In Broker We Trust: A Double-auction Approach for Resource Allocation in NFV Markets
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Abstract—Network function virtualization (NFV) is an emerging scheme to provide virtualized network function (VNF) services for next-generation networks. However, finding an efficient way to distribute different resources to customers is difficult. In this paper, we develop a new double-auction approach named DARA that is used for both service function chain (SFC) routing and NFV price adjustment to maximize the profits of all participants. To the best of our knowledge, this is the first work to adopt a double-auction strategy in this area. The objective of the proposed approach is to maximize the profits of three types of participants: NFV broker, customers and service providers. Moreover, we prove that the approach is a weakly dominant strategy in a given NFV market by finding the Bayesian Nash equilibrium in the double-auction game. Finally, according to the results of the performance evaluation, our approach outperforms the single-auction mechanism with higher profits for the three types of participants in the given NFV market.

Index Terms—Network function virtualization (NFV), Double auction, Virtualized network function (VNF), Service function chain (SFC), Resource allocation

I. INTRODUCTION

NETWORK function virtualization (NFV) is a new scheme for enhancing network scalability and flexibility [1], [2], [3], [4]. Due to the high costs of providing storage space and computational resources for traditional network methods, it is becoming difficult to provide new full services in the current networks [5], [6]. The development of NFV introduces new approaches, along with other emerging technologies, such as software-defined networks (SDNs) [7], [8], [9] and cloud computing [10], to design, schedule and manage network resources. NFV changes the traditional rules for how network operators manage their infrastructure as software instances separate from the hardware platform by using the proven virtualization technology [11]. For example, one example of an open platform for NFV (OPNFV), the Huawei E9000 blade server, is widely applied in industry and is intended to facilitate the commercial adoption of NFV applications [12].

Virtualized network functions (VNFs) defined by the NFV are virtualized tasks that are separated from network hardware, which is provided by network service providers. In fact, NFV distributes VNFs, including firewall, storage and routing, executed on commodity hardware, as shown in Figure 1, because a single VNF instance is not enough to providing a valid service to the required customers [13]. Multiple VNFs, instantiated without delay and equipment installation, can be connected to obtain chains of network services, called a service function chain (SFC) [14], which is made-to-order for different use cases [15], [16], [17]. Accompanying SFCs, an NFV market is an emerging scheme in which the SFC broker has geodistributed information of SFCs and then sells them to users on demand, while SFC suppliers provide one or more SFCs. The participants, including customers, the SFC broker and service suppliers, play a game with each other for their own benefits.

The development of NFV faces also several technical issues in handling VNF, which is the most important component of an SFC. Previous works [18], [19] have shown that although the underlying network is lightly utilized, virtualization may still lead to performance problems such as abnormal latency variations and severe throughput jitter. Therefore, the first problem of NFV is that the hardware and software may be supported by different service providers, resulting in skewed utilization, increased latency or unstable throughput. The second problem is that when multiple suppliers manage the virtualized resources in the network, it is difficult to coordinate with suppliers to provide good service performance [20], [21].

To solve these problems, some industrial projects from commercial companies attempt to define standards for the coordination of suppliers [22], [23], [24]. However, the resource utilization and service performance of these methods are not sufficient to support the distribution of SFC in NFV market due to inefficient scheduling strategies. Finding an efficient method for scheduling available SFCs among independent suppliers and to calculate the applicable price in the NFV market is challenging. In general, an auction-based method can improve the efficiency of resource scheduling in a competitive environment [25]. Compared with simple allocation using fixed pricing, an auction mechanism provides more economical efficiency for suppliers according to customer demands, flexible allocation of SFCs and finer targeting of customers.

Therefore, in this paper, we present an auction-based resource scheduling method for guaranteeing resource utilization and service performance in the NFV market. Although the traditional single-auction method can improve the resource utilization to schedule SFCs, it cannot guarantee the profits of service suppliers. Rather than using single-auction methods, a double-auction model can achieve a higher efficiency with competitive bidding between customers and service suppliers. A double-auction mechanism can model the interaction of two or three parties well, where buyers request SFCs with the bidding price, suppliers provide their services with the asking price, and the broker decides the transaction value [26]. Through competitive bidding and asking, the profit in
the double-auction method is higher than that in the single-auction method. Consequently, we first design an efficient double-auction model in our resource scheduling method in which both service suppliers and customers can participate in the auction market. The main goal of our model is to maximize the profits of the three participants when the auction mechanism is incentive compatibility (IC).

The main contributions of this paper can be summarized as follows.

1) First, we propose the concept of the double-auction NFV market and characterize the mutual effect between the SFC broker, customers and resource suppliers. With this concept, we can combine a large number of VNFs into different service chains and then schedule them separately for a customer in an NFV market.

2) We formulate a double-auction model with constraints of customers and sellers to maximize the profits of the three participants. To solve this model, we use three algorithms, including auction, price adjustment and payment strategy, to schedule network resources. We combine the normal distribution element and the price adjustment to control the auction progress. We also theoretically analyze the effectiveness of our proposed method.

3) We conduct a comprehensive simulation to evaluate the DARA resource scheduling method. The results confirm that the DARA method outperforms the single-auction model with respect to the profits of both participants.

