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ORIGINAL ARTICLES

A ROLE OF GRANULARITY AND BACKGROUND KNOWLEDGE IN REASONING PROCESSES

Towards a Foundation of *Kansei* Representation in Human Reasoning

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Abstract: In this paper, we present a foundation of formulating human reasoning processes with *Kansei* representation based on granularity generated by background knowledge. For the objective, we examine the role of background knowledge and granularity in several kinds of reasoning. We put emphasis on difference between objective and subjective levels of knowledge. We represent subjective knowledge by lower approximation in rough set theory under granularity generated by background knowledge. Then we examine monotonicity and non-monotonicity between the two levels with respect to deduction and non-monotonic reasoning. Next we apply the idea to abduction which gives us which premise is plausible when we are given rules and conclusions. The problem in abduction is how to select one from several possible premises and we solve it by calculating inclusion degree between possible premises and the lower approximation of the given conclusion. Thus we can introduce some ordering into the set of possible premises. Finally we apply a similar idea to conflict resolution typically dealt with in reasoning in expert systems and robot control. When we have plural monotonic rules which can be applied to a given premise, if conclusions are associated with their own action or execution, then we cannot take the conjunction of all conclusions. Again we must select one of the conclusions using the inclusion degree between the premise and the lower approximations of possible conclusions. Thus we can introduce some ordering into the set of possible conclusions associated with actions.

Keywords: *Rough sets, Lower and upper approximations, Modal logic, Granularity, Adjustment of granularity, Deduction, Non-monotonic reasoning, Abduction, Conflict resolution.*

1. INTRODUCTION

The late professor Z.Pawlak[10] proposed rough set theory[10,11] in 1982. Since then, nowadays it becomes one of the most important and remarkable methodologies for imprecise and uncertain data and reasoning from data. Recently in 21st century, the concept of granular computing has newly advocated by several researchers[2,12] in rough set community and the importance of rough set theory more and more increases.

Along this line, the authors in [4-9] have applied the idea of granular computing into reasoning processes. We call the formulation granular reasoning or zooming systems. In order to apply it to human ordinary reasoning, we must introduce *Kansei* representations to reasoning processes.

In this paper, for the purpose, we examine some relationship between several kinds of reasoning processes and granularity generated by background knowledge. We introduce two levels of objective and subjective under background knowledge. In particular, we put much emphasis on the role of lower approximation, whose size depends on the granularity based on background knowledge, in several kinds of reasoning such as deduction, non-monotonic reasoning, abduction, and conflict resolution in expert systems and robot control.

Recently in Japan, *Kansei* engineering provides very interesting and important applications of rough set theory. There the concept of reducts plays an important part in such applications. We expect this paper would give *Kansei* community another aspect of rough set theory, that is, adjustment of granularity.

This paper is organized as follows. In Section 2, we give a brief description on rough set theory and modal logic. In Section 3, we examine the role of granularity generated by background knowledge in several kinds of reasoning processes such as deduction, non-monotonic reasoning, abduction, and conflict resolution. Section 4 concludes the paper.

2. GRANULAR COMPUTING

2.1 Rough Sets

The basic idea of rough set theory[10,11] is to describe unknown objects represented in some set (universe of discourse) in terms of known (usually finite) data, by which we can generate an equivalence relation. In fact, first, some clusters of elements are generated by known

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set of data. Then, unknown objects, which we want to explain, are described using such clusters just like 'building blocks.' We can regard such known set of data as background knowledge.

Formally, let U be a universe of discourse and R be an equivalence relation on U. In general, a relation on U is a subset of the direct (Cartesian) product of U:

$$R \subseteq U \times U.$$

When a pair (x,y) is in R, we write xRy. A relation R on U is said to be an equivalence relation just in case it satisfies the following three properties: for every $x, y, z \in U$,

(1) xRx (reflexivity)
(2) $xRy \Rightarrow yRx$ (symmetry)
(3) xRy and $yRz \Rightarrow xRz$ (transitivity)

where '⇒' means 'imply.'

The set $[x]_R$ defined by

$$[x]_R = \{y \in U \mid xRy\}$$

is called the equivalence class of x with respect to R. The family of all equivalence classes of each element in U with respect to R is denoted U/R, that is,

$$U/R = \{[x]_R \mid x \in U\}.$$

It is called the quotient set of U with respect to R. Equivalence classes satisfy the following properties:

(1) $xRy \Rightarrow [x]_R = [y]_R$,
(2) $\mathrm{not}(xRy) \Rightarrow [x]_R \cap [y]_R = \emptyset$.

