Propagation of Elastic Wave in two Layered Concentric Cylinder Having Different Elastic Constants

著者	MATSUOKA Kenichi, NOMACHI Sumio
journal or publication title	Memoirs of the Muroran Institute of Technology. Science and engineering
volume	10
number	4
page range	619-625
year	1982-11-30
URL	http://hdl.handle.net/10258/3743

Propagation of Elastic Wave in two Layered Concentric Cylinder Having Different Elastic Constants

Kenichi G. MATSUOKA and Sumio G. NOMACHI*

Abstract

Propagation of stress waves in a two concentrically layered cylinder: an inner solid and an outer annular cylinders of different elastic moduli each other, is investigated as an eigen value problem of the coefficients matrices of boundary conditions which are derived from the solution of dynamic equations of cylindrical coordinates by means of Hankel transforms.

The discussions are around the variations of wave velocity with the change of ratio between wave length and the diameter of the outer cylinder. The numerical calculations are performed for several ratio of the diameter of the cylinders.

1. Introduction

A cylinder concentrically layered one with another is supposed to be the simplest example of composite materials. The solution of stress wave in the two layered cylinder, may give us one of the basic properties concerning the stress wave propagation in a fiber reinforeced composite which has become of increasing importance as well as research object.

In this paper, the discussion is specifically focused on the propagation of the axial stress wave in the two layered elastic cylinder. The variations of the wave velocity are shown by the change of ratio between the wave length and the diameter of the outer part, with the different combinations of elastic moduli and densities. The dispersion diagram thus obtained give the bar velocity of the two layered cylinder by letting wave length be infinite, and the modes corresponding to various velocity describe how the composite action works between both layers.

* Department of Civil Engineering, Hokkido University.

2. Fundamental Equations of Stress Wave propagation

The equation of motions written in the cylindrical coordinates r, θ and z, yield the solutions of the harmonic stress wave in the solid and the hollow cylinders by means of finite Hankel transforms in the r direction, as follows;

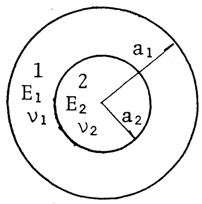

Fig. 1 The two layered cylinder.

$$u=(\tilde{\boldsymbol{A}}_{mr}+\tilde{\boldsymbol{B}}_{mr})\cos m\theta\cdot e^{ip(t-z/V)} \tag{1}$$

$$v=(\tilde{\boldsymbol{A}}_{mr}-\tilde{\boldsymbol{B}}_{mr})\sin m\theta\cdot e^{ip(t-z/V)} \tag{2}$$

$$\begin{aligned}\tilde{\boldsymbol{A}}_{mr}=\sum_{k=1}^{\infty}&[\chi_{mp}^{(k)}(N_\mu r)\{\alpha_{mk}/2\mu+(m+1)A_{mk}+(m-1)B_{mk}-iNE_{mk}/2\}/N_\mu\\&+(\mu N^2/\rho p^2)\{\chi_{mp}^{(k)}(N_\mu r)/N_\mu-N_\alpha\chi_{mp}^{(k)}(N_\alpha r)/N^2\}\{\beta_{mk}/2\mu\\&+(m+1)A_{mk}-(m-1)B_{mk}-iNE_{mk}\}]\end{aligned} \tag{3}$$

$$\begin{aligned}\tilde{\boldsymbol{B}}_{mr}=\sum_{k=1}^{2}&[-\chi_{ms}^{(k)}(N_\mu r)\{\alpha_{mk}/2\mu+(m+1)A_{mk}+(m-1)B_{mk}\\&+iNE_{mk}/2\}N_\mu+(\mu N^2/\rho p^2)\{\chi_{ms}^{(k)}(N_\mu r)/N_\mu-N_\alpha\chi_{ms}^{(k)}(N_\alpha r)/N^2\}\\&\times\{\beta_{mk}/2\mu+(m+1)A_{mk}-(m-1)B_{mk}-iNE_{mk}\}],\end{aligned} \tag{4}$$

