Admissible Sectional Dimensions of R/C Floor Elements to be Designed without Deflection Check Part 2: Floor Slabs

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Abstract

We review the adequacy of the slab thickness equation in the r/c design code by the Architectural Institute of Japan, in comparison with the related data obtained by our recently renovated method which employs the known effective stiffness and an experimentally estimated time-dependent reduced elastic modulus and whose practical accuracy to tolerable degrees has been observed in our earlier reports. Using this method we try series of deflection estimate for slab models of graduated dimensions in a practically chosen wide range, for the purpose of examining the subject matter of defined possibility for rational slab sections to be designed without deflection check.

1. Introduction

The provisions for slab thickness in the current Japanese Code for the design of r/c constructions by the Architectural Institute, here simply called the AIJ Code[1], had been derived from results of long-time experiments and field researches mainly concerning domestic types of two-way floor systems. The derivation is known to have assumed a long-time deflection of built-in slabs of the order of 16 times the elastic deflection and an admissible or limiting long-time deflection of 1/250 of short span Lx of a slab; while both criteria can in fact be regarded more or less too large, respectively implying overestimated longtime deflections and insufficient restriction on them; in view of cases published so far of their field measurement and examples of their code treatment.

Herein made is a systematic recalculation accordingly needed of the above long-time multiplying ratio for slabs as it varies with graduated dimensions in a major practical range, in order for us to clarify main specific aspects of actual distribution of this ratio, when depending on our system. Thus we try to set up a more rational deflection limitation as well as to examine criteria for the slab design as in the title.

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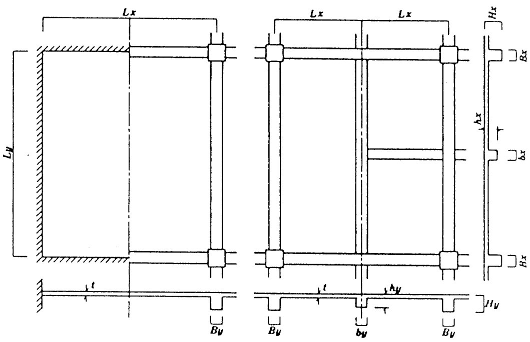
Those attempts are made by applying our procedure to the above set of floor slab models reinforced according to the AIJ Code with their thicknesses observing its cited thickness provision whose latest amendment is said to have largely contributed to reported minimized cases since about that time of large deflection complaint as to a floor system of the later design.

All our present calculation is to depend on our recently modified approach here designated I-effective-E-sustained procedure[2] based on using reduced values of both moment of inertia and elastic modulus respectively due to cracking and time-dependency of the concrete[2].



(a) Slabs without Beams (Types-A)　(b) Slabs with Beams (Types-B)

Fig. 1 Introduced Types of Calculation Models with Identified Notation for their Dimensions.

2. Calculation Models

Table 1 Dimensions of Calculation Models.

System	Restraint/ Beam Types	Geometric Items for Floor Elmnts. in cm		Assumed Overall and Sectional Dimensions in cm	Note (Numerals in cm)
Slabs without Beams or Types A	All-Edge-Fixed	Slab Thickness Short-Span Length Aspect Ratio		Code Vals. & Usual 15, 18, 20 450 to 900 at 50 Intervals 1.0 to 2.0 at 0.2 Intervals	Edge-Fixed and Girder-Restrained Slabs are Assumed to be Equal in Thickness, Short-Span Length and Aspect Ratio; L = Center-to-Center Member Length.
	All-Edge-Restrained on Girders	Girder Width 〃 Depth		40 + 2.5(L - 900)/150 0.1 L + 5(600 - L)/150	
Slabs with Beams or Types B	Transverse only in y-direction at Mid-Bay	Slab Thickness Short-Span Length Long-Span 〃		Code Values 300, 375, 450 450 to 1350 at 150 Intervals	with 12 at its Lowest; Span Length between Centers of Supports.
		Girders: with Beams	Width Depth	40 + 5(2Lx - 750)/150 0.2 Lx + 5(900 - 2Lx)/150	Respective Lx and Ly being Center-to-Center Span Lengths along Short and Long Edges ;
		without Beams	Width Depth	40 + 2.5(Ly - 900)/150 0.1Ly + 5(600 - Ly)/150	
		Beams	Width Depth	40 + 2.5(Ly - 900)/150 Ly/15	Beam Depth less than Girder Depth Hx being Assumed to be Hx.
	Transverse in both x- and y-directions at Mid-Bay (with Two Crossing)	Slab Thickness Short Span Length Aspect Ratio		12 225 to 675 at 75 Intervals 1.0	Center-to-Center Span.
		Girders	Width Depth	40 + 5(2Lx - 750)/150 0.2Lx + 5(900 - 2Lx)/150	Lx being Length of Span of Member over Short Edge.
		Beams	Width Depth	40 - 5(900 - 2Ly)/150 2Lx/15 ; 2Lx/20	

