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A Study on the Reduction of Computing Time of the Monte Carlo Method Applied to the Radiative Heat Transfer *

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A modified Monte Carlo method is suggested to reduce the computing time and improve the convergent stability of the iteration process. This method succeeds the advantages of the usual Monte Carlo method, that is, adaptability to the complex geometry of the heat transfer system and to the variable property problem. In this method, the number of the radiative bundles emitted from the control elements is proportional to the difference of the emissive energy between two successive iterative turns and the other kind of the radiative bundle is defined and used to correct the variation of the radiative property between two iterative turns. Analytical examples show that this method is able to reduce the computing time remarkably and to improve the convergent stability sufficiently.

Key Words : Thermal Radiation, Monte Carlo Method, Variable Property, Computing Time, Convergent Stability, Iteration

1. Introduction

In the industrial fields, there are many kinds of heat transfer systems where it is not possible to explain the characteristics without considering the radiative heat transfer. The energy equation regarding the problems including the radiative heat transfer is an integral equation or an integrodifferential equation which is solved by some methods of numerical analysis in general. There is the Monte Carlo method as one of the numerical methods. The Monte Carlo method that is a probabilistic method has the excellent feature of easy applications to such a case that thermal radiative properties are not uniform and also the geometries of the systems are complex. Also, in this method, the mathematical treatment of numerical analysis is easy. However, the Monte Carlo method has a defect that it requires long computing time when the radiative heat transfer coexists with the other kinds of heat transfer mechanisms or when the energy equation is non-linear on account of the dependency of thermal properties on temperature and so on. To resolve this defect, the author already suggested a method called the Differential Emissive Power Emission method (abbreviated as DPE method[4]) in which the number of radiative bundles emitted from a control element was taken proportional to the difference of emissive power between two successive iterative turns. However, this method had a restriction in an applicability to a variable property problem.

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In this paper, the author modifies this DPE method so that it can be applied to the variable property problem without losing the excellent feature of the DPE method and shows an application example to examine the characteristics of the new modified DPE method called Differential Emissive Power Emission Method Applied to Variable Property Problems (abbreviated as DPEV method).

Nomenclature

- A : total area of wall
- ΔA : area of wall element
- c_p : specific heat at constant pressure
- C_{fs} : coefficient that gives the energy of a bundle
- E : emissive power $=\sigma T^4$
- ΔE : difference of emissive power between successive iterative turns [defined by Eqs.(22),(23)]
- k : incident or reflected time of a bundle
- l : distance
- L : traced distance of a bundle
- N_t : total number of bundle
- q_{in} : heat generating rate
- q_R : heat flux of radiation
- S : radiative energy of a energy correcting bundle
- s : radiative energy of a property correcting bundle
- t : time
- T : temperature
- V : total volume of medium
- ΔV : volume of control element
- y : coordinate
- y_0 : distance between walls
- ΔZ : changing rate of absorbable probability [Eqs.(24),(25)]
- ε : emissivity of wall
- κ : absorption coefficient
- λ : thermal conductivity

ρ : density
σ : Stefan-Boltzman constant
ϕ, ϕ' : incident and refraction angles of radiative ray at wall
Φ : heat generating rate by friction

Suffixes
i : iterative turn g : medium
0 : control element w : wall

2. Calculating Procedure

2.1 Basic equations   Consider the heat transfer field composed of radiation, convection and the other kinds of heat transfer mechanisms. For the sake of facilitaring the discussion, consider a coordinate system shown in Fig.1 where the flow field is already known. Then the energy equations regarding the medium and the wall are expressed as follows:

$$c_p\rho\frac{DT}{Dt}=\lambda\nabla^2T+\Phi-\mathrm{div}\,q_{Rg} \cdots\cdots (1)$$

$$q_w=-\lambda\nabla T-q_{Rw} \cdots\cdots (2)$$

where, D/Dt is the material differential operator of the fluid dynamics, ∇^2 , ∇ are the Laplacian and gradient operators, respectively. Where $-\mathrm{div}\,q_{Rg}$ and $-q_{Rw}$ in Eqs.(1) and (2) are expressed as follows:

$$-\mathrm{div}\,q_{Rg}=-4\kappa_0E_{g0}+\kappa_0\left[\int_V\kappa E_gP_gdV+\int_A\varepsilon E_wP_gdA\right] \cdots\cdots (3)$$

$$-q_{Rw}=-\varepsilon_0E_{w0}+\varepsilon_0\left[\int_V\kappa E_gP_wdV+\int\varepsilon E_wP_wdA\right] \cdots\cdots (4)$$