The remainder of this paper is organized as follows. The related works are discussed in Section II. We present the related preliminaries in Section III. We introduce the resource allocation problem in data center networks. We also propose the DARA resource scheduling and pricing method with an NFV performance constraint in Section IV. In Section V, we design a double-auction mechanism containing the DARA algorithm and a price adjustment to guarantee availability. We present some numerical results in Section VI, and then we demonstrate the feasibility of the proposed DARA method through a mathematical analysis and comparison with the single-auction model. Finally, we present the main conclusions and future research directions in Section VII.

II. RELATED WORK

Several large industrial projects, such as the European Telecommunications Standards Institute (ETSI) and the Internet Engineering Task Force (IETF), propose industry standards on NFV in the form of white papers [27]. To solve the problem of data traffic, the SFC working group of IETF [14] finds a dynamic approach with a series of network functions to guide the physical or virtualized data traffic. The resource scheduling in this paper follows the framework and assumptions in these white papers.

Gember et al. [28] propose programming a network-aware orchestration layer called Stratos to deploy middleboxes in the cloud for a virtual middlebox appliance. Stratos’s process consists of three phases: determining the number of VNFs in each type, deciding a better position for each type of VNF in the cloud, and guiding the data traffic through service chains. To solve the placement problem of VNFs, Ming Xia et al. [29], [13] find a heuristic algorithm that can be efficiently operated with binary integer programming (BIP). Moreover, in their study, it is possible to minimize the cost of optical-electronic-optical (O-E-O) conversions by using NFV chains in optical data centers.
Prior works mainly focus on the VNF deployment problem from the perspective of resource allocation. In fact, social welfare and resource market are the other mechanisms for providing good service performance to competing customers, and auctions have been regarded as a primary method.

Bari et al. [30] present two methods to solve the VNF orchestration problem (VNF-OP). The first method is an integer linear programming (ILP) formulation with an implementation in IBM ILOG CPLEX Optimization Studio (CPLEX) for small-scale networks. Second, for large-scale networks, they also propose a heuristic algorithm based on dynamic programming. Double-auction methods are widely used in distributing resources between competing customers and resource suppliers. In [31], the authors propose an intelligent resource allocation mechanism based on building a double combinatorial auction model based on a reputation system to avoid malicious behavior. They also present a price decision mechanism based on backpropagation (BP) neural network to make decisions scientifically. In [32], the authors introduce a novel resource allocation mechanism for three types of participants: providers, tenants and end users.

To clearly minimize the costs of capital and operation, SDN is first proposed based on the wireless virtualization architecture, which can solve multiple flow transmission problems when there are multiple infrastructure providers and multiple mobile virtual network operators [33]. Moreover, by using a virtual resource allocation algorithm, the authors also solve an optimization problem of social welfare, improving the quality of service (QoS) requirements while reducing transaction costs. Dong et al. [34] find a caching scheme named SRCMN to enhance the network performance under constrained conditions in an SDN-enabled network. In [22], the authors first present an efficient and truthful auction method to distribute resources dynamically and to price the unit of transaction. To connect atomic network functions and provide integrated services, they define NFV service chains in a data center.

VNF orchestration and capital expense problems can be solved as an auction model, and the double-auction model is also an effective way to solve resource allocation problems. In [35], the authors present a novel double-auction scheme to protect the privacy of electric vehicles (EVs) and meet the requirement of demand response. In the double-auction market, the auctioneer matches buyers to sellers to achieve the maximum social welfare. As an auctioneer, the cloud protects the privacy between bidders and the auctioneer. Rather than the traditional double-auction methods, the authors [36] first adopt a game theory method to analyze the profits of the cloud provider and customers, as well as state the profit functions. Dou An et al. [37] propose a novel weakly dominant strategy-based on-line double-auction (SODA) method in the smart grid system to address the energy management issues with microgrids. The theoretical analysis proves that SODA can achieve high performance with (weak) budget balance and computational efficiency.

Fu et al. present a new type of core-selecting virtual machine (VM) combinatorial auction-based allocations [38] that can economically and efficiently calculate bidder charges from the core of the price vector space. In [39], the authors formulate the problem of allocating virtual resources as an optimization problem to maximize the total utility of the system. Then, they transform the transaction cost problem into an iterative double-auction problem. In this process, the bidding prices are changed by the iterative computation according to their own utility until the deal is closed. To solve the problems of SFC positioning and pricing, Zhang et al. [23] propose a novel auction mechanism in which the NFV provider owns resource information and customers can bid stochastically online. This mechanism significantly enhances the performance of existing techniques, while both sellers and buyers occupy or supply the VNF service chain in a limited time. M. Nazif Fajrty et al. [40] introduce a general double-auction scheme to solve the energy distribution problem among competing buyers, sellers and agents in a microgrid. They create a suitable projection objective function to maximize the total welfare of participants, while the agents can sell or procure energy with free bids in a selfish manner. By formulating a double-auction mechanism, our main objective is to maximize the profit of the SFC broker as an auctioneer in the NFV market. In the auction mechanism, we also strive to guarantee the profits of customers and resource suppliers.

### III. Preliminaries

In this paper, we consider a centralized SFC broker who collects resources from distributed service suppliers to obtain maximum overall profit. As shown in Figure 1, customers request a certain ordered chain of VNFs; meanwhile, more than one seller supplies resources for the required service chain. Therefore, the customers are capable of choosing the price for their requirements, while the SFC broker also desires a higher profit. The SFC broker collects all the available service chains and supplies them to the customers. Accordingly, we can formulate a double-auction problem in this market.

