LETTER

Dominating Sets in Two-Directional Orthogonal Ray Graphs*

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SUMMARY A 2-directional orthogonal ray graph is an intersection graph of rightward rays (half-lines) and downward rays in the plane. We show a dynamic programming algorithm that solves the weighted dominating set problem in $O(n^3)$ time for 2-directional orthogonal ray graphs, where n is the number of vertices of a graph.

key words: *Boolean-width, dominating set, dynamic programming, two-directional orthogonal ray graphs*

1. Introduction

A bipartite graph G with bipartition (U, V) is called an *orthogonal ray graph* [10] if there exist a set of disjoint horizontal rays (closed half-lines) R_u, $u \in U$, in the xy-plane and a set of disjoint vertical rays R_v, $v \in V$, such that for any $u \in U$ and $v \in V$, $(u, v) \in E(G)$ if and only if R_u and R_v intersect. The set $\mathcal{R}(G) = \{R_w \mid w \in V(G)\}$ is called an *orthogonal ray representation* of G. Orthogonal ray graphs have been introduced in connection with the defect-tolerant design of nano-circuits [9]. An orthogonal ray graph is called a *2-directional orthogonal ray graph* (*2-DORG* for short) if every horizontal ray R_u, $u \in U$, has the same direction, and every vertical ray R_v, $v \in V$, has the same direction.

For 2-DORGs, various characterizations with an $O(n^2)$-time recognition algorithm are known [10], where n is the number of vertices of a graph. Also, some problems are known to be solvable or approximable in polynomial time for 2-DORGs [5], [7], [8], [11]–[14]. We recently showed in [13] that the weighted dominating set problem can be solved in $O(n^4 \log n)$ time for 2-DORGs by using a new parameter, boolean-width of graphs. Boolean-width of graphs is introduced in [2], [3], and several problems can be solved in polynomial time by dynamic programming algorithms if the graphs has boolean-width $O(\log n)$. In this paper by using dynamic programming techniques directly, we show an $O(n^3)$-time algorithm that solves the weighted dominating set problem for 2-DORGs.

We note that by using boolean-width of graphs, some other kinds of graph problems, such as the independent set problem, can be solved in polynomial time for several classes of graphs. See [1]–[4] for details. We expect that the complexity of the problems can be reduced by using direct dynamic programming approaches, as shown in this paper.

It should also be noted that the complexity of the weighted dominating set problem for orthogonal ray graphs still remains open, whereas the problem can be solved in polynomial time provided that orthogonal ray representations of graphs [13], [15].

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2. Problem

All graphs considered in this paper are finite, simple, and undirected. For a graph G, let $V(G)$ and $E(G)$ denote the set of vertices and edges, respectively, and let $n = |V(G)|$. The *open neighborhood* of a vertex v of G is the set $N_G(v) = \{u \in V(G) \mid (u, v) \in E(G)\}$, and the *closed neighborhood* of v is the set $N_G[v] = \{v\} \cup N_G(v)$. The closed neighborhood of a vertex set $S \subseteq V(G)$ is $N_G[S] = \bigcup_{v \in S} N_G[v]$. If no confusion arises, we will omit the index G.

A vertex v of a graph G is said to *dominate* all vertices of $N[v]$. A vertex set $D \subseteq V(G)$ is said to *dominate* $v \in V(G)$ if D has at least one vertex dominating v. A vertex set $D \subseteq V(G)$ is called a *dominating set* of G if every vertex of G is dominated by D. The *weighted dominating set problem* is to find a dominating set with minimum weight in a given vertex-weighted graph. Previous works of the problem for graphs related to orthogonal ray graphs can be found in [13].