$$P_g=\frac{e^{-\int_0^l\kappa dl}}{\pi l^2}+\sum_{k=1}^{\infty}\left[\Pi_k^*(1-\varepsilon_k)\times\cos\phi_k\frac{e^{-\int_0^l\kappa dl}}{\pi l^2}\right] \cdots\cdots (5)$$

$$P_w=\frac{e^{-\int_0^l\kappa dl}}{\pi l^2}\cos\phi_1'+\sum_{k=2}^{\infty}{}^*\Pi_k^* \cdots\cdots (6)$$

$$\Pi_k^*=\frac{e^{-\int_0^l\kappa dl}}{\pi l_0^2}\cos\phi_1'\left[\prod_{j=1}^{k-1}(1-\varepsilon_j)\times\cos\phi_j\cos\phi_{j+1}'\frac{e^{-\int_0^{l_j}\kappa dl}}{\pi l_j^2}\right] \cdots\cdots (7)$$

where, the symbol Σ^* means that the summation is taken only for k's which correspond to the heat receiving wall considered.

Here, Eqs.(3) and (4) are constituted respectively assuming that the medium and the wall are gray and that the wall is isotropic and diffusive. These assumptions are not restrictive ones for the present method, but this DPEV method has the same flexibilities as the conventional Monte Carlo method.

The following quantities are introduced to facilitate the numerical treatments.

$$B_{g0}=\lambda\nabla^2T+\Phi-c_p\rho\frac{DT}{Dt} \cdots\cdots (8)$$

$$B_{w0}=-\lambda\nabla T-q_w \cdots\cdots (9)$$

Eqs.(1) and (2) are rewritten as follows by substituting of Eqs.(8) and (9).

$$4\kappa_0E_{g0}=\int_V\kappa E_gP_gdV+\int_A\varepsilon E_wP_gdA+B_{g0} \cdots\cdots (10)$$

$$\varepsilon_0E_{w0}=\int_V\kappa E_gP_wdV+\int_A\varepsilon E_wP_wdA+B_{w0} \cdots\cdots (11)$$

2.2 Discussions about the Monte Carlo method

Eqs.(1) and (2) are linear with respect to the emissive power E if the radiative heat transfer dominates the characteristics of the heat transfer system and the radiative properties are assumed constant, that is, the properties do not depend on the other variable. Then, the solutions of the energy equations can be obtained by the conventional Monte Carlo method, used by Howell[2] and Taniguchi[3], where the radiative bundle is traced until it is absorbed by the wall through the successive repetitions of emission-absorption-reemission cycle and the radiative heat of a bundle is calculated from the heat quantity of the heat source. This method does not require any iterative calculation.

However, when the conditions mentioned above are not satisfied, the energy equations become non-linear and the numerical analysis has to be performed by an iterative procedure. Thus the author suggested a Standard Method Applied to Non-linear Problems (abbreviated as N method[4]) evolved from the conventional Monte Carlo method to facilitate application. In this method, the number of the radiative bundles emitted from each control element is proportional to the emissive energy of its own and the individual radiative bundle is traced from the emission point to the absorption point with probabilistic means, which gives easy understanding of the phenomena of radiative heat transfer.

In general, the Monte Carli method is not suitable to the numerical calculation with iteration, because this method needs long computing time to get the approximate solution to probability. Therefore, the author modified it and obtained a method named the DPE method. In this method, the number of the radiative bundles is proportional to the difference of emissive power between two successive iterative turns, the computing time is much shortened and the convergent stability is improved remarkably. However, this method cannot be applied to the iterative calculation of

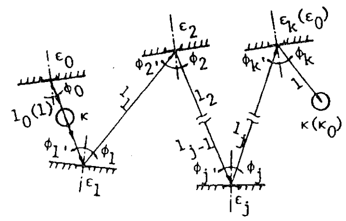

Fig.1 Co-ordinate

the radiative properties which vary at each iterative turn.

In this paper, the author suggests a modified DPE method called the DPEV method which is applicable to variable properties problems and reduces the computing time by the same degree as the DPE method.

2.3 A method suggested in this paper

As the N method is used at the first iterative turn or at first several turns in the execution of the DPEV method suggested in this paper, the outline of the N method is given below.