We present our basic definitions and assumptions to describe the formulation of the auction problem. We assume that there are two or more service suppliers who can provide some objects in a single cloud. Let \( I \) denote the set of sellers, numbered \( 1, 2, \ldots, j, \ldots, J \), and let \( \mathcal{J} = \{1, \ldots, J\} \). Sellers face \( I \) buyers or potential buyers, numbered \( 1, 2, \ldots, i, \ldots, I \). Let \( \mathcal{F} \) represent the set of buyers, and \( \mathcal{F} = \{1, \ldots, I\} \). We use \( i \) to denote typical buyers in \( \mathcal{F} \). In the auction, we assume that all sellers do not know the bidding and asking prices of other buyers and sellers. We provide a precise definition of the double-auction problem as follows.

**Definition 1:** A market consists of sellers, buyers, and a broker, while a single auction consists of an auctioneer and many buyers.

In a single-auction market, such as the English auction market, bidding prices are increasing, and each subsequent bidding price is greater than the previous one. If no buyer is willing to continue bidding, the buyer with the highest bidding price pays the bidding price and the auction ends. In a double-auction market, buyers first present their bidding price, and sellers submit their asking price to the auctioneer. Then, the auctioneer chooses the hammer price, denoted by \( p \), which is
decided by the asking price and bidding price. Finally, price $p$ must satisfy the rule that the hammer price is higher than the bidding price and less than the asking price. Thus, we consider a double-auction approach for scheduling resources in the NFV market. We assume that the double-auction model in our paper is the truthful auction model based on IC.

**Definition 2:** An auction mechanism is IC if the dominant strategy for all customers is to reveal its true valuation, regardless of other buyers’ bidding [41].

Let $b_i$ denote the bidding price of a given buyer except buyer $i$. We use $P(b_i) : R \rightarrow [0, 1]$ to denote the cumulative distribution function corresponding to the density $f_i(\cdot)$. Hence,  

$$P(b_i) = \int_{-\infty}^{b_i} f_i(s_i)ds_i. \quad (1)$$

**Theorem 1:** The equivalent condition of the truthful auction market is

1. The probability $P(b_i)$ of buyer $i$ with price $b_i$ is monotonically nondecreasing in $b_i$;
2. The charge $p_i$ of buyer $i$ is equal to  

$$p_i = b_iP(b_i) - \int_0^{b_i} P_{-i}(b)db, \quad (2)$$

where $P(b_i)$ is the probability that buyer $i$ obtains the instance and $P_{-i}(b)$ is the winning probability of a given buyer except $b_i$ [42], [43].

**Definition 3:** A strategy for player $i \in \mathcal{I}$ is a map $s_i : \mathcal{B} \rightarrow S_i$, where $S_i$ denotes an action for each player $i$, and $\mathcal{B}$ denotes the set of bidding prices, and $S_i$ is the strategy sequence set [44].

**Theorem 2:** The strategy function $s(\cdot)$ is a Bayesian Nash equilibrium (BNE) if, for all $i \in \mathcal{I}$ and for all $b_i \in \mathcal{B}$, we have the following equation:  

$$s_i(b_i) \in \arg\max_{s_i \in \mathcal{S}_i} \sum_{b_{-i}} f(b_{-i}|b_i)u_i(s_i, s_{-i}(b_{-i}), b_i, b_{-i}), \quad (3)$$

where $u_i$ is the utilization function of buyer $i$ [45].

**IV. DOUBLE-AUCTION MODEL**

In this section, we describe the double-auction model for the NFV market. Participants in the market are customers, resource suppliers, and the SFC broker. Customers have one or more independent tasks for execution, and resource suppliers have available resources. The SFC broker possesses geodistributed information of SFCs and distributes and sells SFCs based on the demands of the customer.

We formulate the SFC distribution in the NFV network with a double-auction market to satisfy IC. There are $K$ types of SFCs, such as routing, firewall, and storage. There are three assumptions in our model, as follows:

1. Each seller has enough SFCs for all buyers’ requests in the truthful NFV auction market and sells the same SFC to different buyers.
2. The SFC broker only places those SFCs from sellers that satisfy requests of buyers. Therefore, in the auction process, the constraints of SFCs, such as delay or service capability, are always satisfied.
3. Buyers can purchase all required SFCs from the NFV market after the auction process. Meanwhile, each buyer only obtains one SFC from one seller.

Our model is formulated depending on Definition 2 and Theorem 1. Let $b_i^k$ and $a_i^k$ denote the bidding price and the asking price for the $k$-th SFC, respectively. Every buyer $i$ has a private valuation denoted by $d_i^k$ and a hammer price denoted by $p_i^k$ for the $k$-th SFC that satisfy  

$$b_i^k \leq p_i^k \leq d_i^k. \quad (4)$$

The profit of buyer $i$ for $k$-th SFC is $u_i^k = d_i^k - p_i^k$; thus, the profit of buyer $i$ is $u_i = \sum_{k=1}^{K} u_i^k$. Therefore, the total benefit of buyers is  

$$U_{Buyer} = \sum_{i=1}^{I} \sum_{k=1}^{K} (d_i^k - p_i^k). \quad (5)$$

Seller $j$ has a cost price denoted by $c_j^k$ and a hammer price denoted by $p_j^k$ for the $k$-th SFC that satisfy  