Then the quotient set U/R gives a partition of U. Thus we can deal with equivalence classes as building blocks under background knowledge induced from relation R. In fact, we can approximate a set X (unknown object) in the two ways just illustrated in Figure 1.

One way is to make an approximation from inside using the building blocks of U/R that are contained in X:

$$\underline{R}(X) = \cup\{[x]_R \mid [x]_R \subseteq X\}.$$

It is called the lower approximation of X with respect to R. The other is to make an approximation from outside of X by deleting the building blocks that have no intersection with X. Equivalently, it comes to

$$\overline{R}(X) = \cup\{[x]_R \mid [x]_R \cap X \neq \emptyset\}.$$

It is called the upper approximation of X with respect to R. Obviously we have the following inclusion:

$$\underline{R}(X) \subseteq X \subseteq \overline{R}(X).$$

Further we use the following three terms:

(1) Positive region of X: $Pos(X) = \underline{R}(X)$
(2) Borderline region of X: $Bd(X) = \overline{R}(X) - \underline{R}(X)$
(3) Negative region of X: $Neg(X) = X - \overline{R}(X)$

The pair

$$(\underline{R}(X), \overline{R}(X))$$

is called the rough set of X with respect to R. And the pair

$$(U,R)$$

is referred as an approximation or Pawlak space.

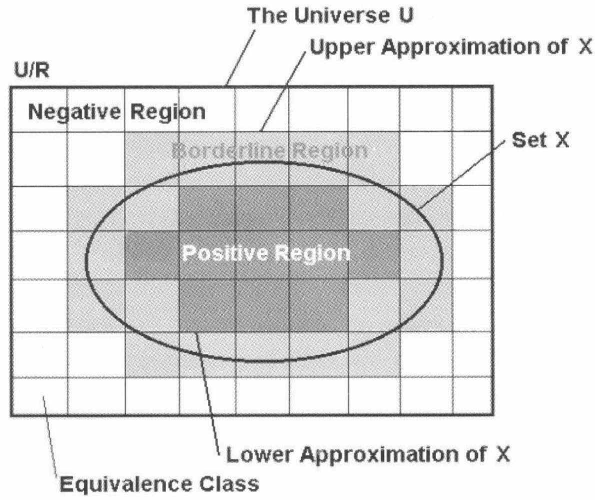

Figure 1: Two kinds of approximations

2.2 Adjustment of Granularity

Intuitively speaking, the size of building blocks depends on the granularity generated by a given approximation space or its quotient set. In Figure 2, U/R has coarser granularity that U/R' has. Thus, in general, we can understand that U/R' gives better approximation than U/R.

In order to deal with degrees of granularity in a quantitative way, several kinds of measures are introduced for the finite universe case. Among them, the following measure is called the accuracy of X with respect to R:

$$\alpha_R(X) = |\underline{R}(X)| / |\overline{R}(X)|.$$

Another well-known measure is

$$\gamma_R(X) = |\underline{R}(X)| / |X|,$$

which is called the quality of X with respect to R. By these measures we can have the degree of granularity of X under background knowledge.

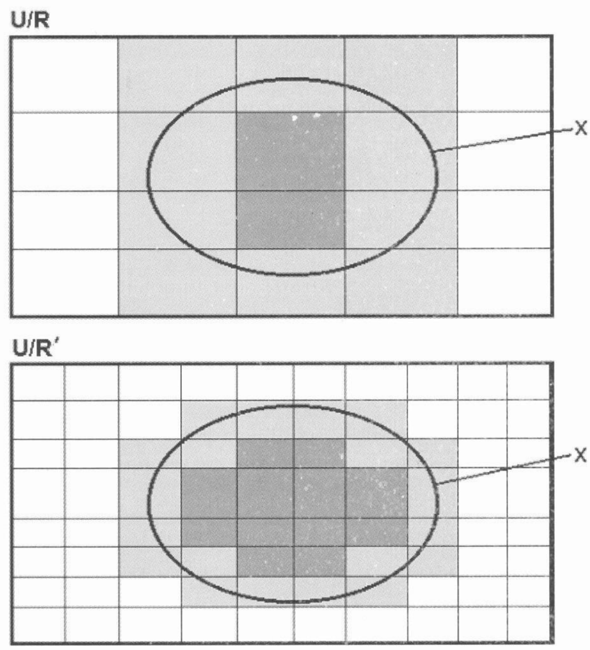

Figure 2: Adjustment of granularity.