Let $c : V(G) \to \mathbb{R}$ be a weight (or cost) function of a graph G, where $\mathbb{R}$ is a set of real numbers, and let $c(v)$ denotes the *weight* of a vertex v of G. For a vertex set $D \subseteq V(G)$, let $c(D) = \sum_{v \in D} c(v)$ be the weight of D. It is shown in [6] that any algorithm that finds a minimum-weight dominating set for graphs with non-negative weights can be extended without loss of efficiency to the algorithm for graphs with negative weights. Hence, in the rest of this paper, we assume that $c(v)$ is non-negative for every $v \in V(G)$.

3. Algorithm

If S is a vertex set of a graph G and v is a vertex of G, we use for convenience $S + v$ and $S - v$ instead of $S \cup \{v\}$ and $S \setminus \{v\}$, respectively. For a family of vertex sets $\{S_1, S_2, \ldots, S_k\}$ of G, we use $\min\{S_1, S_2, \ldots, S_k\}$ to denote a set S_i with minimum weight (Break ties arbitrarily).

Let G be a 2-DORG with bipartition (U, V) and an orthogonal ray representation $\mathcal{R}(G) = \{R_w \mid w \in V(G)\}$. We assume without loss of generality that every R_u, $u \in U$ is a rightward ray and every R_v, $v \in V$ is a downward ray. It is shown in [10] that such an orthogonal ray representation of a 2-DORG can be obtained in $O(n^2)$ time. Let (x_w, y_w) be the endpoint of R_w, $w \in V(G)$. We refer to x_w and y_w as the x- and y-coordinate of w, respectively. Since the graphs are finite, we can see that the endpoints can be perturbed slightly so that the x-coordinates are distinct and the y-coordinates are distinct [12]. Notice that for any $u \in U$ and $v \in V$, $(u, v) \in E(G)$ if and only if $x_u < x_v$ and $y_u < y_v$.

Let G be a 2-DORG with bipartition (U, V) and non-negative weight function c, and let $(w_1, w_2, \ldots, w_n)$ be the total ordering of $V(G)$ such that for any w_i and w_j, $i < j$ if and only if $x_{w_i} < x_{w_j}$. For convenience of algorithm description, we add two isolated dummy vertices w_0 and w_{n+1} with weight 0. Let w_0 be a vertex of U, and we denote it by u_d. Let w_{n+1} be a vertex of V, and we denote it by v_d. We define for any $i \in \{0, 1, \ldots, n\}$ that

$$\begin{aligned} W_i &= \{w_j \in V(G) \cup \{u_d, v_d\} \mid j \leq i\}, \\ U_i &= W_i \cap U, \text{ and} \\ V_i &= \overline{W_i} \cap V, \end{aligned}$$

where $\overline{W_i} = (V(G) \cup \{u_d, v_d\}) \setminus W_i$. Notice that $u_d \in U_i$ and $v_d \in V_i$ for any $i \in \{0, 1, \ldots, n\}$.

For a vertex set $S \subseteq W_i$, let $u_S \in S \cap U$ be the vertex with minimum y-coordinate among $S \cap U$. Any vertex $v \in V_i$ adjacent to a vertex $u \in S \cap U$ is also adjacent to u_S, since $x_{u_S} < x_v$ and $y_{u_S} < y_u < y_v$. Since $N[S] \cap \overline{W_i} \subseteq V_i$, any vertex of $\overline{W_i}$ dominated by S must be dominated by u_S, that is, $N[S] \cap \overline{W_i} = N[u_S] \cap \overline{W_i}$. With this observation, we show in [13] that the weighted dominating set problem can be solved in $O(n^4 \log n)$ time for 2-DORGs. We also use it in this paper, and we refer to u_S as the *representative* of S.