$$\begin{aligned}w=\tilde{\boldsymbol{W}}_{mr}\cos m\theta\cdot e^{ip(t-z/V)}=\sum_{k=1}^{2}&[G_m^{(k)}(N_\mu r)E_{mk}+(2\mu iN/\rho p^2)\\&\times\{G_m^{(k)}(N_\alpha r)-G_m^{(k)}(N_\mu r)\}\{\beta_{mk}/2\mu+(m+1)A_{mk}-(m-1)B_{mk}\\&-iNE_{mk}\}]\cos m\theta\cdot e^{ip(t-z/V)}),\end{aligned} \tag{5}$$

where u, v, w are components of displacements in the r, θ, z directions respectively. μ, λ: Lame's elastic constants, $N=p/V=2\pi=\pi/l$. p: circular frequency, V: propagation velocity of wave in the z direction, L: wave length, l: half of wave length, ρ: density, $m=0, 1, 2,\ldots\ldots$, $N_\mu^2=N^2-\rho p^2/\mu$, $N_\alpha^2=N^2-\rho p^2/(2\mu+\lambda)$.

The following functions are seen in the Eqs. (3)~(5)

$$G_m^{(k)}(Nr)=R_{m\cdot m}^{(k)}(Nr)/R_{m\cdot m}^{(k)}(Na_k),\quad \chi_{mp}^{(k)}(Nr)=R_{m+1\ m}^{(k)}(Nr)/R_{m\cdot m}^{(k)}(Na_k),$$
$$\chi_{ms}^{(k)}(Nr)=R_{m-1\cdot m}^{(k)}(Nr)/R_{m\cdot m}^{(k)}(Na_k),$$

which are for the outer cylinder by letting the outer and inner radii a_1 and a_2, and a_0

$=a_2$, that is

$$R^{(k)}_{j,m}(Nr)=I_j(Nr)K_m(Na_{k-1})-(-1)^{j+m}I_m(Na_{k-1})K_j(Nr),$$

$$j=m-1,\ m,\ m+1, \qquad k=1,\ 2$$

In case of a solid cylinder, letting $k=1$, and the radius $= a$, we have

$$R^{(1)}_{j,m}(Nr)=I_j(Nr), \qquad j=m-1,\ m,\ m+1.$$

The functions I and K are the modified Bessel functions of the lst and 2nd order respectively. The letters α_{mk}, β_{mk}, A_{mk}, B_{mk} and E_{mk}, are unknown coefficients relating to the stresses and the displacements on the outer and inner boundaries.

3. Boundary Conditions

By putting l for the subscription, we have the coefficients of the outer layer and the coefficients of the inner cylinder are given by the subscription of 2, as shown in Fig. 1.

Let r be a_k in the Fgs. (3), (4) we have,

$$\tilde{\boldsymbol{A}}_{mr})_{r=a_k}=a_kA_{mk}, \ (6), \qquad \tilde{\boldsymbol{B}}_{mk})_{r=a_k}=a_kB_{mk}, \ (7).$$

1). Conditions for $r=a_1$.

The Egs. (6), (7) yield,

$$\text{i)} \quad \tilde{\boldsymbol{A}}_{mr\cdot1})_{r=a_1}=a_1A_{m1\cdot1}, \qquad \tilde{\boldsymbol{B}}_{mr\cdot1})_{r=a_1}=a_1B_{m1\cdot1}, \tag{8}$$

The outer surface is free from any stresses, and it follows,

$$\left.\begin{array}{l}\text{ii)} \quad \sigma_{r\cdot a_1})_{r=a_1}=0 \quad \therefore \ \beta_{m1\cdot1}=0, \qquad \tau_{r\theta\cdot1})_{r=a1}=0 \quad \therefore \ \alpha_{m1\cdot1}=0, \\ \qquad \tau_{rz\cdot1})_{r=a_1}=0,\end{array}\right\} \tag{9}$$

2). Conditions for $r=a_2$.