Table 2 Material Properties and Loads for Calculation Models.

Assumed Items		Adopted Values with Supplementary Definition	
Concrete	Compressive Strength :	210 kg/cm^2	: Code Value in Ordinary Use;
	Modulus of Rupture :	26.1 〃	= 1.8 $\sqrt{F_c}$ Suggested in Code;
	Bond Strength :	14 〃	= F_c/15 for End-Top Deformed Steel;
	Elastic Modulus :	210,000 〃	: Code Value;
	Sustained Elastic Modulus:	26,600 〃	at Infinite Age ($t = \infty$);
	Poisson's Ratio :	0.2	: Code Value;
Rein-force-ment	Allowable Tensile Stress :	2,000 kg/cm^2	for Code No. SD300 Steel;
	Elastic Modulus :	2.1x10^6 〃	with Modular Ratio n = 10;
Loads	Construction Load :	---------	= Conventional 2.1 times Weight of an R/C Floor;
	Full Design Live Load :	300, 180 kg/m^2	: Code Values for respective Office and Living Room;
	Long-Time Sustained Portion of Live Load :	100, 60 〃	= 1/3 of do., respectively;
	Wt. of Ceiling & Finish :	80 〃	: Code Value in Ordinary Use.

Shown in Fig. 1, calculation models to be discussed are grouped into: (a) types-A slabs, viz., all-edge-fixed rectangular slabs as well as interior slabs of a slab-girder floor system with an infinite number of identical bays continuous in both x- and y-directions, here respectively to be designated type-Al and type-A2 structures; and (b) types-B structures or interior slabs with beams in a slab-beam-girder floor system with identical bays endlessly continuous as above; in which case (b) a slab is supposed to have a mid-span transverse beam or two crossing, with either case of structure to be called in the following type-Bl or type-B2 slab, respectively.

Table 1 covers the assumed geometric items viz. short spans, aspect ratios, slab thickness and sectional dimensions; where slab thickness is to have the AIJ Code values while customary 12, 15 and 18cm are also used for types-A slabs without beams.

Listed in Table 2 are currently assumed material properies and types and amounts of loads; where design live loads are adopted for here generally supposed office floors but equally for residence floors in the case of types-A slabs; since efforts are then also made to obtain any practical measure of the order of effect of different live load assumption on deflection result.

All deflections are obtained for a long-time sustained load composed of dead load plus 1/3 of design live load; while reinforcing resorts to the design load designated in the Code and the moments provided for in its section 9; in such a way that the restrictive clauses of Section 13 may be observed and any combination of the used D10 and the D13 deformed steel may be made so as to minimize their total weight. However, for calculating reduced moments of inertia of initially cracked slab and beam sections, construction loads are used exclusively, in due consideration of their being deservedly rated as maximum loads in the whole loading history; which treatment is also the case generally in the pertinent literature.[2]

Reinforcement is arranged with a covering depth of the slab main steel of 3cm and with a heaviest limit of the end top reinforcement amounting to the D13 steel spaced @100mm.

The adopted difference mesh is square with a width of Lx/6 or Lx/10 respectively for structures of types-A and B; with Ly then contatining either fraction.

3. Long-Time Slab Deflection

3.1 Rectangular Slabs

Table 3 provides the predicted values of causally different deflections of rectangular floor slabs, to be used for offices and residences when their short-edge lengths and aspect ratios are varied systematically. "LTD", short for "long-time deflection", will be used also in diagrams.