2.3 Rough Sets and Modal Logic

Rough sets have a close relationship with modal logics[1]. In fact, approximation spaces are just modal algebras for modal system KT5 (S5).

In general, modal logic is supplemented by two modal operators $\Box$ and $\Diamond$ which are applied to one sentence. For a sentence p, standard readings of them are as follows:

(1) $\Box$p: p is necessary.

(2) $\Diamond$p: p is possible.

Depending on context, however, there are many other interpretations such as 'obligation', 'knowledge', and 'belief' for operator $\Box$.

There are several kinds of semantics for modal logic and, among them, Kripke semantics is now a most popular one for interpreting sentences in modal logics. A Kripke frame F is a pair

$$(U,R),$$

where U is a non-empty set of possible worlds and R is a relation on U. Apparently, an approximation space is just a Kripke frame when R is an equivalence relation.

A Kripke model M is a triple

$$(U,R,v),$$

where (U,R) is a frame and v is a valuation which assigns 'true' or 'false' to atomic sentences at each world x.

Let us write

$$M,x\models p$$

when a sentence p is true at a world $x \in U$ in a Kripke model M=(U,R,v). Also we use

$$M\models p,$$

which means p is true at every world x in U.

Truth conditions of modal sentences are defined by

$$M,x\models\Box p \Leftrightarrow \forall y(xRy \Rightarrow M,y\models p),$$
$$M,x\models\Diamond p \Leftrightarrow \exists y(xRy \text{ and } M,y\models p).$$

That is, p is necessary if and only if p is true at every world we can access while p is possible just in case there is at least one world at which p is true. The set of worlds at which p is true in M, denoted $\|p\|^M(\subseteq U)$, is defined by

$$\|p\|^M = \{x \in U \mid M,x\models p\},$$

which is often called the proposition of p in M.

Let us define the set of worlds which can be accessible from x in U by the following set

$$U_x=\{y \mid xRy\}.$$

We can rewrite the truth conditions of modal sentences in a simpler way:

$$M,x\models\Box p \Leftrightarrow U_x \subseteq \|p\|^M,$$
$$M,x\models\Diamond p \Leftrightarrow U_x \cap \|p\|^M \neq \emptyset.$$

When R is an equivalence relation, then U_x becomes an equivalence class $[x]_R$. Now readers can find the right-hand sides of the above formulas correspond to the definition of lower and upper approximations, respectively. Thus we have

$$\underline{R}(\|p\|^M) = \|\Box p\|^M,$$
$$\overline{R}(\|p\|^M) = \|\Diamond p\|^M.$$

In this paper, we adopt the following notation if no confusion arises: for a sentence p (in small letter), we use its capital letter P for the proposition of p:

$$P = \|p\|^M.$$

In particular, we use

$$\Box P=\|\Box p\|^M,$$
$$\Diamond P=\|\Diamond p\|^M$$

for modal sentences.

Finally in this section, we sketch proof theory of modal logic KT5. KT5 is axiomatized by the following special rule and axiom schemas for modal sentences

RN. $p \Rightarrow \Box p$ (From p infer $\Box$p),

Def$\Diamond$. $\Diamond p \leftrightarrow \neg\Box\neg p$,

K. $\Box(p\rightarrow q)\rightarrow(\Box p\rightarrow\Box q)$,

T. $\Box p\rightarrow p$,

5. $\Diamond p\rightarrow\Box\Diamond p$,

as well as the usual rule and axioms of propositional logic. KT5 is proved to be both sound and complete with respect to the class of Kripke models with equivalence relations (cf.[1]).

3. GRANULARITY GENERATED BY BACKGROUND KNOWLEDGE IN REASONING PROCESSES

In this section, we examine several kinds of reasoning under granularity generated from background knowledge.

3.1 Objective and Subjective Levels of Knowledge

First we note objective and subjective levels of knowledge.

When a fact p is given, its proposition P is just the maximum set of accessible worlds. In ordinary reasoning, however, we cannot enumerate the total of them when carrying out reasoning processes. In general, we could imagine some proper subset of P at most.

One possible way of specifying such subset is that we can consider some relevant worlds under background knowledge to be the lower approximation $\Box$P of P. This is based on the idea that background knowledge formulates its own context with some granularity, in which our way of observing worlds is determined.