Likewise, we have from the Egs. (6) and (7),

$$\left.\begin{array}{ll}\text{iii)} \quad \tilde{\boldsymbol{A}}_{mr\cdot1})_{r=a_2}=a_2\cdot A_{m2\cdot1}, & \tilde{\boldsymbol{A}}_{mr\cdot2})_{r=a_2}=a_2A_{m1\cdot2}, \\ \qquad \tilde{\boldsymbol{B}}_{mr\cdot1})_{r=a_2}=a_2B_{m2\cdot1}, & \tilde{\boldsymbol{B}}_{mr\cdot2})_{r=a_2}=a_2B_{m1\cdot2},\end{array}\right\} \tag{10}$$

and the continuity of the displacements is written by

$$\left.\begin{array}{lll}\text{iv)} \quad u_1)_{r=a_2}=u_2)_{r=a_2}, & v_1)_{r=a_2}=v_2)_{r=a_2}, & w_1)_{r=a_2}=w_2)_{r=a_2} \\ \qquad \therefore \ A_{m2\cdot1}=A_{m1\cdot2}, & B_{m2\cdot1}=B_{m1\cdot2}, & E_{m2\cdot1}=E_{m1\cdot2} \\ \sigma_{r\cdot1})_{r=a_2}=\sigma_{r\cdot2})_{r=a_2}, & \tau_{r\theta\cdot1})_{r=a_2}=\tau_{r\theta\cdot2})_{r=a_2} & \\ \qquad \therefore \ \alpha_{m1\cdot1}=\alpha_{m1\cdot2}, & \beta_{m1\cdot1}=\beta_{m1\cdot2} & \end{array}\right\} \tag{11}$$

and

$$\tau_{rz\cdot1})_{r=a_2}=\tau_{rz\cdot2})_{r=a_2}, \tag{12}$$

The Egs. (8)~(12), lead to the matrix for the eigen value from which phase velocity of stress wave may be obtained.

4. Numerical Example

Taking the ratio of the elastic moduli and the densities between both layer as $E_2/E_1=$ 7. 0 and $\rho_2/\rho_1=3.2$ with the Poisson's ratio: $\nu_1=1/6$, $\nu_2=0.3$, we carried out the numerical calculation of the phase velocity cocerning axially symmetrical wave for $a_2/a_1=0.0$, 0. 25, 0. 5 and 1. 0. The case of $a_2/a_1=0.0$ and 1. 0 coincide with the cases a single solid cylinder. The calculation of the eigen values from the prescribed matrix, was performed by means of iteration keeping an accuracy for a number of five figures. The eigen value is nothing but the phase velocity, from which the group velocity is derived by numerical differentiation.

Fig. 2 shows the dispersion diagram concerning the phase velocity of the lst order. The ordinate of the figure is measured by the ratio between the phase velocity and the

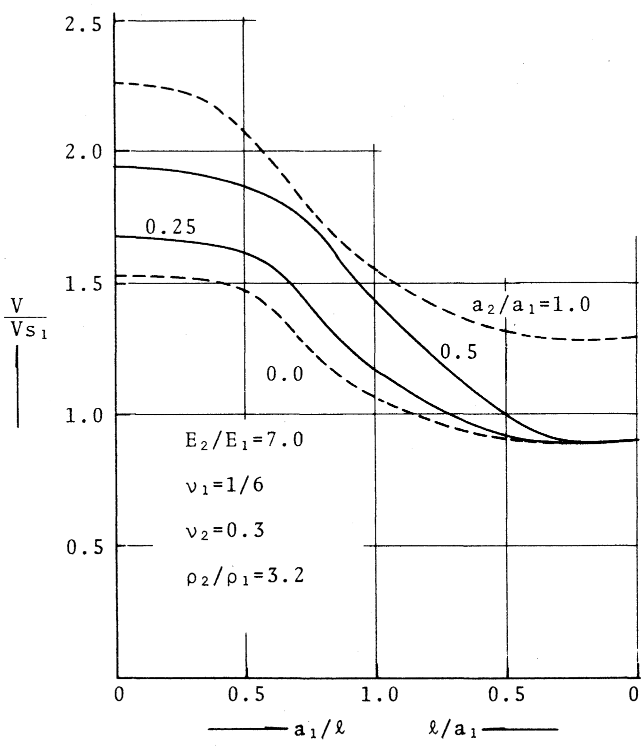

Fig. 2 The dispersion diagram of phase velocity.