Compared office with residence floor slab thicknesses both due to the AIJ Code show the former smaller by about one or two centimeters than the latter provided any two slabs of the respective types have equal short-span lengths and aspect ratios. Accordingly in all cases long-time deflection values at an infinite structural age or terminative deflection values for former slabs prove to be larger than those for the latter strutures despite rather smaller loads applied on them.

Incidentally, there are some few entries of seemingly incoherent calculations of terminative deflection larger for slabs with smaller, not as usually thought larger aspect ratios, although they are equal in short-span length and thickness; in fact being minor irregularity attributable to different amounts of reinforcement between any two compaired cases.

Though this table relates to the cases of structures with long-span lengths within 13.5m and slab thickness less than 30cm due to the AIJ Code, other more practical cases with conventional 12, 15 and 18cm of thickness are taken up in Fig. 2 provided reinforcement then has a quantity maximum of the steel spaced @ 10cm.

Fig. 2 shows for each of the prior introduced type-Al slabs the distrbution of ratios of long-time deflection to its elastic portion, or long-time deflection multipliers for short. Here both maximum and average values are noted of long-time deflection multipliers of respectively 12.5 and 9.1 for offices as well as 13.8 and 9.8 for living floor parts of residences, all being values considerably smaller than 16 on which the AIJ Code depends for its provision for slab thickness as referred to above.

Table 3 Predicted Deflections under Working Loads of Type-A1 Slabs of the AJJ Code Design