A pair (S, v) of a vertex set $S \subseteq W_i$ and a vertex $v \in V_i$ is said to *dominate* W_i if all vertices of W_i are dominated by S or v, that is, $W_i \subseteq N[S + v]$. In the algorithm, we use the two-dimensional table D_i for each $i \in \{0, 1, \ldots, n\}$ that has index set $U_i \times V_i$. The contents of $D_i[u][v]$ for every $u \in U_i$ and $v \in V_i$ are defined as follows:

$$\begin{aligned} \mathcal{S}_i[u][v] &= \left\{ S \subseteq W_i \;\middle|\; \begin{array}{l} u \text{ is the representative of } S \\ \text{and } (S, v) \text{ dominates } W_i, \end{array} \right\}; \\ D_i[u][v] &= \min\{\mathcal{S}_i[u][v]\}. \end{aligned}$$

In other words, $D_i[u][v]$ stores one of the minimum-weight subsets S of W_i such that the representative of S is u and (S, v) dominates W_i. Notice that since u_d is isolated, $u_d \in D_i[u][v]$ for any $i \in \{0, 1, \ldots, n\}$, $u \in U_i$, and $v \in V_i$. We can see that $D_n[u][v_d] - u_d$ with minimum weight over all $u \in U_n$ is the minimum-weight dominating set of an input 2-DORG. We compute the contents of table D_{i+1} from table D_i by the following relationships, which are proved in the next section. In the rest of this paper, we use ∞ to denote a vertex set of sufficiently large weight so that $D_i[u][v] = \infty$ means that there is no such vertex set in the graph. We assume that a sum containing a ∞ equals ∞.

Algorithm 1: An $O(n^3)$-time algorithm to find a minimum-weight dominating set in 2-DORGs

Input: An orthogonal ray representation $\mathcal{R}(G)$ of a 2-DORG G.
Output: A minimum-weight dominating set of G.
Add two isolated dummy vertices u_d and v_d;
$D_0[u_d][v] \leftarrow u_d$ for all $v \in V$;
$D_i[u][v] \leftarrow \infty$ for all $i \in \{1, 2, \ldots, n\}$, $u \in U_i$, and $v \in V_i$;
for $i \leftarrow 0$ **to** $n - 1$ **do**
  foreach $u \in U_i$ *and* $v \in V_{i+1}$ **do**
    if $w_{i+1} \in U$ **then**
      $u' \leftarrow w_{i+1}$;
      if $y_{u'} < y_u$ **then**
        $D_{i+1}[u'][v] \leftarrow \min\{D_{i+1}[u'][v],\ D_i[u][v] + u'\}$;
        if $(u', v) \in E(G)$ **then**
          $D_{i+1}[u][v] \leftarrow D_i[u][v]$;
        else
          $D_{i+1}[u][v] \leftarrow \infty$;
      else
        if $(u', v) \in E(G)$ **then**
          $D_{i+1}[u][v] \leftarrow D_i[u][v]$;
        else
          $D_{i+1}[u][v] \leftarrow D_i[u][v] + u'$;
    if $w_{i+1} \in V$ **then**
      $v' \leftarrow w_{i+1}$;
      if $y_{v'} < y_v$ **then**
        if $(u, v') \in E(G)$ **then**
          $D_{i+1}[u][v] \leftarrow D_i[u][v]$;
        else
          $D_{i+1}[u][v] \leftarrow D_i[u][v] + v'$;
      else
        if $(u, v') \in E(G)$ **then**
          $D_{i+1}[u][v] \leftarrow \min\{D_i[u][v], D_i[u][v'] + v'\}$;
        else
          $D_{i+1}[u][v] \leftarrow D_i[u][v'] + v'$;
return $D_n[u][v_d] - u_d$ with minimum weight over all $u \in U_n$;