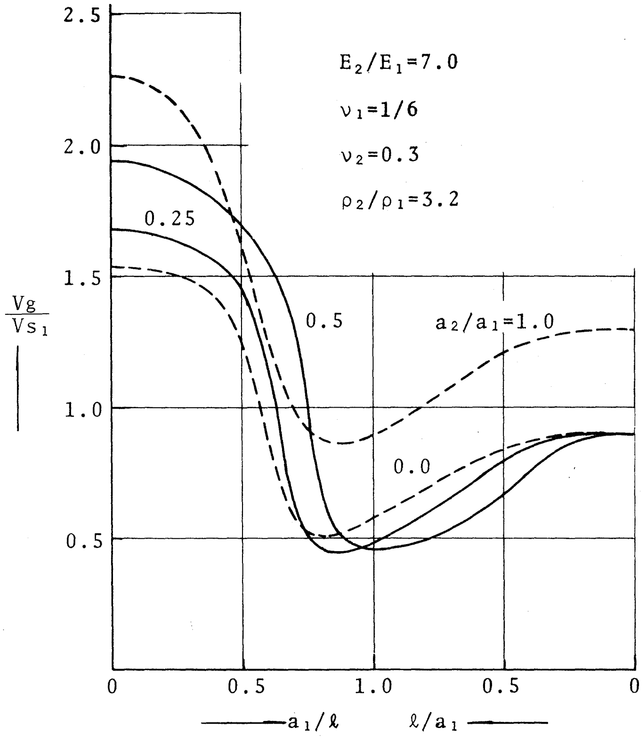

Fig. 3 The dispersion diagram of group velocity.

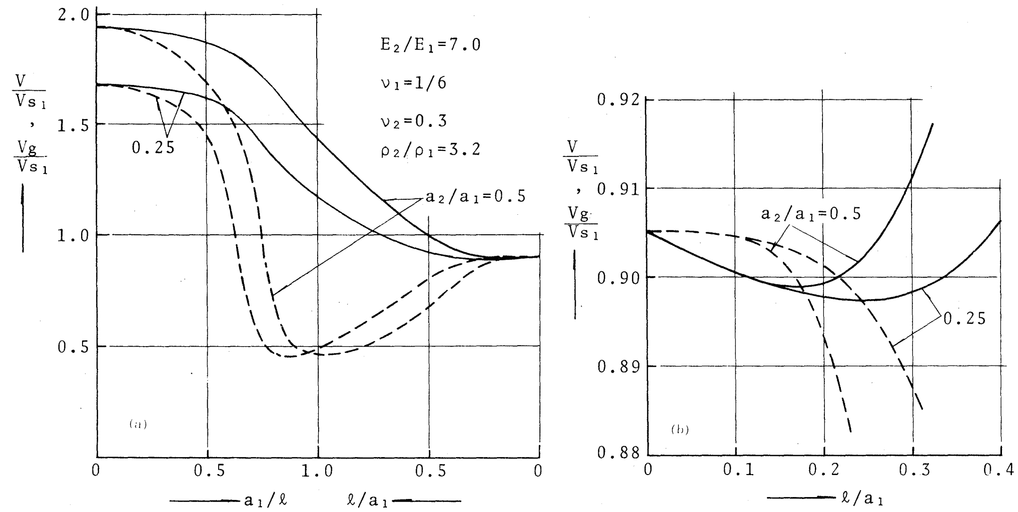

Fig. 4 The comparison between phase and group velocity of two layered cylinder.

shear wave velocity of the outer material, and abssisa, by a_1/l on the left and by l/a_1 on the right part in which l is the half of wave length. Thus the figure covers all the wave length from zero to infinite. The dotted lines are for the solid cylinders,