Short Span	Aspect Ratio	Slab Thickness t (cm)		Elastic Deflection Δ_e (mm)		Immediate Deflection Δ_i (mm)		Additional Deflection Δ_s (mm)		Long-Time Deflection, LTD $\Sigma\Delta=\Delta_s+\Delta_i$		LTD Multiplier $\Sigma\Delta/\Delta_e$		Recip. of LTD Ratio $L_x/\Sigma\Delta$		Note
L_x(cm)	L_y/L_x	Ofc.	Res.	Ofc.	Res.	Ofc.	Res.	Ofc.	Res.	Ofc.	Res.	Ofc.	Res.	Ofc.	Res.	
450	1.0	13	12	0.7	0.8	0.7	0.8	0.0	0.0	4.9	5.6	7.0	7.0	918	804	Δ_s = Additional
	1.2	14	13	0.8	0.9	1.0	1.2	1.7	3.0	8.0	10.2	10.0	11.3	563	441	Deflection due
	1.4	15	14	0.8	0.9	1.1	1.3	2.1	3.3	8.5	10.6	10.6	11.8	529	425	to Bond-Slip of
	1.6	15	14	0.9	1.0	1.4	1.7	2.8	4.2	10.1	12.5	11.2	12.5	446	360	Steel Anchorage;
	1.8	15	15	1.0	0.9	1.6	1.6	3.0	4.0	10.7	11.5	10.7	12.8	421	391	
	2.0	16	15	0.9	0.9	1.4	1.7	2.7	4.2	9.5	11.9	10.6	13.2	474	378	$\Delta_t=\Delta_{i+cp+sh}$;
500	1.0	15	14	0.8	0.8	0.8	0.8	0.0	0.0	5.1	5.7	6.4	7.1	980	877	
	1.2	16	15	0.9	1.0	1.3	1.4	2.8	3.8	9.8	11.7	10.9	11.7	510	427	LTD = Long-Time
	1.4	17	16	0.9	1.0	1.4	1.6	2.7	4.2	9.9	12.3	11.0	12.3	505	407	Deflection (Ab-
	1.6	17	16	1.1	1.1	1.7	1.9	3.2	4.0	11.1	12.9	10.1	11.7	450	388	breviated);
	1.8	18	17	1.0	1.0	1.6	1.9	3.2	4.2	10.6	12.4	10.6	12.4	472	403	
	2.0	18	17	1.0	1.1	1.8	2.1	3.1	4.1	10.6	12.5	10.6	11.4	472	400	Subhead "Ofc."
550	1.0	16	15	1.0	1.1	1.4	1.5	3.3	3.8	11.1	12.4	11.1	11.3	495	444	or "Res." re-
	1.2	18	17	1.0	1.1	1.5	1.6	3.5	4.2	11.3	12.7	11.3	11.5	487	433	spectively re-
	1.4	19	18	1.1	1.1	1.6	1.8	3.1	3.8	11.0	12.5	10.0	11.4	500	440	lates to the
	1.6	20	18	1.1	1.3	1.7	2.2	3.2	4.1	11.1	13.8	10.1	10.6	495	399	Cases of Office
	1.8	20	19	1.1	1.2	1.9	2.2	3.1	4.0	11.3	13.0	10.3	10.8	487	423	or Residence
	2.0	20	19	1.2	1.2	2.1	2.4	2.9	3.9	11.1	13.0	9.3	10.8	495	423	Floors;
600	1.0	18	17	1.1	1.1	1.5	1.7	3.4	4.7	11.7	14.0	10.6	12.7	513	429	
	1.2	20	19	1.1	1.2	1.7	1.9	3.5	4.6	12.1	14.1	11.0	11.8	496	426	
	1.4	21	20	1.2	1.3	1.9	2.1	3.3	4.4	12.1	14.1	10.1	10.8	496	426	
	1.6	22	20	1.2	1.4	2.0	2.5	3.1	4.0	11.7	14.3	9.8	10.2	513	420	
	1.8	22	21	1.3	1.4	2.3	2.5	2.9	4.0	11.7	13.8	9.0	9.9	513	435	
	2.0	22	21	1.3	1.4	2.5	2.8	3.1	3.8	12.2	13.4	9.4	9.6	492	448	
650	1.0	20	19	1.2	1.2	1.7	1.9	3.8	4.9	12.8	14.8	10.7	12.3	508	439	
	1.2	22	21	1.3	1.3	1.9	2.1	3.5	4.7	12.8	14.8	9.8	11.4	508	439	
	1.4	23	22	1.4	1.4	2.2	2.4	3.4	4.3	12.9	14.7	9.2	10.5	504	442	
	1.6	24	23	1.4	1.4	2.4	2.6	3.4	4.0	12.9	14.2	9.2	10.1	504	458	
	1.8	25	23	1.3	1.5	2.4	2.9	3.1	3.8	11.9	14.0	9.2	9.3	546	464	
	2.0	25	23	1.4	1.6	2.6	3.3	3.2	4.0	12.2	14.4	8.7	9.0	533	451	
700	1.0	22	21	1.2	1.3	1.8	2.0	3.5	4.6	13.1	15.1	10.9	11.6	534	464	Cases of L_y
	1.2	25	23	1.3	1.4	1.9	2.3	3.5	4.1	12.7	14.9	9.8	10.6	551	470	over 13.5 m
	1.4	26	24	1.4	1.6	2.2	2.7	3.1	4.4	12.6	15.7	9.0	9.8	556	446	Excluded;
	1.6	27	25	1.4	1.6	2.5	3.0	3.4	4.1	13.0	15.1	9.3	9.4	538	464	
	1.8	27	26	1.5	1.6	2.8	3.0	3.1	3.8	12.7	14.0	8.5	8.8	551	500	
750	1.0	24	23	1.4	1.4	2.0	2.2	3.2	4.3	13.4	15.4	9.6	11.0	560	487	
	1.2	27	26	1.4	1.5	2.2	2.4	3.2	4.0	13.3	14.8	9.5	9.9	564	507	
	1.4	28	27	1.6	1.6	2.5	2.7	3.1	3.7	13.4	14.7	8.4	9.2	560	510	
	1.6	29	28	1.6	1.6	2.8	3.1	3.2	4.1	13.7	15.1	8.6	9.4	547	497	
	1.8	30	28	1.6	1.8	2.9	3.5	2.9	3.7	12.6	15.1	7.9	8.4	595	497	
800	1.0	27	25	1.3	1.5	2.0	2.4	3.1	4.2	13.1	16.0	10.1	10.7	611	500	Cases of t
	1.2	29	28	1.6	1.6	2.4	2.6	3.1	4.0	14.1	15.6	8.8	9.8	567	513	over 30 cm
	1.4	—	29	—	1.8	—	3.1	—	3.6	—	15.5	—	8.6	—	516	Excluded.
	1.6	—	30	—	1.8	—	3.4	—	3.8	—	15.8	—	8.8	—	506	
850	1.0	29	27	1.5	1.6	2.2	2.6	3.0	3.5	13.7	15.8	9.1	9.9	620	538	

Fig. 3 represents the last graph of distribution of long-time deflection multipliers; here for structures of A2-type, designed for offices, different in aspect ratio and dimensions. In this case, the respective maximum and average of the long-time multipliers are 9.7 and 6.9; amounts both reflecting significantly less bond-slip than edge-fixed cases, mitigated by girder deflection to lower both indices as much.

