A Table of the Coefficients of the Interpolation Formula of Steffensen

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Abstract

In this report the present writer gives a table of the numerical values for the coefficients of the interpolation formula of Steffensen. The values are computed to ten signicant figures and odd differences are also included.

The interpolation formula of Steffensen is one of the most useful formulas for interpolation. Representing by $y(x)$ the function to be tabulated, by x the argument, and by h the tabular interval, the formula may be written as follows:[1]

$$y(x+ph) = y(x) - \bar{S}_1(p)\delta_{-\frac{1}{2}} + S_1(p)\delta_{\frac{1}{2}} - \bar{S}_2(p)\delta^2_{-\frac{1}{2}} + S_2(p)\delta^2_{\frac{1}{2}} - \bar{S}_3(p)\delta^5_{-\frac{1}{2}} + S_3(p)\delta^5_{\frac{1}{2}} - \cdots,$$

where $\bar{S}_n(p) = \binom{-p+n}{2n}$ and $S_n(p) = \binom{p+n}{2n}$ $(n = 1, 2, 3 \cdots)$. This formula employs odd differences only according to the scheme:

$x-h$	y_{-1}				
		$\delta_{-\frac{1}{2}}$	$\delta^3_{-\frac{1}{2}}$	$\delta^5_{-\frac{1}{2}}$	$\cdots$
x	y_0				
		$\delta_{\frac{1}{2}}$	$\delta^3_{\frac{1}{2}}$	$\delta^5_{\frac{1}{2}}$	$\cdots$
$x+h$	y_1				

So far as the writer remembers, the numerical values for these coefficients seem to have not been given as yet notwithstanding the practical value of this formula. The object of the present report is to give the numerical values of these coefficients. A table of these values are given on the following pages.

This table contains the numerical values over the range form $p = 0.00$ to $p = 1.00$ by intervals of 0.01 for the first six coefficients $\bar{S}_n(p)$, $S_n(p)$ $(n = 1, 2, 3)$. The values of the first four coefficients $\bar{S}_n(p)$, $S_n(p)$ $(n = 1, 2)$ are exact, and those of $\bar{S}_3(p)$ and $S_3(p)$ are computed to ten significant figures with the possible errors less than half one unit in the last place. For convenience in the application of this formula, difference-table is also included in the modified form:

x	y_0	$\delta_{-\frac{1}{2}}$	$\delta^3_{-\frac{1}{2}}$	$\delta^5_{-\frac{1}{2}}$ $\cdots$
$x+h$	y_1	$\delta_{\frac{1}{2}}$	$\delta^3_{\frac{1}{2}}$	$\delta^5_{\frac{1}{2}}$ $\cdots$

(1) See for example, L.M. Milne-Thomson: The Calculus of Finite Differences, (1933), p. 74; K. Hayashi: Interpolation and Numerical Computation (in Japanese) (1943), pp. 47–48.