$$c_j^k \leq p_j^k \leq a_j^k. \quad (6)$$

The profit of seller $j$ for $k$-th SFC is $u_j^k = p_j^k - c_j^k$; thus, the profit of seller $j$ is $u_j = \sum_{k=1}^{K} u_j^k$. Therefore, the total benefit of sellers is  

$$U_{Seller} = \sum_{j=1}^{J} \sum_{k=1}^{K} (p_j^k - c_j^k). \quad (7)$$

Consider the case in which buyer $i$ submits their first bidding price $b_i^k$ for the $k$-th SFC and seller $j$ simultaneously submits their first asking price $d_j^k$ for the $k$-th SFC. If $b_i^k \geq a_j^k$, buyer $i$ and seller $j$ make a deal for the $k$-th SFC with price $p_{ij}^k$ decided by the SFC broker in the range of $[a_j^k, b_i^k]$. If $b_i^k < a_j^k$, buyer $i$ and seller $j$ have to adjust the initial price. Buyer $i$ has to increase their bidding price $b_i^k$, and seller $j$ has to bring the price down until the deal ends, as shown in Figure 2(a) and Figure 2(b), respectively. The rate of price adjustment in Figure 2(a) changes more slowly than the rate in Figure 2(b). Thus, in these two price adjustments, the range of hammer price decided by the SFC broker is different. Clearly, the fast price adjustment is better for the broker to select the appropriate hammer price from the larger range. Therefore, we choose a fast price adjustment function from the normal distribution as  

$$a_j^k(n + 1) = a_j^k(n) (1 + \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-(t-\mu)^2} dt), \quad (8)$$

$$\hat{b}_j^k(n + 1) = b_j^k(n) (1 - \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-(t-\mu)^2} dt), \quad (9)$$

where $a_j^k$ and $\hat{b}_j^k$ are the prices after the price adjustments, respectively. The number of auction rounds is denoted as $n$. Let $\mu$ denote a constant parameter, and when $t = \mu$, we can obtain the maximum number of trades. With a suitable parameter, we can achieve a tradeoff between the time of the trade and the profits of the three participants in the market.

Let $r_{ij}^k$ denote whether buyer $i$ and seller $j$ make a deal ($r_{ij}^k = 1$) or do not make a deal ($r_{ij}^k = 0$). The profit of the
The range of hammer price
(b) Fast price adjustment

Fig. 2. Price adjustments in double auction

SFC broker from buyer $i$ and seller $j$ for the $k-$th SFC is $(b_i^k - a_j^k)r_{ij}^k$. Thus, the profit of the SFC broker is as follows:

$$U_{NFV} = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} (b_i^k - a_j^k)r_{ij}^k.$$  \hspace{1cm} (10)

Let $\gamma$ denote the service charge per auction round to prevent the price adjustment of buyers or sellers from having an endless loop. When we acquire an appropriate service charge, buyers and sellers can only choose to adjust the original price to avoid overpaying for $\gamma$. Therefore, our objective is to maximize the profits of the three participants in this paper as follows:

$$\max \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} (b_i^k - a_j^k)r_{ij}^k + 2N\gamma,$$  \hspace{1cm} (11)

$$\max \sum_{k=1}^{K} \sum_{i=1}^{I} (d_i^k - p_{ij}^k)r_{ij}^k,$$  \hspace{1cm} (12)

$$\max \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} r_{ij}^k,$$  \hspace{1cm} (13)

where $N$ is the total number of auction rounds.

Equation (12) is to maximize the profit of buyers, and Equation (13) is to maximize the trading ratio for sellers. From Equations (5) and (7), we can obtain the profits of buyers and sellers as

$$U_{Buyer} = \sum_{i=1}^{I} \sum_{j=1}^{J} (d_i^k - p_{ij}^k)r_{ij}^k - N\gamma, \hspace{1cm} (14)$$

$$U_{Seller} = \sum_{i=1}^{I} \sum_{j=1}^{J} (p_{ij}^k - c_j^k)r_{ij}^k - N\gamma. \hspace{1cm} (15)$$

To satisfy all requests from buyers, we assume that at least one buyer provides a bidding price that is higher than the minimum asking price, given by

$$\max_i b_i^k \geq \min_j a_j^k. \hspace{1cm} (16)$$

Finally, the profit maximization problem in the double-auction model is given by

$$\max \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} (b_i^k - a_j^k)r_{ij}^k + 2N\gamma,$$  \hspace{1cm} (17.1)

$$\max \sum_{k=1}^{K} \sum_{j=1}^{J} (d_i^k - p_{ij}^k)r_{ij}^k, \hspace{1cm} \forall j \in \mathcal{J} (17.2)$$

$$\sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} r_{ij}^k. \hspace{1cm} (17.3)$$
s.t. \[
\sum_{j=1}^{K} \sum_{k=1}^{J} (d_i^k - p_{ij})r_{ij}^k - N\gamma \geq 0, \quad \forall i \in \mathcal{I}; \quad (17a)
\]
\[
\sum_{i=1}^{I} \sum_{k=1}^{K} (p_{ij}^k - c_{ij}^k)r_{ij}^k - N\gamma \geq 0, \quad \forall j \in \mathcal{J}; \quad (17b)
\]
\[
r_{ij}^k \in \{0, 1\}, \quad \forall i, j \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}; \quad (17c)
\]
\[
b_{ij}^k \leq p_{ij}^k \leq d_{ij}^k, \quad \forall i, j \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}; \quad (17d)
\]
\[
c_{ij}^k \leq p_{ij}^k \leq a_{ij}^k, \quad \forall i, j \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}. \quad (17e)
\]

V. ALGORITHM

In this section, we present three algorithms to optimize the profit in data center networks by adjusting the bidding price and asking price. In our model, customers require SFCs with corresponding VNFS, while service suppliers want the maximum profit. This problem is considered as a noncooperative game, which is proven to be a weakly dominant strategy in our algorithms. In Algorithm 1, we present the details of the double-auction process. Algorithm 2 is the process of price adjustment, which can output a feasible solution for guaranteeing the profits of every buyer and seller. Algorithm 3 illustrates the process of searching the real value of SFCs, and it is able to calculate the appropriate price to attract customers. We also prove that three algorithms guarantee the performance of the auction process.