3.2 Slabs with Beams

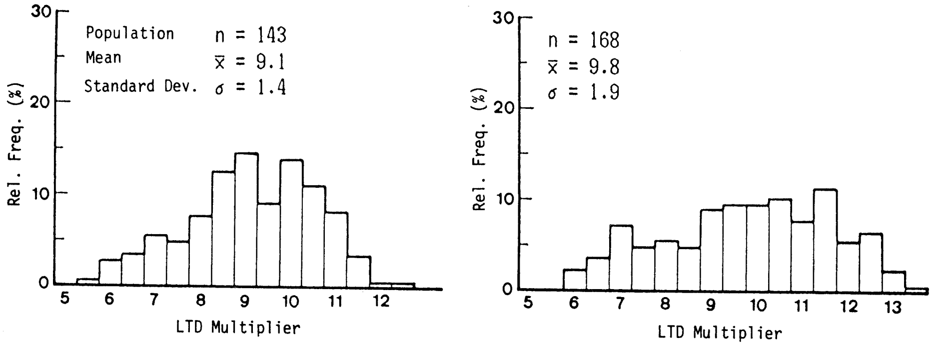
For either set of slabs of B1- or B2-type, distribution of long-time deflection multipliers is shown in Fig. 4 where their maximal and mean values are 5.3 and 4.5 for the former slabs and 8.1 and 6.3 for the latter, respectively, all being evidently smaller than those preceding, showing a dominant edge restraining effect of peripheral girder frames.

4. Admissible Deflections of Floor Slabs

Deflection problems on reinforced concrete floor structure related to its design are said to have almost ceased to arise, presumably in consequence to a large extent of the latest Code revision.

Here we attempt to introduce an admissible floor deflection, based on the largest of the ratios of long-time predictions to span length, hereafter called long-time deflection ratios.

The rightmost figure column of Table 3 shows the reciprocals of the relevant long-time deflection ratios and Fig. 5 shows two curves of cumulative frequency distribution of these values each for either set of slabs for office or residence floors. Correspondingly noted is a minimum of the



(a) Office　　(b) Residence

Fig. 2 Frequency Distributions of Long-Time Deflection Multiplier for Slabs of Type-A1.

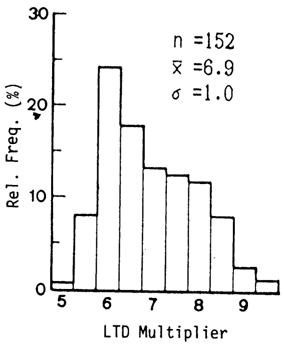
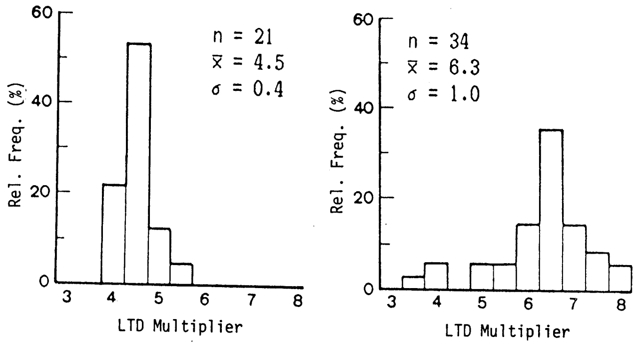


Fig. 3 Frequency Distribution of Long-Time Deflection Multipliers for Slabs of Type-A2.