p	$\bar{S}_1$	δ	S_1	δ	p
0.00	0.00000	−0.00505	0.00000	0.00495	0.00
0.01	−0.00495	−0.00495	0.00505	0.00505	0.01
0.02	−0.00980	−0.00485	0.01020	0.00515	0.02
0.03	−0.01455	−0.00475	0.01545	0.00525	0.03
0.04	−0.01920	−0.00465	0.02080	0.00535	0.04
0.05	−0.02375	−0.00455	0.02625	0.00545	0.05
0.06	−0.02820	−0.00445	0.03180	0.00555	0.06
0.07	−0.03255	−0.00435	0.03745	0.00565	0.07
0.08	−0.03680	−0.00425	0.04320	0.00575	0.08
0.09	−0.04095	−0.00415	0.04905	0.00585	0.09
0.10	−0.04500	−0.00405	0.05500	0.00595	0.10
0.11	−0.04895	−0.00395	0.06105	0.00605	0.11
0.12	−0.05280	−0.00385	0.06720	0.00615	0.12
0.13	−0.05655	−0.00375	0.07345	0.00625	0.13
0.14	−0.06020	−0.00365	0.07980	0.00635	0.14
0.15	−0.06375	−0.00355	0.08625	0.00645	0.15
0.16	−0.06720	−0.00345	0.09280	0.00655	0.16
0.17	−0.07055	−0.00335	0.09945	0.00665	0.17
0.18	−0.07380	−0.00325	0.10620	0.00675	0.18
0.19	−0.07695	−0.00315	0.11305	0.00685	0.19
0.20	−0.08000	−0.00305	0.12000	0.00695	0.20
0.21	−0.08295	−0.00295	0.12705	0.00705	0.21
0.22	−0.08580	−0.00285	0.13420	0.00715	0.22
0.23	−0.08855	−0.00275	0.14145	0.00725	0.23
0.24	−0.09120	−0.00265	0.14880	0.00735	0.24
0.25	−0.09375	−0.00255	0.15625	0.00745	0.25
0.26	−0.09620	−0.00245	0.16380	0.00755	0.26
0.27	−0.09855	−0.00235	0.17145	0.00765	0.27
0.28	−0.10080	−0.00225	0.17920	0.00775	0.28
0.29	−0.10295	−0.00215	0.18705	0.00785	0.29
0.30	−0.10500	−0.00205	0.19500	0.00795	0.30
0.31	−0.10695	−0.00195	0.20305	0.00805	0.31
0.32	−0.10880	−0.00185	0.21120	0.00815	0.32
0.33	−0.11055	−0.00175	0.21945	0.00825	0.33
0.34	−0.11220	−0.00165	0.22780	0.00835	0.34
0.35	−0.11375	−0.00155	0.23625	0.00845	0.35
0.36	−0.11520	−0.00145	0.24480	0.00855	0.36
0.37	−0.11655	−0.00135	0.25345	0.00865	0.37
0.38	−0.11780	−0.00125	0.26220	0.00875	0.38
0.39	−0.11895	−0.00115	0.27105	0.00885	0.39
0.40	−0.12000	−0.00105	0.28000	0.00895	0.40
0.41	−0.12095	−0.00095	0.28905	0.00905	0.41
0.42	−0.12180	−0.00085	0.29820	0.00915	0.42
0.43	−0.12255	−0.00075	0.30745	0.00925	0.43
0.44	−0.12320	−0.00065	0.31680	0.00935	0.44
0.45	−0.12375	−0.00055	0.32625	0.00945	0.45
0.46	−0.12420	−0.00045	0.33580	0.00955	0.46
0.47	−0.12455	−0.00035	0.34545	0.00965	0.47
0.48	−0.12480	−0.00025	0.35520	0.00975	0.48
0.49	−0.12495	−0.00015	0.36505	0.00985	0.49
0.50	−0.12500	−0.00005	0.37500	0.00995	0.50

p	$\bar{S}_1$	δ	S_1	δ	p
0.50	−0.12500	−0.00005	0.37500	0.00995	0.50
0.51	−0.12495	0.00005	0.38505	0.01005	0.51
0.52	−0.12480	0.00015	0.39520	0.01015	0.52
0.53	−0.12455	0.00025	0.40545	0.01025	0.53
0.54	−0.12420	0.00035	0.41580	0.01035	0.54
0.55	−0.12375	0.00045	0.42625	0.01045	0.55
0.56	−0.12320	0.00055	0.43680	0.01055	0.56
0.57	−0.12255	0.00065	0.44745	0.01065	0.57
0.58	−0.12180	0.00075	0.45820	0.01075	0.58
0.59	−0.12095	0.00085	0.46905	0.01085	0.59
0.60	−0.12000	0.00095	0.48000	0.01095	0.60
0.61	−0.11895	0.00105	0.49105	0.01105	0.61
0.62	−0.11780	0.00115	0.50220	0.01115	0.62
0.63	−0.11655	0.00125	0.51345	0.01125	0.63
0.64	−0.11520	0.00135	0.52480	0.01135	0.64
0.65	−0.11375	0.00145	0.53625	0.01145	0.65
0.66	−0.11220	0.00155	0.54780	0.01155	0.66
0.67	−0.11055	0.00165	0.55945	0.01165	0.67
0.68	−0.10880	0.00175	0.57120	0.01175	0.68
0.69	−0.10695	0.00185	0.58305	0.01185	0.69
0.70	−0.10500	0.00195	0.59500	0.01195	0.70
0.71	−0.10295	0.00205	0.60705	0.01205	0.71
0.72	−0.10080	0.00215	0.61920	0.01215	0.72
0.73	−0.09855	0.00225	0.63145	0.01225	0.73
0.74	−0.09620	0.00235	0.64380	0.01235	0.74
0.75	−0.09375	0.00245	0.65625	0.01245	0.75
0.76	−0.09120	0.00255	0.66880	0.01255	0.76
0.77	−0.08855	0.00265	0.68145	0.01265	0.77
0.78	−0.08580	0.00275	0.69420	0.01275	0.78
0.79	−0.08295	0.00285	0.70705	0.01285	0.79
0.80	−0.08000	0.00295	0.72000	0.01295	0.80
0.81	−0.07695	0.00305	0.73305	0.01305	0.81
0.82	−0.07380	0.00315	0.74620	0.01315	0.82
0.83	−0.07055	0.00325	0.75945	0.01325	0.83
0.84	−0.06720	0.00335	0.77280	0.01335	0.84
0.85	−0.06375	0.00345	0.78625	0.01345	0.85
0.86	−0.06020	0.00355	0.79980	0.01355	0.86
0.87	−0.05655	0.00365	0.81345	0.01365	0.87
0.88	−0.05280	0.00375	0.82720	0.01375	0.88
0.89	−0.04895	0.00385	0.84105	0.01385	0.89
0.90	−0.04500	0.00395	0.85500	0.01395	0.90
0.91	−0.04095	0.00405	0.86905	0.01405	0.91
0.92	−0.03680	0.00415	0.88320	0.01415	0.92
0.93	−0.03255	0.00425	0.89745	0.01425	0.93
0.94	−0.02820	0.00435	0.91180	0.01435	0.94
0.95	−0.02375	0.00445	0.92625	0.01445	0.95
0.96	−0.01920	0.00455	0.94080	0.01455	0.96
0.97	−0.01455	0.00465	0.95545	0.01465	0.97
0.98	−0.00980	0.00475	0.97020	0.01475	0.98
0.99	−0.00495	0.00485	0.98505	0.01485	0.99
1.00	0.00000	0.00495	1.00000	0.01495	1.00