Lemma 1. *Suppose $w_{i+1} \in U$, and let $u' = w_{i+1}$. Then,*

$$D_{i+1}[u'][v] = \min\{D_i[u][v] + u' \mid u \in U_i \text{ s.t. } y_{u'} < y_u\},$$

and $D_{i+1}[u][v]$ is

$$\begin{array}{ll} D_i[u][v] & \text{if } y_{u'} < y_u \text{ and } (u', v) \in E(G), \\ \infty & \text{if } y_{u'} < y_u \text{ and } (u', v) \notin E(G), \\ D_i[u][v] & \text{if } y_{u'} > y_u \text{ and } (u', v) \in E(G), \text{ and} \\ D_i[u][v] + u' & \text{if } y_{u'} > y_u \text{ and } (u', v) \notin E(G), \end{array}$$

for any $u \in U_i$, $v \in V_{i+1}$, and $i \in \{0, 1, \ldots, n-1\}$. □

Lemma 2. *Suppose $w_{i+1} \in V$, and let $v' = w_{i+1}$. Then, $D_{i+1}[u][v]$ is*

$$\begin{array}{ll} D_i[u][v] & \text{if } y_{v'} < y_v \text{ and } (u, v') \in E(G), \\ D_i[u][v] + v' & \text{if } y_{v'} < y_v \text{ and } (u, v') \notin E(G), \\ \min\{D_i[u][v] & \\ \quad D_i[u][v'] + v'\} & \text{if } y_{v'} > y_v \text{ and } (u, v') \in E(G), \text{ and} \\ D_i[u][v'] + v' & \text{if } y_{v'} > y_v \text{ and } (u, v') \notin E(G), \end{array}$$

for any $u \in U_i$, $v \in V_{i+1}$, and $i \in \{0, 1, \ldots, n-1\}$. □

Lemmas 1 and 2 establish Algorithm 1 shown above by using dynamic programming techniques for computing table D_i in the increasing order of $i \in \{0, 1, \ldots, n-1\}$.

Theorem 3. *Algorithm 1 solves the weighted dominating set problem in $O(n^3)$ time for 2-DORGs.*

Proof. Correctness of the algorithm is shown by Lemmas 1 and 2. Since the algorithm consists of three nested loops and each loop index ($i \in \{0, 1, \ldots, n-1\}$, $u \in U_i$, and $v \in V_{i+1}$) takes at most $n+1$ values, the algorithm runs in $O(n^3)$ time. Since the the orthogonal ray representation of a 2-DORG can be obtained in $O(n^2)$ time [10], we have the theorem. □

4. Proof of Lemmas

4.1 Proof of Lemma 1

We will compute the content of $D_{i+1}[u][v]$ from table D_i. Recall that $w_{i+1} \in U$ and $u' = w_{i+1}$.

We first show how to compute $D_{i+1}[u'][v]$ for every $v \in V_{i+1}$. Notice that $u' \in D_{i+1}[u'][v]$ by definition, and $D_{i+1}[u'][v]$ has no vertex of U_i whose y-coordinate is lower than that of u', for otherwise u' is no longer the representative of the vertex set.

Claim 4. $D_{i+1}[u'][v] - u' = D_i[u][v]$*, where u is the representative of $D_{i+1}[u'][v] - u'$, that is, the vertex in $D_{i+1}[u'][v] \cap U$ with the second-minimum y-coordinate.*

Proof. Since W_i has no vertex dominated by u', $D_{i+1}[u'][v] - u' \in \mathcal{S}_i[u][v]$. There is no vertex set $D \in \mathcal{S}_i[u][v]$ such that $c(D) < c(D_{i+1}[u'][v] - u')$, for otherwise we have $D + u' \in \mathcal{S}_{i+1}[u'][v]$ and $c(D + u') < c(D_{i+1}[u'][v])$, a contradiction. Thus, $D_{i+1}[u'][v] - u'$ is a minimum-weight vertex set in $\mathcal{S}_i[u][v]$. □

From Claim 4, we can compute $D_{i+1}[u'][v]$ as follows.

Lemma 5.

$$D_{i+1}[u'][v] = \min\{D_i[u][v] + u' \mid u \in U_i \text{ s.t. } y_{u'} < y_u\}. \quad \square$$

We next show how to compute $D_{i+1}[u][v]$ for every $u \in U_i$ and $v \in V_{i+1}$. We first show the following.