(a) Slabs of B1-Type　　(b) Slabs of B2-Type

Fig. 4 Frequency Distribution of Long-Time Deflection Multipliers for Slabs of Type-B.

cited reciprocal as large as ca. 360 for Ly/Lx=1.6, with the values less than 400 remaining only within 10 percent of the total entries even for residence floors and the rest mostly ranging from ca. 420 to 560 for both floor types.

Thus, we may be allowed to suggest an admissible value of floor slab deflection limit in practical design of 1/350 of short-span length, based on the judgement that 1/360 cannot be exceeded in most cases of the AIJ Code floor design. It may be safe to say that there will be no significant change in order of those values even if the whole analysis are again tried including partial more elaborate calculation and allowing for reliable extents of certain sectional dimensions relatively prone to field deviation in the execution process.

In the subsequent consideration we are to use these crieria coupled with another required measure of absolute deflection limit of 20mm on a totaled deflections of both slabs and frame elements joining them, which value we adopt regarding it as most frequently used of the available code deflection limits; and this total will be called, more practically, bay maximum deflection.

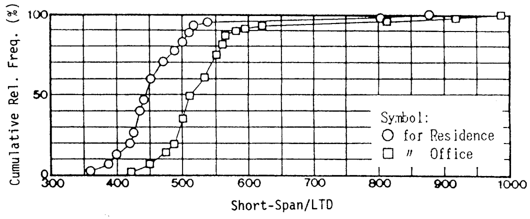


Fig. 5 Short-Span/Long-Time Deflection Ratios for Type-A1 Slabs of Code Thickness.

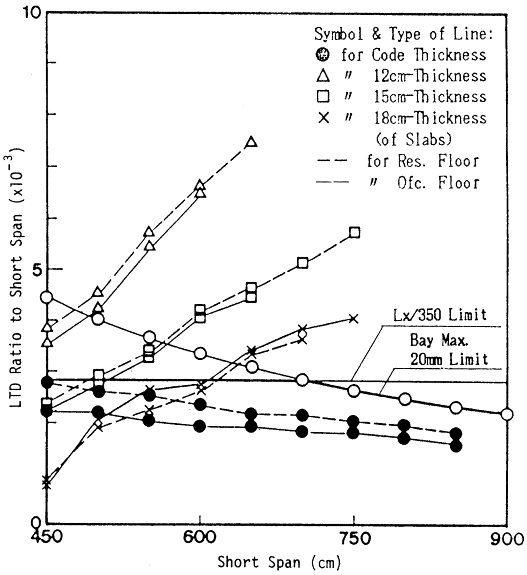


Fig. 6 Long-Time Deflection Ratios to Their Short-Spans for Type-A1 Slabs when Ly/Lx=1.6.

5. Floor Slabs Capable of Design without Deflection Check

The smallest of the slab thicknesses allowed to be designed without deflection check may easily been obtained as the result of our plotting long-time deflections of types-A slabs or rectangular slabs with all edge rigidly fixed or restrained with girders, as respectively in Fig. 6 or 7, and then drawing into them serviceability deflection limits introduced in the preceding section.

Likewise, Fig. 8 refers to introduced types-B interior slabs with beams; provided that Fig. 6

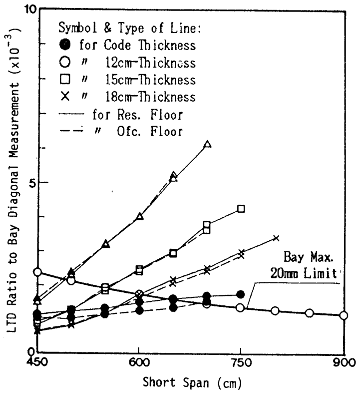
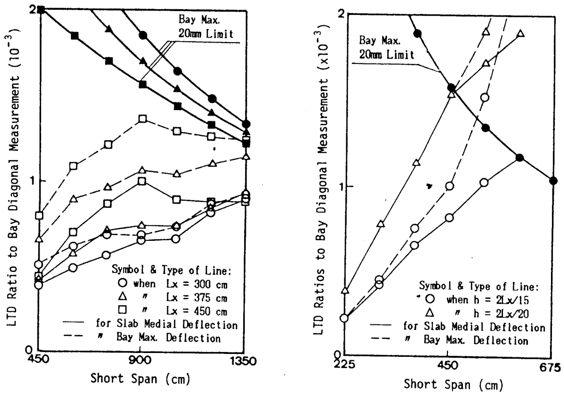


Fig. 7 Long-Time Deflections to Bay Diagonal Length for Slabs of Type A1 when Ly/Lx=1.6.