p	$\bar{S}_2$	δ	δ^3	p
0.00	0.00000 00000 0	0.00083 74162 5	−5050 0	0.00
0.01	0.00082 90837 5	0.00082 90837 5	−4950 0	0.01
0.02	0.00164 93400 0	0.00082 02562 5	−4850 0	0.02
0.03	0.00246 02837 5	0.00081 09437 5	−4750 0	0.03
0.04	0.00326 14400 0	0.00080 11562 5	−4650 0	0.04
0.05	0.00405 23437 5	0.00079 09037 5	−4550 0	0.05
0.06	0.00483 25400 0	0.00078 01962 5	−4450 0	0.06
0.07	0.00560 15837 5	0.00076 90437 5	−4350 0	0.07
0.08	0.00635 90400 0	0.00075 74562 5	−4250 0	0.08
0.09	0.00710 44837 5	0.00074 54437 5	−4150 0	0.09
0.10	0.00783 75000 0	0.00073 30162 5	−4050 0	0.10
0.11	0.00855 76837 5	0.00072 01837 5	−3950 0	0.11
0.12	0.00926 46400 0	0.00070 69562 5	−3850 0	0.12
0.13	0.00995 79837 5	0.00069 33437 5	−3750 0	0.13
0.14	0.01063 73400 0	0.00067 93562 5	−3650 0	0.14
0.15	0.01130 23437 5	0.00066 50037 5	−3550 0	0.15
0.16	0.01195 26400 0	0.00065 02962 5	−3450 0	0.16
0.17	0.01258 78837 5	0.00063 52437 5	−3350 0	0.17
0.18	0.01320 77400 0	0.00061 98562 5	−3250 0	0.18
0.19	0.01381 18837 5	0.00060 41437 5	−3150 0	0.19
0.20	0.01440 00000 0	0.00058 81162 5	−3050 0	0.20
0.21	0.01497 17837 5	0.00057 17837 5	−2950 0	0.21
0.22	0.01552 69400 0	0.00055 51562 5	−2850 0	0.22
0.23	0.01606 51837 5	0.00053 82437 5	−2750 0	0.23
0.24	0.01658 62400 0	0.00052 10562 5	−2650 0	0.24
0.25	0.01708 98437 5	0.00050 36037 5	−2550 0	0.25
0.26	0.01757 57400 0	0.00048 58962 5	−2450 0	0.26
0.27	0.01804 36837 5	0.00046 79437 5	−2350 0	0.27
0.28	0.01849 34400 0	0.00044 97562 5	−2250 0	0.28
0.29	0.01892 47837 5	0.00043 13437 5	−2150 0	0.29
0.30	0.01933 75000 0	0.00041 27162 5	−2050 0	0.30
0.31	0.01973 13837 5	0.00039 38837 5	−1950 0	0.31
0.32	0.02010 62400 0	0.00037 48562 5	−1850 0	0.32
0.33	0.02046 18837 5	0.00035 56437 5	−1750 0	0.33
0.34	0.02079 81400 0	0.00033 62562 5	−1650 0	0.34
0.35	0.02111 48437 5	0.00031 67037 5	−1550 0	0.35
0.36	0.02141 18400 0	0.00029 69962 5	−1450 0	0.36
0.37	0.02168 89837 5	0.00027 71437 5	−1350 0	0.37
0.38	0.02194 61400 0	0.00025 71562 5	−1250 0	0.38
0.39	0.02218 31837 5	0.00023 70437 5	−1150 0	0.39
0.40	0.02240 00000 0	0.00021 68162 5	−1050 0	0.40
0.41	0.02259 64837 5	0.00019 64837 5	− 950 0	0.41
0.42	0.02277 25400 0	0.00017 60562 5	− 850 0	0.42
0.43	0.02292 80837 5	0.00015 55437 5	− 750 0	0.43
0.44	0.02306 30400 0	0.00013 49562 5	− 650 0	0.44
0.45	0.02317 73437 5	0.00011 43037 5	− 550 0	0.45
0.46	0.02327 09400 0	0.00009 35962 5	− 450 0	0.46
0.47	0.02334 37837 5	0.00007 28437 5	− 350 0	0.47
0.48	0.02339 58400 0	0.00005 20562 5	− 250 0	0.48
0.49	0.02342 70837 5	0.00003 12437 5	− 150 0	0.49
0.50	0.02343 75000 0	0.00001 04162 5	− 50 0	0.50