Claim 6. *If $u' \notin D_{i+1}[u][v]$, then we have $(u', v) \in E(G)$ and* $D_{i+1}[u][v] = D_i[u][v]$.

Proof. Recall that $(D_{i+1}[u][v], v)$ dominates u'. Since $u' \notin D_{i+1}[u][v]$ and W_i has no vertex dominating u', v must dominate u'. Hence, $(u', v) \in E(G)$.

Since $u' \notin D_{i+1}[u][v]$, we have $D_{i+1}[u][v] \in \mathcal{S}_i[u][v]$. There is no vertex set $D \in \mathcal{S}_i[u][v]$ such that $c(D) < c(D_{i+1}[u][v])$, for otherwise $(u', v) \in E(G)$ implies that $D \in \mathcal{S}_{i+1}[u][v]$, contradicting the minimality of $D_{i+1}[u][v]$ in $\mathcal{S}_{i+1}[u][v]$. Thus, $D_{i+1}[u][v]$ is a minimum-weight vertex set in $\mathcal{S}_i[u][v]$. □

We distinguish two cases, each of which corresponds to Lemma 7 and 8, respectively.

Lemma 7. *Let $u \in U_i$ be a vertex with $y_{u'} < y_u$. Then,*

$$D_{i+1}[u][v] = \begin{cases} D_i[u][v] & \textit{if } (u', v) \in E(G), \\ \infty & \textit{otherwise.} \end{cases}$$

Proof. We have that $u' \notin D_{i+1}[u][v]$, for otherwise $y_{u'} < y_u$ implies that the representative of the vertex set is u', a contradiction. The lemma is derived from Claim 6. □

Lemma 8. *Let $u \in U_i$ be a vertex with $y_{u'} > y_u$. Then,*

$$D_{i+1}[u][v] = \begin{cases} D_i[u][v] & \textit{if } (u', v) \in E(G), \\ D_i[u][v] + u' & \textit{otherwise.} \end{cases}$$

Proof. We first show that

$$\text{if } u' \in D_{i+1}[u][v], \text{ then } D_{i+1}[u][v] - u' = D_i[u][v]. \quad (1)$$

Notice that the representative of $D_i[u][v] + u'$ is still u, since $y_{u'} > y_u$. Since W_i has no vertex dominated by u', $D_{i+1}[u][v] - u' \in \mathcal{S}_i[u][v]$. There is no vertex set $D \in \mathcal{S}_i[u][v]$ such that $c(D) < c(D_{i+1}[u][v] - u')$, for otherwise we have $D + u' \in \mathcal{S}_{i+1}[u][v]$ and $c(D + u') < c(D_{i+1}[u][v])$, a contradiction. Thus, $D_{i+1}[u][v] - u'$ is a minimum-weight vertex set in $\mathcal{S}_i[u][v]$.

Now, we have from Claim 6 and (1) that $D_{i+1}[u][v] = \min\{D_i[u][v], D_i[u][v] + u'\}$ if $(u', v) \in E(G)$, and $D_{i+1}[u][v] = D_i[u][v] + u'$ otherwise. Since we assume that the weights of vertices are non-negative, we have the lemma. □

Lemmas 5, 7, and 8 prove Lemma 1.

4.2 Proof of Lemma 2

We show how to compute $D_{i+1}[u][v]$ for every $u \in U_i$ and $v \in V_{i+1}$ from table D_i. Recall that $w_{i+1} \in V$ and $v' = w_{i+1}$. Notice that v' does not appear in the index of D_{i+1}, since $v' \notin V_{i+1}$. We first show the following.

Claim 9. *If $v' \notin D_{i+1}[u][v]$, then we have $(u, v') \in E(G)$ and* $D_{i+1}[u][v] = D_i[u][v]$.

Proof. Since u is the representative of $D_{i+1}[u][v]$ and $v' \notin D_{i+1}[u][v]$, u must dominate v'. Hence, $(u, v') \in E(G)$.

