. i.e., is a *meta-object* in comparison with p and q, since tha objects of observation with respect to (2.1) are p and q themselves.

Even when $R(p)$ is certainly asserted to be a determinate aggregate (i.e., a set), it is possible that whether $R(p)$ be finite or not is essentially dubious, or so is whether $R(p) \cap M$ be an empty set or not for a special set $M \subset L(p)$. In such cases, if the assumption that $R(p)$ is finite leads to a contradiction, then $R(p)$ must be infinite, and if the assumption that $R(p) \cap M = \emptyset$ leads to a contradiction, then it must be that $R(p) \cap M \neq \emptyset$. Thus, if only it is certainly promised that $R(p)$ should give a determinate aggregate, we may use the concept of $R(p)$ in our (objectivist) logical calculus, though we do not know it in all its details. However, this may not necessarily be a fact directly ascertained in connection with the axioms postulated on the primitive universe, but it seems rather correct to say that the admission of $R(p)$ to be used in the calculus is simultaneously demanded with the admission of the concept $R(p)$ as a set in the annexed set theory.

When a proof of a proposition P (to be true) is made by showing the validity of P' the interpretation of P in the annexed set theory, if any inappropriateness is to be inserted in the course of the proof, it is possibly expected that the annexed set theory is responsible for that inappropriateness. However, as the annexed set theory is also set under the regulation of the empiricist pragmatism, no pure set-theoretical inappropriateness can methodologically be expected to stray into. Therefore, if any inappropriateness is to be inserted, it may not be other than some sets or classes cannot be perfectly defined out in the original proto-construction.

About two propositions S_1 and S_2, if it is provable that if S_1 is true so is S_2 and if S_1 is false then S_2 cannot be fully true, then we write

$$S_1 (\Rightarrow) S_2 \tag{2.2}$$

and say "S_1 *implies* S_2". If we have, in addition to (2.2),

$$S_2(\Rightarrow)S_1 \tag{2.3}$$

S_1 and S_2 are said to be *equivalent* and we then take them as stating the same thing. If (2.2) is the case but (2.3) is not, then there must be at least one event for which is found

$$S_2 \wedge \sim S_1$$

not to be a fallacy, so that we may in effect have that

$$S_2 = S_1 \vee (S_2 \wedge \sim S_1) \tag{2.4}$$

The logical decomposition (2.4) may be considered to give a quantitative decomposition of S_2. Let us discuss this point more in detail. If a proposition S may be convinced to be possible, there must exist at least one evidence to verify it. Moreover, in our view, this evidence is requisitely demanded to be concretely ascertained, in any way, in connection with the primitive universe, that is, to be an event concretely defined on the universe. Such an evidence is called an *objectivist* one. If there exists the maximum aggregate of objectivist evidences of S as a determinate one (ie. a set), we denote it by $E(S)$ and call it the *example range* of S. We may say S is verfiable in $E(S)$ on condition that $E(S) \neq \varnothing$. If $E(S) = \varnothing$ S is false, and if $E(S) \neq \varnothing$ then S is possible.

If S_1 and S_2 are possible propositions, (2.4) may be considered to be equivalent to the relation

$$E(S_2) = E(S_1) \cup E(S_2 \wedge \sim S_1).$$

So, if (2.2) holds, we have

$$E(S_2) \supseteq E(S_1).$$

Thus being the conditions, if we take S_1 and S_2 as properties, with regard to the definition of (2.1) we have

$$S_1 \Rightarrow S_2 \tag{2.5}$$

because we may then take as $R(S_k) = \mathrm{E}(S_k)$ $(k = 1, 2)$. Thus we may generally adopt the notation (2.5) instead of (2.2).

Thus accumulating definitions and distinctions, we will have various implicative relations to be applied to analysis. The study of these relations may, closely related to the annexed set theory, be developed on. The thus promised theory of relations is referred as the *annexed methodology* for the intended objectivism.