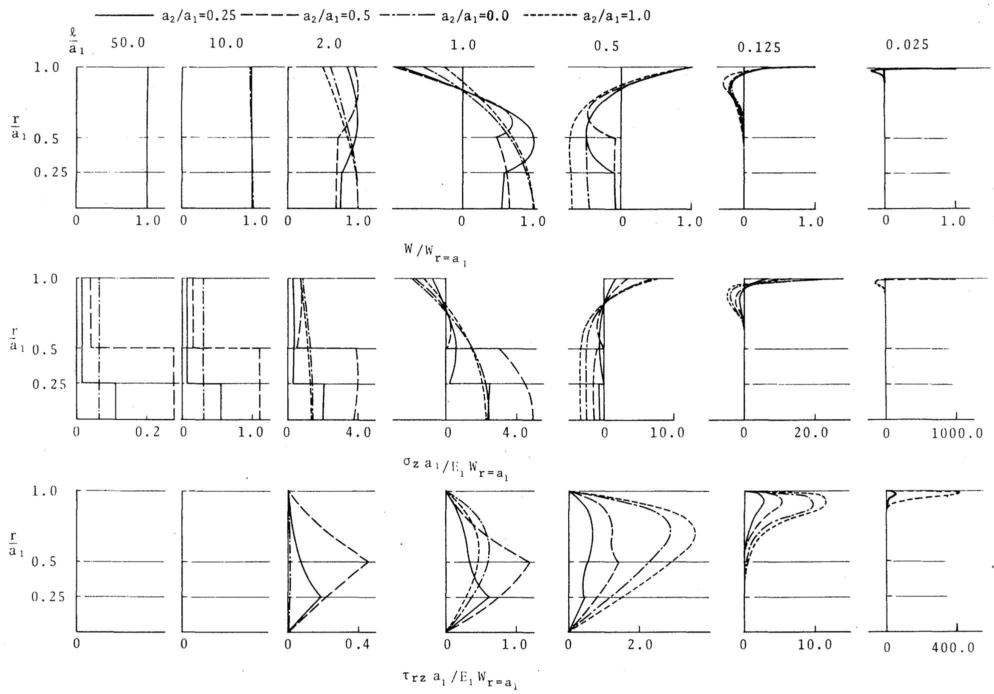

Fig. 5 The wave modes of w, σ_z, and τ_{rz}.

one with E_1, ρ_1, ν_1 and another with E_2, ρ_2, ν_2. The full lines are for the two layered cylinder, and they are drawn between both dotted lines in Fig. 2. In case of infinite wave length, V/V_{s_1}=1. 681 for a_2/a_1=0. 25 and V/V_{s_1}=1. 942 for a_2/a_1=0. 5. The bar velocities corresponding to the above, which are calculated by use of material constants averaged over cross sections, are 1. 699 and 1. 940, respectively. They are of fairly good coincidence. However, as wave length tends to zero the velocity become to that of Rayleigh wave on the outer free suface.

Fig. 3 shows the dispersion diagram of group velocities. The lines in the figure represent same discrimination with those in Fig. 2. The group velocities of the layered cylinder are not always evaluated between those of solid cylinders. The minimum values for the layered cylinder are smaller than those of solid cylinders.

Fig. 4 shows the phase and the group velocities of the two layered cylinder, and the group velocity is always lower than the phase velocity except in the short wave range.

Fig. 5 includes many figures showing the wave modes of the axial displacement W, the stresses σ_z and τ_{rz}, with l/a_1=50. 0, 10. 0, 2. 0, 1. 0, 0. 5, 0. 125, and 0. 025. We see that the modes uniformly distribute over the section when the wave length in large, however, the modes vary over the section as the wave legth becomes short. The mode in solid cylinder tends to surface wave style as the wave length becomes shorter. Whereas the mode in the two layered cylinder, keeps uniform distribution on the section of inner cylinder until a considerably short wave length occurs, a concentration of the mode to the outer surface takes place at very short wave length.

5. Conclusions

Investigating the propagation of stress wave in the concentrical two layered cylinder of different elastic moduli and densities in the way of three dimensional stress problem, we have the results as follows:

1). The phase velocity of the two layered cylinder is in good agreement with the bar velocity by use of statical equivalent rigidity.

2). By tending the wave length to zero, the phase velocity becomes the Rayleigh wave's one on the outer free surface.

3). The group velocity of the two layered cylinder is not always in between those of the solid cylinders of each elastic moduli and densities.

4). The elastic wave concentrates to the outer boundary for the short wave length.

The numerical example was calculated using FACOM 230-75 of Hokkaido University and MELCOM 9100 of Muroran Institute of Technology.

References

1) K. Matsuoka and S. Nomachi, Procs. JSCE, No. 258, (1977) p. 13.

2) K. G. Matsuoka and S. G. Nomachi, Theoretical and Applied Mechanics, Vol. 24. (1976) p. 399.