2.3.1 N method In general, a solution at the i-th iterative turn is calculated from the following equations which are rewritten as Eqs.(10) and (11) in the form of iterative calculation when the 0-th value of temperature T^0 is given, the values of properties and Φ^i's are calculated with T^i's the values of dependent variables or the values of B^i's defined by Eqs.(8) and (9) are calculated by some numerical method, for example the finite difference method.

$$4\kappa_0^{i-1}E_{g0}^i=\int_V \kappa^{i-1}E_g^{i-1}P_g^i dV+\int_A \varepsilon^{i-1}E_w^{i-1}P_g^i dA + B_{g0}^{i-1} \quad \cdots\cdots (12)$$

$$\varepsilon_0^{i-1}E_{w0}^i=\int_V \kappa^{i-1}E_g^{i-1}P_w^i dV+\int_A \varepsilon^{i-1}E_w^{i-1}P_w^i dA + B_{w0}^{i-1} \quad \cdots\cdots (13)$$

where

$$B_{g0}^{i-1}=\lambda^{i-1}\nabla^2 T^{i-1}+\Phi^{i-1}-c_p\rho\frac{DT^{i-1}}{Dt} \quad \cdots (14)$$

$$B_{w0}^{i-1}=-\lambda^{i-1}\nabla T^{i-1}-q_w^{i-1} \quad \cdots\cdots (15)$$

$$i\geqq 1 \quad \cdots\cdots (16)$$

and, P^i's are the transfer probability of the radiative heat calculated by the Monte Carlo method through Eqs.(5) and (6). In the DPEV method, the radiative heat onto the wall is absorbed in proportion to the emissivity of the wall and this treatment of the absorption of radiative energy differs from those by Howell[2] and Taniguchi[3]. This special treatment is introduced to reduce the use of random number and the quantity of radiative energy carried by a radiative bundle. The radiative energy of a bundle emitted S^i is calculated by

$$S^i=\left[\sum_{\mathrm{all}\,\Delta V}4\kappa^{i-1}E_g^{i-1}\Delta V+\sum_{\mathrm{all}\,\Delta A}\varepsilon^{i-1}E_w^{i-1}\Delta A\right]\Big/N_t \quad \cdots\cdots (17)$$

where, N_t is the total number of the radiative bundles, and ΔV and ΔA are the volume of the control volume and the area of the control area, respectively. From this equation, the number of bundles emitted from a control element considered N^i is calculated by the following equations.

$$N_g^i=4\kappa^{i-1}E_g^{i-1}\Delta V/S^i \quad \cdots\cdots (18)$$

$$N_w^i=\varepsilon^{i-1}E_w^{i-1}\Delta A/S^i \quad \cdots\cdots (19)$$

The radiative energy transported to the medium or the wall S^{*i} is calculated by the following equations.

$$S_g^{*i}=S^i\left[\prod_{j=1}^{k}(1-\varepsilon_j^{i-1})\right] \quad \cdots\cdots (20)$$

$$S_w^{*i}=\sum_{k'=1}^{k}{}^{*}\left\{S^i\left[\prod_{j=1}^{k'}(1-\varepsilon_j^{i-1})\right]\right\} \quad \cdots\cdots (21)$$

2.3.2 DPEV method The iterative calculation is performed bv the DPEV method except that the first several turns at which the calculation is performed by the N method.

a. Calculation procedure Define ΔE^i and ΔZ^i's by

$$\Delta E_g^i=\kappa^{i-1}E_g^{i-1}-\kappa^{i-2}E_g^{i-2} \quad \cdots\cdots (22)$$

$$\Delta E_w^i=\varepsilon^{i-1}E_w^{i-1}-\varepsilon^{i-2}E_w^{i-2} \quad \cdots\cdots (23)$$

$$\Delta Z_g^i=1-P_g^{i-1}/P_g^i \quad \cdots\cdots (24)$$

$$\Delta Z_w^i=1-P_w^{i-1}/P_w^i \quad \cdots\cdots (25).$$

The radiative energy transfered C_0^i's are calculated by the following equations using ΔE^i, ΔZ^i and P^i's.

$$C_{g0}^i=C_{g0}^{i-1}+\int_V[\Delta E_g^i+\kappa^{i-2}E_g^{i-2}\Delta Z_g^i]P_g^i dV+\int_A[\Delta E_w^i+\varepsilon^{i-2}E_w^{i-2}\Delta Z_g^i]P_g^i dA \quad \cdots\cdots (26)$$

$$C_{w0}^i=C_{w0}^{i-1}+\int_V[\Delta E_g^i+\kappa^{i-2}E_g^{i-2}\Delta Z_w^i]P_w^i dV+\int_A[\Delta E_w^i+\varepsilon^{i-2}E_w^{i-2}\Delta Z_w^i]P_w^i dA \quad \cdots\cdots (27)$$

$$i\geqq 2 \quad \cdots\cdots (28)$$

Eqs.(10) and (11) are rewritten into the following equation by means of Eqs.(26) and (27).