(a) Type-B1 Slabs　　(b) Type-B2 Slabs

Fig. 8 Long-Time Deflection Ratios to Bay Diagonal Length for Slabs of Type-B.

treats long-time deflection ratios as ratios of deflections to short-span length, in which case a line and a curve respectively mark the coupled limits of 1/350 and 20mm; while in Figs. 7 and 8 only the latter limit is drawn, when defined inclusive of girder deflection, and relevant deflection ratios are taken in proportion to diagonal measurements across bays of a slab-girder floor or a slab-beam-girder floor.

Further, plotted intersections of the curves of long-time deflection ratio and the serviceability limiting curves defined now give limiting curves in Fig. 9, showing the limiting aspect ratios and overall dimensions, with which a slab is designed without care of deflection in our currently defined context. Namely, Fig. 9(a) and 9(b) respectively provide the thickness of designed slabs within the admissible long-time deflection ratio as well as slab thicknesses satisfying the absolute limit on bay maximum deflections.

Connectedly, long-time deflections of slabs each designed with the corresponding AIJ Code thickness are kept within the associated admissible deflection ratios but are not always below the above absolute 20mm limit on bay maxima.

On the contrary, as for types-B of structures, this 20mm limit will not be exceeded if corresponding span/beam depth ratios are less than 15 in either case of the slab with one mid-bay tranverse beam or two crossing. In this latter case, a slab with crossed beams may be designed with span/beam depth ratios assumed up to 20 without deflection check if the beam span or corresponding column-to-column distance is less than 9m.

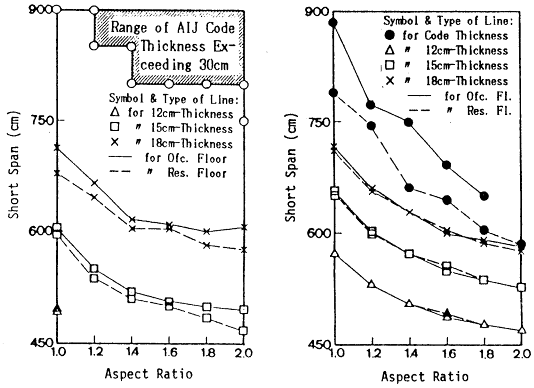
6. Summary

The foregoing observations may be summarized as follows:

(1) long-time deflection multipliers as ratios of long-time deflections to their elastic portions fall in a wide range of 6 to 12;

(2) since being, if implicitly, based on its cited ratio of 16 in the same context, the AIJ's provisions for slab thickness are conservative enough and

(3) there is scarcely any possibility that long-time deflections of slabs of the AIJ Code design under working loads may exceed 1/360 of its span length;



(a) Case of Type-A1 Slabs (b) Case of Type-A2 Slabs

Fig. 9 Limitation on Dimensions vs. Aspect Ratios of Slabs Designed without Deflection Check.

while, in view of floor slabs of the latest revised AIJ Code design being reported to have experienced almost no deflection damage:

(4) we assume an admissible deflection ratio of 1/350, construing it as most usually acceptable;

(5) with this limit observing we can design slabs with even less thickness than those provided for in the AIJ Code;

(6) long-time deflection check is needless if a slab thickness is more than that required in the Code;

(7) in this case, however, there is the other possibility that the above 20mm limit is exceeded; notably, this can be true for slabs with significantly greater panel sizes with correspondingly large thicknesses required in the Code, due to excessive increases in self-weight of the floor system; and

(8) for interior slabs with beams, long-time deflection check is dispensable if beam depths are larger than 1/15 of spans.

While this report is exclusively concerned with interior slabs with their combined action with that of girders taken into account, in an immediate further writing we will examine either exterior slabs or one-span cases of floor systems.

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[1] Architectural Institute of Japan: Standard Code with Commentary for the Design of Reinforced Concrete Building Constructions, 5th Ed., Maruzen, 1988.

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