Since $v' \notin D_{i+1}[u][v]$, we have $D_{i+1}[u][v] \in \mathcal{S}_i[u][v]$. There is no vertex set $D \in \mathcal{S}_i[u][v]$ such that $c(D) < c(D_{i+1}[u][v])$, for otherwise $(u, v') \in E(G)$ implies that $D \in \mathcal{S}_{i+1}[u][v]$, contradicting the minimality of $D_{i+1}[u][v]$ in $\mathcal{S}_{i+1}[u][v]$. Thus, $D_{i+1}[u][v]$ is a minimum-weight vertex set in $\mathcal{S}_i[u][v]$. □

We distinguish two cases, each of which corresponds to Lemma 10 and 11, respectively.

Lemma 10. *Let $v \in V_{i+1}$ be a vertex with $y_{v'} < y_v$. Then,*

$$D_{i+1}[u][v] = \begin{cases} D_i[u][v] & \textit{if } (u, v') \in E(G), \\ D_i[u][v] + v' & \textit{otherwise.} \end{cases}$$

Proof. We first show that

$$\text{if } v' \in D_{i+1}[u][v], \text{ then } D_{i+1}[u][v] - v' = D_i[u][v]. \quad (2)$$

Since $y_{v'} < y_v$, we have $N(v') \cap W_i \subseteq N(v) \cap W_i$. It follows that $(D_{i+1}[u][v] - v', v)$ dominates W_i, and hence, $D_{i+1}[u][v] - v' \in \mathcal{S}_i[u][v]$. There is no vertex set $D \in \mathcal{S}_i[u][v]$ such that $c(D) < c(D_{i+1}[u][v] - v')$, for otherwise we have $D + v \in \mathcal{S}_{i+1}[u][v]$ and $c(D + v') < c(D_{i+1}[u][v])$, a contradiction. Thus, $D_{i+1}[u][v] - v'$ is a minimum-weight vertex set in $\mathcal{S}_i[u][v]$.

Now, we have from Claim 9 and (2) that $D_{i+1}[u][v] = \min\{D_i[u][v], D_i[u][v] + v'\}$ if $(u, v') \in E(G)$, and $D_{i+1}[u][v] = D_i[u][v] + v'$ otherwise. Since we assume that the weights of vertices are non-negative, we have the lemma. □

Lemma 11. *Let $v \in V_{i+1}$ be a vertex with $y_{v'} < y_v$. Then,*

$$D_{i+1}[u][v] = \begin{cases} \min\{D_i[u][v], \\ \qquad D_i[u][v'] + v'\} & \text{if } (u, v') \in E(G), \\ D_i[u][v'] + v' & \text{otherwise.} \end{cases}$$

Proof. We first show that

$$\text{if } v' \in D_{i+1}[u][v], \text{ then } D_{i+1}[u][v] - v' = D_i[u][v']. \quad (3)$$

Since $y_{v'} > y_v$, we have $N(v') \cap W_i \supseteq N(v) \cap W_i$. It follows that $(D_{i+1}[u][v] - v', v')$ dominates W_i, and hence, $D_{i+1}[u][v] - v' \in \mathcal{S}_i[u][v']$. There is no vertex set $D \in \mathcal{S}_i[u][v']$ such that $c(D) < c(D_{i+1}[u][v] - v')$, for otherwise we have $D + v' \in \mathcal{S}_{i+1}[u][v]$ and $c(D + v') < c(D_{i+1}[u][v])$, a contradiction. Thus, $D_{i+1}[u][v] - v'$ is a minimum-weight vertex set in $\mathcal{S}_i[u][v']$.

Now, we have from Claim 9 and (3) that $D_{i+1}[u][v] = \min\{D_i[u][v], D_i[u][v'] + v'\}$ if $(u, v') \in E(G)$, and $D_{i+1}[u][v] = D_i[u][v'] + v'$ otherwise. □

Lemmas 10 and 11 prove Lemma 2.

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