$$4\kappa_0^{i-1}E_{g0}^i=C_{g0}^i+B_{g0}^{i-1} \quad \cdots\cdots (29)$$

$$\varepsilon_0^{i-1}E_{w0}^i=C_{w0}^i+B_{w0}^{i-1} \quad \cdots\cdots (30)$$

That is to say, the values of the first term in the right-hand side of Eqs.(26) and (27) are calculated just as in the N method except that the numbers of radiative bundles are proportional to ΔE^i given by Eqs.(22) and (23) ; this kind of bundles is called the energy correcting bundle because the quantities of emissive power emitted at each iterative turn are corrected with these bundles. In the calculation of the second term, the radiative bundles and the quantities involved in ΔZ^i are used; this kind of bundles is called the property correcting bundle because the values of the radiative property used at each iterative turn are corrected with these bundles. The temperature or the emissive power of the control element at the i-th iterative turn is calculated by Eqs.(29) and (30) just as in the N method, after the calculations are performed over the region considered. Furthermore, in the case that $P^i=P^{i-1}$, that is, if the radiative properties are kept constant at each iterative turn, the relation $\Delta Z^i=0$ is satisfied and the DPEV method is reduced to the DPE method.

b. Some quantities used in the calculation First of all, define the radiative energy of the correcting energy bundle S^i by the following equaton.

$$S^i=S^{i-1}/C_{fs}^i,\ \ C_{fs}^i\geqq 1 \quad \cdots\cdots (31)$$

where, $C_{f_S}^i$ is a variable that controls the quantity of S^i. The number of radiative bundles N^i is assigned to each element through the following equation which is deduced from Eqs.(31),(22) and (23).

$$N_g^i=|\Delta E_g^i|\Delta V/S^i \quad \cdots\cdots (32)$$

$$N_w^i=|\Delta E_w^i|\Delta A/S^i \quad \cdots\cdots (33)$$

where, S^i takes the following values according to the sign of ΔE^i.

$$\left.\begin{array}{l}\Delta E^i\geqq 0:\ S^i=|S^i|\\ \Delta E^i<0:\ S^i=-|S^i|\end{array}\right\} \quad \cdots\cdots (34)$$

The radiative heat transfered by a radiative bundle S^{*i} is calculated from Eq.(20) or (21).

Next, consider the correcting property bundle. In the Monte Carlo method, the most time consuming parts are those of the trace of the radiative bundles and of the generation of the random number. Then the number of the property correcting bundles is set the same values as that the energy correcting bundles and it traces the same trajectory as that of the correcting energy bundle in order to avoid the overlap of the use of radiative bundles and to reduce the computing time. That is to say, the calculation of ΔZ^i is done in accordance with that of P^i. If the radiative bundle is reflected k-times until it is absorbed, then ΔZ_g^i is given as follows.

$$\Delta Z_g^i=1-\frac{\kappa_0^{i-2}\left[\prod_{j=1}^{k}(1-\varepsilon_j^{i-2})\cos\phi_j^{i-2}\cos\phi_j'^{i-2}\right.}{\kappa_0^{i-1}\left[\prod_{j=1}^{k}(1-\varepsilon_j^{i-1})\cos\phi_j^{i-1}\cos\phi_j'^{i-1}\right.}$$
$$\frac{\left./\pi(l_{j-1}^{i-2})^2\right]\left[e^{-\int_0^L \kappa^{i-2}dL}/\pi(l^{i-2})^2\right]}{\left./\pi(l_{j-1}^{i-1})^2\right]\left[e^{-\int_0^L \kappa^{i-1}dL}/\pi(l^{i-1})^2\right]} \quad \cdots\cdots (35)$$

As the second term in the right hand side equals to P_g^i/P_g^{i-1} and the locus of P_g^{i-1} is the same as that of P_g^i, Eq.(35) is transformed as follows,

$$\Delta Z_g^i=1-\left(\frac{\kappa_0^{i-2}}{\kappa_0^{i-1}}\right)\left[\prod_{j=1}^{k}\frac{(1-\varepsilon_j^{i-2})}{(1-\varepsilon_j^{i-1})}\right]$$
$$\times\exp\left[-\int_0^L(\kappa^{i-2}-\kappa^{i-1})dL\right] \quad \cdots\cdots (36)$$

Similarly, the next equation is obtained for ΔZ_w^i.