A. DARA Algorithm for ILP Problem (17)

We propose the DARA auction algorithm to solve the ILP problem (17). First, we prove the following theorem that states that there is no polynomial-time dynamic algorithm for the ILP problem (17).

Theorem 3: The SFC broker profit maximization problem, as shown in the ILP problem (17), is NP-hard.

Proof: An example of an NP-hard problems is the $0-1$ knapsack problem. As a typical optimization problem, it is proven to be an NP-hard problem, given by

\[
\max \sum_{i=1}^{n} w_i x_i
\]

s.t. \[
\sum_{i=1}^{n} w_i x_i \leq W, \quad \forall x_i \in \{0, 1\}.
\]

From the above equations, the $0-1$ knapsack problem is a special form of the ILP problem (17) with one constraint. Thus, the SFC broker profit maximization problem is an NP-hard problem.

From Theorem 3, we apply a game theory method to find the equilibrium solution as Algorithm 1. First, we initialize the demands of customers for different SFCs. As shown on Lines 3-10, the algorithm resets set $\mathcal{I}$ of buyers for different instances. Then, as the auctioneer in the market, the SFC broker distinguishes buyers who want to buy the $k$–th SFC as the while loop in Algorithm 1. In the distinguishing process, when the SFC broker finds that one buyer bids a price greater than the minimum asking price, the process of the auction will continue, and two participants will be chosen to stop the auction.

In the auction process, the customer needs to ask the SFC broker for the geographical information of VNFS to know the corresponding servers for the required SFCs. Only if the bidding price is higher than other bidding prices, the lowest asking price and the private value can the customer buy the required SFC from the NFV market. At the same time, the SFC broker records this purchase as Line 15 in Algorithm 1. First, we initialize customer $i$ in range $(I)$ do

\[
\mathcal{I} = \emptyset ;
\]

\[
\text{if } \text{customer\_demand}(i, k) \text{ is truth then}
\]

\[
\mathcal{I} = \mathcal{I} + i;
\]

\[
\text{end if}
\]

\[
\text{end for}
\]

\[
\text{while any($\mathcal{I}$) do}
\]

\[
\text{if } \max b_{ij}^k \geq \min a_{ij}^k \text{ then}
\]

\[
i^* = \arg \max b_{ij}^k;
\]

\[
j^* = \arg \min a_{ij}^k;
\]

\[
\text{price}_{i^*, j^*}^k \in [a(k, i^*), b(k, j^*)] \text{ decided by SFC broker ;}
\]

\[
r_{i^*, j^*}^k = 1;
\]

\[
customer\_price_{i^*, j^*}^k = price_{i^*, j^*}^k + r_{i^*, j^*}^k;
\]

\[
\text{if } customer\_price_{i^*, j^*}^k \neq c_{i^*, j^*}^k \text{ then}
\]

\[
seller\_price_{i^*, j^*}^k = price_{i^*, j^*}^k + (1 - charge) \times price_{i^*, j^*}^k;
\]

\[
\text{end if}
\]

\[
\text{end while}
\]

\[
\text{The sellers and buyers adjusting price within their limits;}
\]

\[
\text{end if}
\]

\[
\text{end while}
\]

\[
\text{Algorithm 1 DARA auction algorithm for ILP problem (17)
Input: The numbers of buyers, sellers and SFCs; the demand of buyers; the set of SFCs for sellers.
Output: The total profit of SFC.
1: for $k$ in range($K$) do
2: $\mathcal{I} = \emptyset ;$
3: for $i$ in range $(I)$ do
4: if customer\_demand($i, k$) is truth then
5: $\mathcal{I} = \mathcal{I} + i;$
6: else
7: $\mathcal{I} = \mathcal{I} + 0;$
8: end if
9: end for
10: while any($\mathcal{I}$) do
11: if max $b_{ij}^k \geq \min a_{ij}^k$ then
12: $i^* = \arg \max b_{ij}^k;$
13: $j^* = \arg \min a_{ij}^k;$
14: $price_{i^*, j^*}^k \in [a(k, i^*), b(k, j^*)] \text{ decided by SFC broker ;}$
15: $r_{i^*, j^*}^k = 1;$
16: $customer\_price_{i^*, j^*}^k = price_{i^*, j^*}^k + r_{i^*, j^*}^k;$
17: if customer\_price_{i^*, j^*}^k $\neq c_{i^*, j^*}^k$ then
18: $seller\_price_{i^*, j^*}^k = price_{i^*, j^*}^k + (1 - charge) \times price_{i^*, j^*}^k;$
19: else
20: $seller\_price_{i^*, j^*}^k = price_{i^*, j^*}^k + r_{i^*, j^*}^k;$
21: end if
22: Profit of SFC broker is $(b_{ij}^k - a_{ij}^k) \times r_{ij}^k + (1 - charge) \times price_{ij}^k;$
23: $\mathcal{I} = \mathcal{I} / i;$
24: else
25: The sellers and buyers adjusting price within their limits;
26: end if
27: end while
28: end for
\]
follow Equations (8) and (9), and we need to prove that there must be an intersection point between adjusting the functions of bidding and selling prices. From Equation (8), we know that