Size of lower approximation $\Box$P depends on granularity generated by background knowledge. P is in objective level while $\Box$P is in subjective level. There are several kinds of meaning of $\Box$P in each context such as a set of 'essential' or 'typical' elements

3.2 Deduction

Logical reasoning in the usual sense, that is, deduction does not consider background knowledge. A typical rule of inference is well-known modus ponens:

$$p, p \to q \Rightarrow q \text{ (From p and } p \to q \text{ infer q).}$$

This means that we can obtain a conclusion q from a fact p and a rule p→q. We examine modus ponens in the framework of possible world semantics.

Let M=(U,R,v) be a Kripke model. Then, in M, rule p→q is represented as set inclusion between propositions:

$$M \models p \to q \Leftrightarrow P \subseteq Q.$$

Then the rule means the following procedure:

(1) Fact p restricts the set of possible worlds that we can access under p.

(2) Then, by rule p→q, we can find that conclusion q is true at every world in the restricted set. That is, q is necessary under the fact p.

Thus the rule implies the monotonicity of deduction. In fact we have

$$p \to q \Rightarrow \Box p \to \Box q$$

which holds in every Kriple models. We can rewrite it in a propositional level as follows:

$$P \subseteq Q \Rightarrow \Box P \subseteq \Box Q.$$

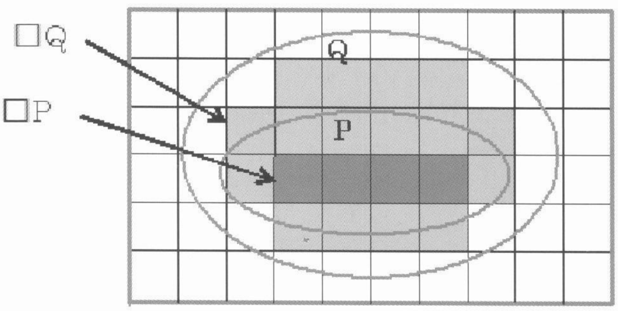

Figure 3: Deduction.

3.3 Non-monotonic Reasoning

Non-monotonic reasoning is one of the most typical kind of ordinary reasoning. The Tweety example is well-known:

(1) Tweety is a bird.
Most birds fly.
Then he flies.

(2) Tweety is a penguin.
Penguins do not fly.
Then he does not fly.

Thus the conclusion in (1) is withdrawn in (2).

Thus the set of conclusions in non-monotonic reasoning no longer increases in a monotonic manner and in this sense the above kind of reasoning is said to be non-monotonic.

As stated in Section 3.2, the usual monotonic reasoning satisfies the monotonicity

$$P \subseteq Q \Rightarrow \Box P \subseteq \Box Q,$$

while in non-monotonic reasoning in general,

$$P \not\subseteq Q, \text{ but } \Box P \subseteq \Box Q.$$

In the Tweety case, let BIRD and FLYING be the set of birds and flying objects, respectively. Then in the objective level,

$$\text{BIRD} \not\subseteq \text{FLYING}$$

but in the subjective level, the inclusion

$$\Box\text{BIRD} \subseteq \Box\text{FLYING}$$

holds. (see Figure 4)

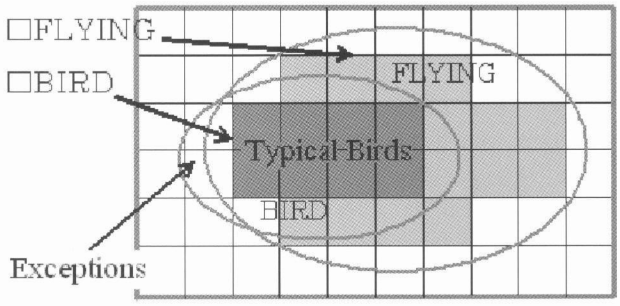

Figure 4: Non-monotonic reasoning.

3.4 Abduction

Abduction has the form of reasoning

$$q, p \to q \Rightarrow p,$$

thus apparently it is not valid because, in general, there are many sentences which imply q. Peirce, however, argued that abduction plays a very important role in scientific discovery in the 19th century.

Also abduction is important in human plausible reasoning, which is not necessary correct, like fortune-telling. When possible candidates of sentences which imply q are given, we can give an order between the candidates using lower approximation of Q.