$$\Delta Z_w^i=1-\left(\frac{\varepsilon_0^{i-2}}{\varepsilon_0^{i-1}}\right)\left[\prod_{j=1}^{k'-1}\frac{(1-\varepsilon_j^{i-2})}{(1-\varepsilon_j^{i-1})}\right]$$
$$\times\exp\left[-\int_0^{L_{k'-1}}(\kappa^{i-2}-\kappa^{i-1})dL\right] \quad \cdots\cdots (37)$$

where, $L_{k'-1}$ indicates the total length of the locus of the radiative bundle to the (k'-1)th incident on the wall. Through Eqs.(26),(27),(36) and (37), the quantities of the heat transfered to the medium and wall by the property correcting bundle s^{*i}'s are written as Eqs.(40) and (41) when the energy of a bundle s^i's are expressed by Eqs.(38) and (39).

$$s_g^i=\kappa^{i-2}E_g^{i-2}\Delta V/N_g^i \quad \cdots\cdots (38)$$

$$s_w^i=\varepsilon^{i-2}E_w^{i-2}\Delta A/N_w^i \quad \cdots\cdots (39)$$

$$s_g^{*i}=s_g^i\left[\prod_{j=1}^{k}(1-\varepsilon_j^{i-1})\right]\Delta Z^i \quad \cdots\cdots (40)$$

$$s_w^{*i}=\sum_{k'=1}^{k}\left\{s_w^i\left[\prod_{j=1}^{k'}(1-\varepsilon_j^{i-1})\right]\Delta Z^i\right\} \quad \cdots\cdots (41)$$

From the comparison of the radiative energy of the energy correcting bundle S^i and of the property correcting bundle s^i, it is understood that the truncation error of the property correcting bundle is larger than that of energy correcting bundle. The reason that s^i becomes larger than S^i as the iteration advances, for s^i is proportional to the absolute value of radiative energy while S^i is proportional to the difference of the radiative energy calculated at successive iterative turns. This truncation error tends to accumulate for the asymptote formulae of Eqs.(26) and (27) are used. Therefore, this error has to be reduced with the aid of the heat balance of the system. The following equation is obtained from the definition of Eqs.(24) and (25) if the number of bundles is large enough to estimate the probability.

$$\int_V \kappa^{i-2}E_g^{i-2}\Delta Z_g^i dV+\int_A \varepsilon^{i-2}E_w^{i-2}\Delta Z_w^i dA=0 \quad \cdots\cdots (42)$$

It is assumed that the truncation error mentioned above is proportional to the radiative energy of each control element in order to make the calculation easy. Then the correcting rate Δz^i is deduced from Eq.(42) and given by Eq.(43). The corrections $\Delta\delta^i$'s are to be added to C_0^i's are expressed by the following Eqs.(44) and (45).

$$\Delta z^i=\frac{\int_V \kappa^{i-2}E_g^{i-2}\Delta Z_g^i dV+\int_A \varepsilon^{i-2}E_w^{i-2}\Delta Z_w^i dA}{\int_V \kappa^{i-2}E_g^{i-2}dV+\int_A \varepsilon^{i-2}E_w^{i-2}dA} \quad \cdots\cdots (43)$$

$$\Delta\delta_g^i=-\kappa^{i-2}E_g^{i-2}\Delta V\Delta z^i \quad \cdots\cdots (44)$$

$$\Delta\delta_w^i=-\varepsilon^{i-2}E_w^{i-2}\Delta A\Delta z^i \quad \cdots\cdots (45)$$

c. The proof of the equivalence of the DPEV method with the N method

To simplify the discussion, the following quantities are introduced here.

$$C_{gg0}^i=C_{gg0}^{i-1}+\int_V[\Delta E_g^i+\kappa^{i-2}E_g^{i-2}\Delta Z_g^i]P_g^i dV \quad \cdots\cdots (46)$$

$$C_{wg0}^i=C_{wg0}^{i-1}+\int_A[\Delta E_w^i+\varepsilon^{i-2}E_w^{i-2}\Delta Z_g^i]P_g^i dA \quad \cdots\cdots (47)$$

$$C_{gw0}^i=C_{gw0}^{i-1}+\int_V[\Delta E_g^i+\kappa^{i-2}E_g^{i-2}\Delta Z_w^i]P_w^i dV \quad \cdots\cdots (48)$$

$$C_{ww0}^i=C_{ww0}^{i-1}+\int_A[\Delta E_w^i+\varepsilon^{i-2}E_w^{i-2}\Delta Z_w^i]P_w^i dA \quad \cdots\cdots (49)$$