3. Critical Topics

(**1**) *Level of a Proposition.* For a given proposition P, if $E(P)$ is to be a determinate class, it is thought requisite that there exists a unversal set of events $L(P)$ which promises the relation

$$E(P) \subseteq L(P)$$

and satisfies the following conditions;

i) every objectivist evidence of P is contained in $L(P)$;

ii) every objectivist counterevidence of P (i.e., evidence by which will be verified that P does not wholly hold) is contained in $L(P)$;

iii) if there is any event e such that, if e can neither be an evidence nor a counter-vidence of P, P can neither be decided as wholly true nor wholly false, then e is contained in $L(P)$;

iv) $L(P)$ contains no other event beyond the stipulations i), ii), and iii).

If L(P) is determinate, P is said to be *objectivistly quantified* by $L(P)$ and then $L(P)$ is called the *level* of P. If $L(P)$ may not be thought as determinate, then is regarded as meaningless and is renounced. The previously suggested interpretation P' of P in the annexed set theory may be expressed as a proposition on $L(P)$.

(**2**) *Supplemental Leap for Objectivist Conjecturing*. In order to treat the proposition "There are infinitely many prime numbers" set-theoretically, we will arrange the prime numbers totally in an increasing sequence as $p_1(=2)$. $p_2(=3)$, $p_3(=5)$, $\cdots$. Then, if $U=$ the total set of natural integers, $E_n=\{p_1,\cdots, p_n\}$, and $L_n=U-E_n$, the assumption that there exists an integer n such that no prime number can be found in L_n has, as well known, led to a contradiction. In this case, indicating by P the above proposition we may have

$$L(P)=\{L_n \mid n=1,\ 2,\ \cdots\}.$$

This $L(P)$ seems to be very naturally conceived in connection with the universe U. However, the assumption

$$(\exists\, n)(L_n|(\forall\, k=1,\ 2,\ \cdots)(L_n \cap E_k=\varnothing)) \qquad (3.1)$$

is merely an abstract imagination because it is practically impossible to examine over all of L_n.

We could fortunately reach a contradiction so that the proposition was concluded to be true. But, if we could never reach any contradiction, the inquiry whether or not finite is the set of prime numbers might never be given any answer. Eventually, the set of prime numbers is essentially an unknowable object (in practice). In effect, the assumptive possibilitiy of (3.1) may be found only when our eyes are cast upon the annexed methodological field provided there. Thus the above conclusion by means of the method of absurdity is considererd as a *supplemental leap* favored by fortune.

(**3**) *Imperfectness of Axiomatics.* There had been a prospect that the fifth axiom of the euclidean geometry (i.e., the axiom of parallels) might be derived from the other part of the system. However, no geometrician could lead to the conviction that the fifth axiom should be requisite. Nowadays, we are convinced that geometry may yet consistently hold if the fifth axiom is altered, However, we also say that, in the euclidean system, the notion of parallelism may not be ferfectly given if the fifth axiom is unused.

The infiniteness of the set of prime numbers should not be changed by any additional postulate, that is, that infiniteness is an objectivistly destined property. If pome property of an event is undecidable in the proto-construction, it is said to be objectivistly undestined. In this context, it may be said the the prote-system is only imperfectly accomodated about the undestined properties.

(**4**) *Elimination of Unmeasurability.* If A is a subset of a set M, it may be thought essentialy natural if we claim that A then may not be larger than M.[*] According to the literature, this claim was set in Euclid's 'Element' (= Stoicheia) as a common notion.[2] However, in the theory of sets, the notion of a determinate aggregate M is read but vague. It is firstly demanded that, if M is determinate, then we have

$$\forall x \in U : x \in M . \vee . x \notin M. \tag{3.2}$$

But, in using the quantifier $\forall$, this relation may not practically be traced. So, it may not be applied beyond the formal use.

If M and G are determinate aggregates of points in a euclidean space E, it seems very natural if we claim

$$\tilde{m} M \leqslant \tilde{m} G \tag{3.3}$$

($\tilde{m}$ meaning the a priori measure) when $M \subseteqq G$. However, as well known, we here are not allowed to assert the relation (3.3) but for the condition that both M and G are $\tilde{m}$-measurable.