\[
\frac{a_k(n+1)}{a_k(n)} = 1 + \int_{-\infty}^{n} \frac{1}{\sqrt{2\pi}} e^{-(t-\mu)^2} dt, \tag{19}
\]

where \(\int_{-\infty}^{n} \frac{1}{\sqrt{2\pi}} e^{-(t-\mu)^2} dt\) is the cumulative function of the Gaussian distribution, which is always greater than 0. Therefore, the price adjusting function of sellers is monotonically decreasing. Similarly, the adjusting function of buyers is monotonically increasing. Because of the assumption in Equation (16), we conclude that there must be an intersection point between the price adjusting functions for bidding and selling.

Therefore, the total profit function of buyers without charge, given by

\[
U_{buyer} = \sum_{j=1}^{J} \sum_{k=1}^{K} (d_{ij}^k - p_{ij}^k)r_{ij}, \tag{20}
\]

is strictly greater than 0. Thus, there is at least one \(\epsilon > 0\) satisfying \(U_{buyer} \geq \epsilon\). If \(\epsilon\) does not exist, then \(U_{buyer} < \epsilon\) is true for all \(\epsilon\). When we choose \(\epsilon\) given by

\[
\epsilon = \frac{\sum_{j=1}^{J} \sum_{k=1}^{K} (d_{ij}^k - p_{ij}^k)r_{ij}}{2}, \tag{21}
\]

we can find the contradiction as

\[
\sum_{j=1}^{J} \sum_{k=1}^{K} (d_{ij}^k - p_{ij}^k)r_{ij} < 0. \tag{22}
\]

Therefore, there is at least one value to satisfy Equation (17a), which is a charge for buyers in our model. For the same reason, we also find the charge for sellers from Equation (17b).

Then, we prove that our algorithm can find a feasible solution in polynomial time.

**Theorem 5:** Algorithm 1 can find a feasible solution in polynomial time.

**Proof:** The loop from Line 1 to Line 28 in Algorithm 1 has \(K\) iterations. During the iteration, the loop from 12 to 27 has at most \(I\) rounds. Thus, the complexity of Algorithm 1 is \(O(n^2)\).

From Theorem 5, it is easy to know the time complexity of Algorithm 1 based on the double-auction is \(O(n^2)\). Compared with the time complexity \(O(n^2)\) of the single-auction-based algorithm [22], Algorithm 1 is a slightly slower than methods based on single auction in a general scale market. Therefore, the increased profit with Algorithm 1 is able to cover the additional computational cost.

**B. Price Adjustment Algorithm for Algorithm 1**

We propose a price adjustment algorithm to guarantee the profits of all participants. In Algorithm 2, Line 3 and Line 11 are the main adjustment functions from Equation (8) and Equation (9), considering the service charge \(\gamma\). Due to the significance of the normal distribution in statistics with random variables [46], we utilize the normal distribution function to improve the flexibility of the price adjustment. If the bidding price exceeds the sum of the private valuation and the service charge, the SFC broker will terminate the transaction. Similarly, if the asking price exceeds the difference between the cost price and the service charge, the transaction will be terminated.

We use an example to illustrate Algorithm 2 for a better understanding. The bidding prices of buyers are presented in Table II, where “−” denotes that the buyer has no request for the corresponding resource.

![Table II](image)

**TABLE II**

**REQUESTS OF BUYERS**

<table>
<thead>
<tr>
<th>Resource 1</th>
<th>Buyer 1</th>
<th>Buyer 2</th>
<th>Buyer 3</th>
<th>Buyer 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>13295</td>
<td></td>
<td>12874</td>
<td>-</td>
<td>13671</td>
</tr>
<tr>
<td>Resource 2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Resource 3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>13366</td>
</tr>
<tr>
<td>Resource 4</td>
<td>13072</td>
<td>-</td>
<td>12617</td>
<td>-</td>
</tr>
</tbody>
</table>

**TABLE III**

**RESOURCES OF SELLERS**

<table>
<thead>
<tr>
<th>(a_k)</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resource 1</td>
<td>19106</td>
<td>15845</td>
</tr>
<tr>
<td>Resource 2</td>
<td>15756</td>
<td>18682</td>
</tr>
<tr>
<td>Resource 3</td>
<td>19668</td>
<td>-</td>
</tr>
<tr>
<td>Resource 4</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Similarly, the asking prices are shown in Table III, where the container \{1, 2, 0, 4\} for resource 1 means that the buyers numbered \{1, 2, 4\} need to buy resource 1. Because the bidding prices are less than the lowest asking price, buyers and sellers have to enter the price adjustment process. Then, the buyers adjust the original prices of resource 1, as shown in Table IV. Similarly, the asking price for resource 1 is shown in Table V. Thus, buyer 4 and resource supplier 2 reach an
agreement for resource 1.

Then, we prove the correctness of the price adjustment strategy.