Then, Eqs.(26) and (27) can be rewritten as follows:

$$C_{g0}^i=C_{gg0}^i+C_{wg0}^i \quad \cdots\cdots (50)$$

$$C_{w0}^i=C_{gw0}^i+C_{ww0}^i \quad \cdots\cdots (51)$$

Here, for example, the following equation can be deduced from expansion of Eq.(46)

using Eqs.(22) and (24).

$$C^{i}_{\rho\rho 0}=\int_V[\varDelta E^{i}_{\rho}+\kappa^{i-2}E^{i-2}_{\rho}\varDelta Z^{i}_{\rho}]P^{i}_{\rho}dV$$
$$+\int_V[\varDelta E^{i-1}_{\rho}+\kappa^{i-3}E^{i-3}_{\rho}\varDelta Z^{i-1}_{\rho}]P^{i-1}_{\rho}dV+\cdots+$$
$$+\int_V[\varDelta E^{2}_{\rho}+\kappa^{0}E^{0}_{\rho}\varDelta Z^{2}_{\rho}]P^{2}_{\rho}dV$$
$$+\int_V\kappa^{0}E^{0}_{\rho}P^{0}_{\rho}dV$$
$$=\int_V\kappa^{i-1}E^{i-1}_{\rho}P^{i}_{\rho}dV+\int_V[\kappa^{i-2}E^{i-2}_{\rho}P^{i-1}_{\rho}$$
$$-\kappa^{i-2}E^{i-2}_{\rho}P^{i}_{\rho}+\kappa^{i-2}E^{i-2}_{\rho}\varDelta Z^{i}_{\rho}]dV+\cdots+$$
$$+\int_V[\kappa^{0}E^{0}_{\rho}P^{1}_{\rho}-\kappa^{0}E^{0}_{\rho}P^{0}_{\rho}+\kappa^{0}E^{0}_{\rho}\varDelta Z^{2}_{\rho}P^{2}_{\rho}]dV$$
$$=\int_V\kappa^{i-1}E^{i-1}_{\rho}P^{i}_{\rho}dV+\sum_{i'=2}^{i}\left\{\int_V[\kappa^{i'-2}E^{i'-2}_{\rho}\right.$$
$$\left.\times(P^{i'-1}_{\rho}/P^{i'}_{\rho}-1+\varDelta Z^{i'}_{\rho})P^{i'}_{\rho}dV]\right\}\quad\cdots\cdots\cdots(52)$$

From Eq.(24), the second term in the right hand side of Eq.(52) becomes zero. Therefore, Eq.(52) reduces to

$$C^{i}_{\rho\rho 0}=\int_V\kappa^{i-1}E^{i-1}_{\rho}P^{i}_{\rho}dV\quad\cdots\cdots\cdots(53)\ .$$

Similarly, Eqs.(47)-(49) are rewritten as follows:

$$C^{i}_{w\rho 0}=\int_A\varepsilon^{i-1}E^{i-1}_{w}P^{i}_{\rho}dA\quad\cdots\cdots\cdots(54)$$

$$C^{i}_{\rho w 0}=\int_V\kappa^{i-1}E^{i-1}_{\rho}P^{i}_{w}dV\quad\cdots\cdots\cdots(55)$$

$$C^{i}_{ww0}=\int_A\varepsilon^{i-1}E^{i-1}_{w}P^{i}_{w}dA\quad\cdots\cdots\cdots(56)\ .$$

By substitution of Eqs.(53)-(56) into Eqs.(50) and (51) the following equations are obtained.

$$C^{i}_{\rho 0}=\int_V\kappa^{i-1}E^{i-1}_{\rho}P^{i}_{\rho}dV+\int_A\varepsilon^{i-1}E^{i-1}_{w}P^{i}_{\rho}dA\quad(57)$$

$$C^{i}_{w0}=\int_V\kappa^{i-1}E^{i-1}_{\rho}P^{i}_{w}dV+\int_A\varepsilon^{i-1}E^{i-1}_{w}P^{i}_{w}dA\quad(58)$$

By substitution of Eqs.(57) and (58) into Eqs.(29) and (30), it is made clear that the solution by the DPEV method coincides with that by the N method.

3. An Example of Application of the DPEV Method

A one-dimensional heat transfer model with internal heat generation shown in Fig.1 is considered here to discuss the reduction of the computing time and the improvement of the convergent stability by the DPEV method. In this model, a radiative medium fills the space between two parpllel plates with infinite length and a heat generating zone is placed in the center of the medium.