For example, in Figure 5, we have three candidates, that is, we have three possible implications:

$$p_1 \to q,$$
$$p_2 \to q,$$
$$p_3 \to q.$$

Now when q is given, we must select one of them.

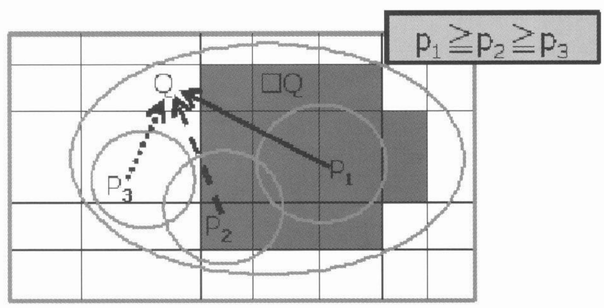

Figure 5: Abduction

For the purpose, let us consider the inclusion measure between P_i and □Q defined by

$$Inc(P_i, \Box Q) = |P_i \cap \Box Q| / |P_i|.$$

Then we can calculate

$$0 = Inc(P_3, \Box Q) < Inc(P_2, \Box Q) < Inc(P_1, \Box Q) = 1,$$

hence we can introduce the following ordering,

$$p_1 \geq p_2 \geq p_3.$$

Therefore we can first choose p_1 as possible premise of abduction.

Note that, in the example, the following implications with modality hold:

$$p_1 \rightarrow \Box q,$$
$$p_2 \rightarrow q,$$
$$p_3 \rightarrow (q \wedge \neg \Box q).$$

We can see the difference of strength of each possible premise for the same conclusion q.

3.5 Conflict Resolution

The similar idea in abduction can be applied to conflict resolution in expert systems. We are given, for example, three monotonic rules:

$$p \rightarrow q_1,$$
$$p \rightarrow q_2,$$
$$p \rightarrow q_3.$$

Then, in a usual logical framework, we have conclusion

$$q_1 \wedge q_2 \wedge q_3.$$

In many application areas, however, like expert systems and robot control, each conclusion is associated with action or execution, thus more than two conclusions cannot be carried out. Hence we must choose one of them.

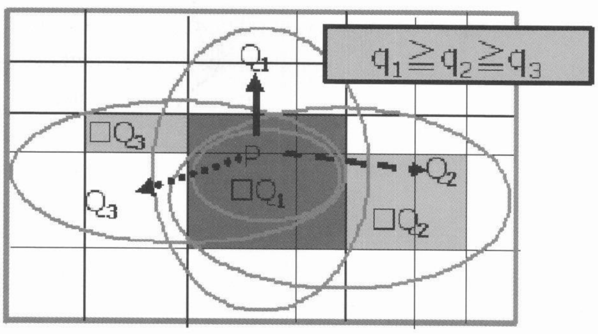

Figure 6: Conflict resolution.

Again let us consider the inclusion measure between P and $\Box Q_i$ defined by

$$Inc(P, \Box Q_i) = |P \cap \Box Q_i| / |P|.$$

Then we can calculate

$$0 = Inc(P, \Box Q_3) < Inc(P, \Box Q_2) v < Inc(P, \Box Q_1) = 1$$

hence we can introduce the following ordering,

$$q_1 \geq q_2 \geq q_3.$$

Therefore it is plausible that we first select q_1 as possible conclusion with execution.

Note that, in the example, the following implications with modality hold:

$$p \rightarrow \Box q_1,$$
$$p \rightarrow q_2,$$
$$p \rightarrow (q_3 \wedge \neg \Box q_3).$$

We can see the difference of strength of each possible conclusion for the same premise p.

4. CONCLUDING REMARKS

In this paper, we have argued an important role of background knowledge and granularity in several kinds of reasoning such as (monotonic) deduction, non-monotonic reasoning, abduction, and conflict resolution. Thereby we have a foundation of applying *Kansei* representation into reasoning. Adjustment of granularity with topology could provide us another important characteristic of rough set theory in *Kansei* engineering as well as reducts. Because rough set theory has a close relationship with topological spaces[3] and adjustment of granularity can be regarded as homomorphism between topological spaces, we may introduce several useful concepts in topological spaces into *Kansei* engineering via rough set theory. A future task is to implement a *Kansei* reasoning system and its application to recommendation and image retrieval systems.

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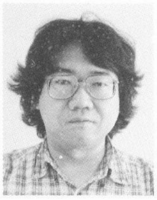

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