**Theorem 6:** For customers and resource suppliers, the price adjustment strategy is the weakly dominant strategy.

**Proof:** In this game, the SFC broker has a simultaneous price adjustment strategy that includes cooperation (C) and defection (D) for buyers and sellers. Cooperation means that the player decides to accept the given strategy, and defection means that the given strategy is rejected. If all buyers and sellers choose cooperation, they both earn profits with probabilities of \( P(C_{buyer}|C_{seller}) \) and \( P(C_{seller}|C_{buyer}) \), respectively.

If buyers and sellers reject the strategy, the probabilities of the profits for buyers and sellers are given by \( P(D_{buyer}|D_{seller}) \) and \( P(D_{seller}|D_{buyer}) \), respectively. If one participant accepts the strategy while the other rejects the strategy, then the probability that the cooperative participant can obtain the profit is given by \( P(C_{buyer}|D_{seller}) \) or \( P(C_{seller}|D_{buyer}) \). Similarly, the probability that the defective participant can obtain the profit is \( P(D_{buyer}|C_{seller}) \) or \( P(D_{seller}|C_{buyer}) \).

According to the prisoner’s dilemma game, we need to prove the following aspects:

(a) The probability that a buyer accepts the pricing adjustment strategy is always higher than the probability of rejection, given by

\[
P(C_{buyer}|D_{seller}) \geq P(C_{buyer}|C_{seller}) \\
\geq P(D_{buyer}|D_{seller}) \geq P(D_{buyer}|C_{seller}).
\]  

(b) The probability that a seller accepts the pricing adjustment strategy is always higher than the probability of rejection, given by

\[
P(C_{seller}|D_{buyer}) \geq P(C_{seller}|C_{buyer}) \\
\geq P(D_{seller}|D_{buyer}) \geq P(D_{seller}|C_{buyer}).
\]

(c) The probability that a cooperative buyer can obtain the profit with a cooperative seller or a cooperative seller with a noncooperative buyer is higher than the probability of a noncooperative buyer with a cooperative seller or a cooperative seller with a noncooperative buyer, given by

\[
P(C_{buyer}|C_{seller}) + P(C_{seller}|C_{buyer}) \\
\geq P(C_{buyer}|D_{seller}) + P(C_{seller}|D_{buyer}).
\]

Let \( A = \{C_{buyer}, D_{seller}\} \) and \( B = \{C_{seller}, D_{seller}\} \) denote the sets of strategies for the buyer and seller, respectively. From Bayes formula [47], we know that

\[
P(C_{buyer}|D_{seller}) = \frac{P(C_{buyer})P(D_{seller}|C_{buyer})}{P(D_{seller})},
\]

where

\[
P(D_{seller}) = P(C_{buyer})P(D_{seller}|C_{buyer}) \\
+ P(D_{buyer})P(D_{seller}|D_{buyer}).
\]

Accordingly, we show the probability matrix in Table VI. For all buyers and sellers, the auction success is not only based on their own prices but also others’ prices. Thus, every participant adjusts the price in the dominant strategy. We determine the equilibrium strategy of every participant based on Bayes game theory.

<table>
<thead>
<tr>
<th>Buyer, Seller</th>
<th>Cooperation (C)</th>
<th>Defection (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperation (C)</td>
<td>( P(C_{buyer}</td>
<td>C_{seller}) )</td>
</tr>
<tr>
<td></td>
<td>( P(C_{seller}</td>
<td>C_{buyer}) )</td>
</tr>
<tr>
<td>Defection (D)</td>
<td>( P(C_{buyer}</td>
<td>D_{seller}) )</td>
</tr>
<tr>
<td></td>
<td>( P(C_{seller}</td>
<td>D_{buyer}) )</td>
</tr>
</tbody>
</table>

In the model, because buyers and sellers only have a common knowledge, our model is based on a symmetric independent private value (SIPV) [48] model with a typical static Bayesian game. Hence, the probability of buyer \( i^* \) with bidding price \( b_{k^*} \) for the \( k \)-th SFC is based on price \( b_i \) or \( a_j \), \( i \neq i^*, j = 1, \ldots, I; j = 1, \ldots, J \). Generally, we need to prove items (a) and (c).

We propose the expectation profit function of buyer \( i \) as

\[
U_i = (d_i - b_i)Pr(b_i \geq b_j, j \neq i),
\]

where \( Pr(.) \) is the extreme probability.

Thus, we can calculate that the probability that a cooperative buyer can obtain the profit and the probability that a cooperative seller can obtain the profit as

\[
P(C_{buyer}) = \prod_{i=1}^{I} Pr(b_i \geq b_{i^*}, i^* \neq i) = \prod_{i=1}^{I} \frac{U_i(d_i)}{(d_i - b_i)},
\]

\[
P(C_{seller}) = \prod_{i=1}^{I} Pr(a_j \leq a_{j^*}, j^* \neq j) = \prod_{j=1}^{J} \frac{U_j(a_j)}{(c_j - a_j)}.
\]

\[
U_i(d_i) = [d_i - p_i]P^{i-1}(d_i),
\]

\[
U_j(a_j) = [p_j - a_j]P^{j-1}(a_j),
\]

where \( P^{i-1}(\cdot) \) is the probability distribution function.

Then, we compute the value of \( P(.) \) as

\[
P(d_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(d_i - a_{i^*})^2}{2\sigma^2}},
\]

where \( \sigma^2 \) is the variance of the price distribution.
Thus, Theorem 6 is proven based on Equations (37) and (36).