It is assumed that the medium is a gray gas as to thermal radiation and its value differs depending on whether there is the heat generation or not and that the walls are isothermal and are diffuse as to the thermal radiation. Furthermore, the system is in the steady state and the convection, conduction and energy dissipation can be ignored. By substitution of these conditions into Eqs.(1) and (2), the following energy equations are deduced.

$$q_{in}-\mathrm{div}\ q_{R\rho}=0\quad\cdots\cdots\cdots(59)$$

$$q_{w}+q_{Rw}=0\quad\cdots\cdots\cdots(60)$$

where, $-\mathrm{div}\ q_{R\rho}$ and $-q_{Rw}$ are used after the transformation into the one-dimensional form from Eqs.(3) and (4) respectively is performed. Although there are many kinds of the functions proposed for the temperature dependence of the radiative absorption coefficient κ and the emissivity ε, in this paper, the following functions are employed.

$$\left.\begin{array}{rl}\text{Heat Generating Zone} & :\ \kappa=0.5(T_{\rho}/1\,500)^{1.5}\ \mathrm{m^{-1}}\\ \text{Non-heat Generating Zone} & :\ \kappa=0.2(T_{\rho}/1\,000)^{-1}\ \mathrm{m^{-1}}\\ \text{Wall} & :\ \varepsilon=0.8(T_{w}/600)^{0.1}\end{array}\right\}\quad\cdots\cdots\cdots(61)$$

Furthermore, the heat generating rates are assumed with a parabolic distribution taking maximum value at the center and zero at the interfaces of the heat generating zone and no heat generating zone as shown in Fig.3; the mean value of the heat generating rate is 2.09×10^{6} kJ/(m^{3}h) [5×10^{5} kcal/(m^{3}h)] , the wall temperature 600K, the distance between the two plates y_0 1m, the total number of elements in medium n 20, the total number of the radiative bundles N_t 100 000 and $C_{f\rho}$ unity.

The solutions by the methods used in this paper are verified with the comparison with the solutions by Usisikin[(1)] for the constant radiative properties and heat generating rate and with the solutions by Taniguchi[(2)] which took into account of the temperature dependence of the emissivity of the wall in addition to the properties considered in this chapter.

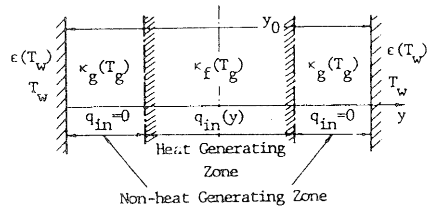

Fig.2 Heat transfer model with the internal heat generation

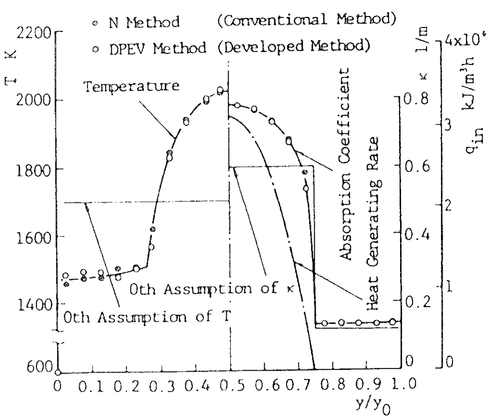

Fig.3 Temperature distribution and values used in calculation

An example of the temperature distribution calculated here is shown in the left half of Fig.3. The distribution of temperature by the N method is the result of the 28th iterative turn at which the iterative solution by the DPEV method is considered to converge.

Although, there are some peculiar values as to the probability calculation in the vicinity region of the wall, the solutions of DPEV method agree on the whole with those of the N method irrespective of the presence of heat generation or not. Fig.3 also shows the 0-th profiles of the temperature and radiative absorption coefficient.

The maximum values of the relative error of the temperature in each iterative turn are shown in Fig.4. As the relative error corresponding to the N method suffered a vibration at the vicinity of the error of 10^{-2} even the iterative turn advanced, the convergence cannot be achieved in the mathematical sense. Thus the N method does not seem to be suitable method applicable to the numerical analysis that needs higher convergent accuracy. The convergent process of the relative error concerning to the DPEV method is very smooth and stable nevertheless the same quantity of radiative energy of a radiative bundle is used as the N method, that is $C_{fs}=1$.

In Fig.5, the result of the traced bundle number and the computing time at each iterative turn are shown in the form of the ratio with the mean values of those in the N method. In the DPEV method, the traced bundle number decreases as an exponential function with advance of iterative turn and this value becomes less than 1/1000 at the 16th iterative turn. It is obvious that the computing time decreases as the traced bundle number decreases. At the second iterative turn, the computing time increases in spite of the decrease of the traced bundle number. This is because it imposes double tasks of the energy correcting and property for individual radiative bundles. Therefore, the computing time of a radiative bundle by the DPEV method being 2.5-3 times that by the N method, the computing time of the DPEV method can be reduced by using the N method at the second iterative turn too.