Assume that M is determinate but is not promised to be $\tilde{m}$-measurable. Then, if $\boldsymbol{M}$ is the total class of $\tilde{m}$-measurable subsets of M, and if

$$b = \sup \tilde{m} A \ (A \in \boldsymbol{M}), \tag{3.4}$$

it appears very plausible that

$$\tilde{m} M \leqslant \text{b}. \tag{3.5}$$

In effect, it is clear that we have (3.5) if M is $\tilde{m}$-measurable.

To tell the truth, "M is unmeasurable" is a very obscure statement. Let us take the case of $\tilde{m}$-measure. This measure is but an extension of Lebesgue measure m. When M is Lebesgue measurable, then M is also $\tilde{m}$-measurable and

$$\tilde{m} M = m M. \tag{3.6}$$

Except for the relation (3.6) (in case of m-measurable M), $\tilde{m}$ is not in advance restricted in any positive way. We thereupon only require that $\tilde{m}$ should be thought as the most natural measure to be associated with the space E.

Since thus no other positive restriction than (3.6) is imposed to $\tilde{m}$, if it is convinced that an aggregate M is a determinate one in E, there is no positive objection against the $\tilde{m}$-measurability of M, so that we may only wait for any device to assign M an adequate value $\tilde{m}M$. Thus, it appears that $\tilde{m}$-unmeasurability is as much unascertainable a notion as that of indeterminateness (of an aggregate). This being so, the difference between the two notions may, as it is, be said to have been left as imperfect in the present system. Therefore, if we eliminate the difference, it will be that we make the difference absorbed in the imperfection of the present system.

Consequently, we decide to make it be an objectivist assertion that if M is a determinate aggregate (i.e., a set) M is $\tilde{m}$-measurable too and

$$b = \tilde{m} M$$

b being given by (3.4). Moreover, the following logical dogma may, in this context, be found very convenient :

Principle of $\tilde{m}$-Measure Destination. When A is a determinate aggregate of points (in a

*) Incidentally, Zeno's paradox "Half a given time is equal to double the time" may be regarded as of an abnormal construction to be rejected from our measure-theoretical discourse when the original set is of positive measure.

euclidean space), if the assumption that A is $\widetilde{m}$-measurable, leads to no other value than α to be assigned as $\widetilde{m}A$, then A is $\widetilde{m}$-measurable and

$$\alpha = \widetilde{m}A.$$

(**5**) *Empiricist Pragmatism.* Taking some proposition as a hypothesis, if there may be found neither evidence to verify it nor contradictory circumstance to deny it out, that hypothesis may give only an illusion of wanton announcing. If we yet allow such an illusion to pretend any regard to be connected with our inference, it is simply a consumption because no concrete event is thereby promised. Thus it is found to make a demand of the empiricist pragmatism that we should renounce any such illusional thesis as a removable noise.

For instance, we have removed the notion of an ordinal number to correspond to a continuum from our list, because it never promises any summable[3] sequence of sets corresponding to its sections to reach a limiting set of positive $\widetilde{m}$-measure. However, an assertion of renouncement should not be so simply used. It is also notable that an apparent (or *prima facie*) illusional thesis may possibly be utilized for some extensional renovation of axiomatics. In suce cases, the theses shall not be considered as mere noises.

(**6**) *Epistemo-logical System.* Euclidean geometry may be preferable to Bolyai-Lobachewskian geometry in respect that the former admits similar figures. Such a preference may be considered to follow from epistemological reflections.

It will be insufficinent for a theory of measure in an euclidean space, if the space is simply given as a mere total aggregate of points, because then may be found no treatment for the fact that any linear intervals of different lengths can be put into one-to-one correspondence (between the points in them.). That is, the notion of a point, if it is independently presented of any other constructive relations to the space than the one of its mere situs, will half lose its spatial meaning.

Such being the state, it would be well first to show infinite sequences of partitions of the space itself and next define the points as the limiting elements of cells of partition. In effect, points may not be understood to make up any set of positive $\widetilde{m}$-measure without the notion of *point measure* (i.e., the abstract size of a point).

Eventually, the contents of reflections or renovations above-stated may be said to establish an *epistemo-logical* system to be associated with the intended theory.

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