Accordingly, we change item (c) as follows:

\[ 2P(C_{\text{buyer}}|C_{\text{seller}}) \geq P(C_{\text{buyer}}|D_{\text{seller}}) + P(D_{\text{seller}}|C_{\text{buyer}}). \]  

(37)

Since the bidding prices of buyers are generally different, buyer \( i \) has a bidding function \( b_i = B_i(d_i) \), where \( d_i \) is the private valuation. Thus, the strategy of buyer \( i \) is \( \{B_1(d_1), B_2(d_2), ..., B_I(d_I)\} \). According to the BNE theorem, if buyer \( i \) knows that other competitors adopt strategy \( B_i^*(d_I) \in \{B_1^*(d_1), B_2^*(d_2), ..., B_I^*(d_I)\} \), \( j \neq i, j = 1, 2, ..., I \), buyer \( i \) will also adopt strategy \( B_i^*(d_I) \). We can obtain \( P(D_{\text{buyer}}|C_{\text{seller}}) = 0 \) and \( P(C_{\text{buyer}}|D_{\text{seller}}) = 1 \). Thus, Theorem 6 is proven based on Equations (37) and (36).

C. DARA Payment Strategy Algorithm

Next, we are supposed to guarantee that our method is able to obtain the accurate price of the SFC for every buyer. This algorithm focuses on the real value of the SFC. From Equation (2), we just need to find a threshold value \( b_i^* \) in our scheduling.

\[ p_i = \begin{cases} b_i^*, & b_i \geq b_i^*; \\ 0, & \text{otherwise}. \end{cases} \]  

(38)

We compute the real value based on the following Algorithm 3.

Because the buyer only makes a requirement in the truthful auction mechanism, the SFC broker should find the real value of the SFC for attracting customers. First, we compute the difference between the high value and the low value in the NFV market. If the difference is greater than a constant value \( \epsilon \), we need to halve the price and then run Algorithm 1 until we find the real value of the SFC.

VI. PERFORMANCE EVALUATION

In this section, we study the NFV service chain in the double-auction market where customers can share SFCs. Each SFC contains three types of resources: storage, routing and firewall. We generate the matrix of bidding prices and asking prices following a random distribution.

First, we compare the resource allocation performance between the DARA model and the single-auction model with different numbers of buyers. Figure 3(a) shows the profit of the SFC broker when the number of buyers is changed with different auction models. It shows the profit of the SFC broker with 100 service suppliers and 100 types of SFCs. From the results, the profit of the SFC broker is increased with more buyers. This result occurs because the profit of the SFC broker mainly comes from the buyers. The solid blue line shows the optimal solution of the profit of the SFC broker maximization problem, and the solid red line is the profit of the SFC broker calculated by the single-auction model. As shown, the profit of the SFC broker in the DARA model is always higher than the profit in the single-auction model.

In addition to the results shown in Figure 3(a), we also compare the resource allocation performance between the DARA model and the single-auction model with different numbers of SFCs. This result occurs because the sellers are not the main factor of the SFC broker profit. It also shows that the profit of the SFC broker in the DARA model is always higher than the profit in the single-auction model.

As shown in Figure 3(c), the profit of the SFC broker is increased by more VNFs. This figure shows the profit of the SFC broker in the DARA model is always higher than the profit in the single-auction model.

Figure 4(a), Figure 4(b) and Figure 4(c) show the proportionality factors between the DARA model and the single-auction model. In Figure 4(a) and Figure 4(c), the line is irregular due to the random training data, while the value of proportionality is always greater than 1. From Figure 4(b), compared to the results in the other figures, the value of proportionality is decreased with more sellers. This result
occurs because the number of sellers cannot impact the profit of the SFC broker.

We finally compare the profits of sellers and buyers between the DARA model and the single-auction model. The empirical cumulative distribution function (CDF) of the profit of sellers and buyers is shown in Figure 5. From the results shown in Figure 5(a), the proposed scheme outperforms the single-auction model. Furthermore, we observe that the probability of the profits for sellers who earn nothing in the DARA model is approximately equal to 0.1. Moreover, the proportion of the seller profit increases gradually, and approximately 90% of sellers obtain $2.5 \times 10^6$ profit. Thus, every seller can obtain some benefits in the DARA model. However, in the single-auction model, the proportion of low-income sellers accounts for up to 90% of the population. Because we assume that one seller can provide resources to a different customer, the blue line is like a “stair-step” graph. Similar to the seller profit analysis, in Figure 5(b), we also present the CDF of buyer profit to show that the profit of more than 80% buyers is less than $1 \times 10^6$ in the single-auction model, while more than 95% buyers can achieve more than $1 \times 10^6$ profit in the DARA model.

In general, we find that the DARA model performs better than the single-auction model due to the limitation of the single-auction model in which only customers can change the price. Our results show that we can achieve the main goal, which is to maximize the profits of the three participants.

VII. CONCLUSION

In this paper, we formulate an auction model that connects VNFs to maximize the profits of the three participants. We use a double-auction method called the DARA mechanism to schedule resources in the NFV market. The DARA method is effective according to the theoretical analysis and performance evaluation. Compared with the single-auction model, the DARA model increases the profits of customers and resource suppliers in NFV markets.

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REFERENCES


Fig. 5. Empirical CDFs of buyers and sellers

(a) Empirical CDF of the seller profit

(b) Empirical CDF of the buyer profit


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