In the result shown in Fig.5, a total computing time for the radiative heat transfer by DPEV method is less than 15% of that by N method.

The convergent processes of the temperature at different zones are shown in Fig.6. At the highest temperature zone, the convergent process of the N method is smooth and shows the same tendency as that of the DPEV method, but at the lower temperature part, that of the N method shows a peculiar vibration of the probability calculation, This is because the number of the absorbed radiative bundles in the lower temperature region being small for the absorbed bundle which is proportional to the absolute value of the emissive power in the case of the N method. However, in case of the DPEV method, there is not any peculiar vibration, for the number of absorbed bundles is proportional to the difference of the emissive power.

This stability of convergence is an excellent feature of the DPEV method and this feature means that the DPEV method can be applied easily to multi-dimensional problem even when the total radiative bundle number is small.

In Fig.7, the convergent processes of the heat balance of the system are shown. In this example, the stability of convergent process seems to be satisfactory except from the first period to middle period even in the N method because of

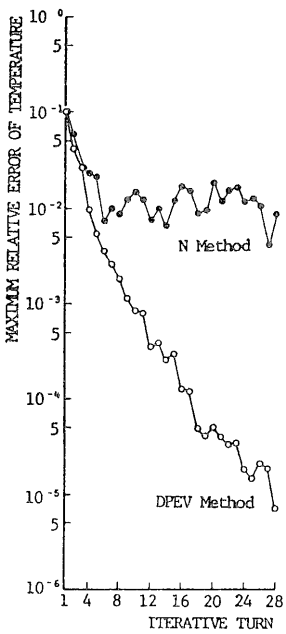

Fig.4 Convergency of maximum relative error of temperature

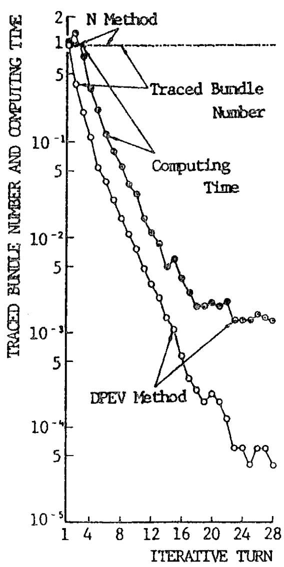

Fig.5 Traced bundle number and computing time

many radiative bundles being used but the relative error of the N method is converging to the value 10^{-3} which is 1 figure smaller than that of the DPEV method.

From the above discussions, it is clear that the DPEV method is excellent with stability of convergence and reduction of computing time except in the first few iterative turns. The stability of convergence and the reduction of computing time by the DPEV method would be great benefit when the simultaneous equations composed of the momentum equation of complex flow, the energy equation including the radiative heat transfer and so on are to be solved by numerical analysis.

4. Conclusions

In this paper, the author presents a modified Monte Carlo method to reduce the calculating time and to improve the convergent stability without losing the excellent feature of the Monte Carlo method such as flexibility, that is, easiness application to the numerical calculation of the variation of the thermal properties or to the numerical analysis about a heat transfer model with complex geometry. In this method, the number of the radiative bundles, which are emitted from the control element and transfer the radiative energy by the probabilistic means, are proportional to the difference of the emissive power between two successive iterative turns. And at the same time, those radiative bundles correct the radiative properties changing at each iterative turn.

An example of its application showes that this modified Monte Carlo method is excellent with reduction of calculation time and the stability of convergence. This method can be used with great benefit when the heat transfer combined with radiation and the other kinds of heat transfer mechanisms is to be solved by numerical analysis or when the radiative properties depend on the other variables.

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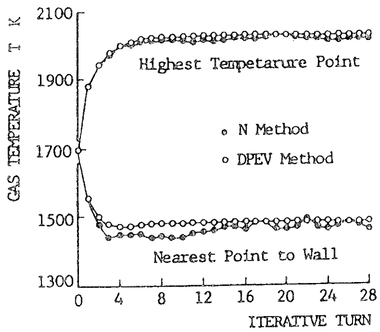

Fig.6 Convergency of temperature

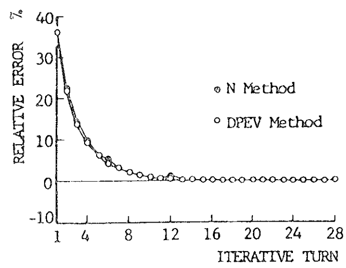

Fig.7 Convergency of heat balance