p	$\mathfrak{S}_2$	δ	δ^3	p
0.50	0.02343 75000 0	0.00001 04162 5	− 50 0	0.50
0.51	0.02342 70837 5	−0.00001 04162 5	50 0	0.51
0.52	0.02339 58400 0	−0.00003 12437 5	150 0	0.52
0.53	0.02334 37837 5	−0.00005 20562 5	250 0	0.53
0.54	0.02327 09400 0	−0.00007 28437 5	350 0	0.54
0.55	0.02317 73437 5	−0.00009 35962 5	450 0	0.55
0.56	0.02306 30400 0	−0.00011 43037 5	550 0	0.56
0.57	0.02292 80837 5	−0.00013 49562 5	650 0	0.57
0.58	0.02277 25400 0	−0.00015 55437 5	750 0	0.58
0.59	0.02259 64837 5	−0.00017 60562 5	850 0	0.59
0.60	0.02240 00000 0	−0.00019 64837 5	950 0	0.60
0.61	0.02218 31837 5	−0.00021 68162 5	1050 0	0.61
0.62	0.02194 61400 0	−0.00023 70437 5	1150 0	0.62
0.63	0.02168 89837 5	−0.00025 71562 5	1250 0	0.63
0.64	0.02141 18400 0	−0.00027 71437 5	1350 0	0.64
0.65	0.02111 48437 5	−0.00029 69962 5	1450 0	0.65
0.66	0.02079 81400 0	−0.00031 67037 5	1550 0	0.66
0.67	0.02046 18837 5	−0.00033 62562 5	1650 0	0.67
0.68	0.02010 62400 0	−0.00035 56437 5	1750 0	0.68
0.69	0.01973 13837 5	−0.00037 48562 5	1850 0	0.69
0.70	0.01933 75000 0	−0.00039 38837 5	1950 0	0.70
0.71	0.01892 47837 5	−0.00041 27162 5	2050 0	0.71
0.72	0.01849 34400 0	−0.00043 13437 5	2150 0	0.72
0.73	0.01804 36837 5	−0.00044 97562 5	2250 0	0.73
0.74	0.01757 57400 0	−0.00046 79437 5	2350 0	0.74
0.75	0.01708 98437 5	−0.00048 58962 5	2450 0	0.75
0.76	0.01658 62400 0	−0.00050 36037 5	2550 0	0.76
0.77	0.01606 51837 5	−0.00052 10562 5	2650 0	0.77
0.78	0.01552 69400 0	−0.00053 82437 5	2750 0	0.78
0.79	0.01497 17837 5	−0.00055 51562 5	2850 0	0.79
0.80	0.01440 00000 0	−0.00057 17837 5	2950 0	0.80
0.81	0.01381 18837 5	−0.00058 81162 5	3050 0	0.81
0.82	0.01320 77400 0	−0.00060 41437 5	3150 0	0.82
0.83	0.01258 78837 5	−0.00061 98562 5	3250 0	0.83
0.84	0.01195 26400 0	−0.00063 52437 5	3350 0	0.84
0.85	0.01130 23437 5	−0.00065 02962 5	3450 0	0.85
0.86	0.01063 73400 0	−0.00066 50037 5	3550 0	0.86
0.87	0.00995 79837 5	−0.00067 93562 5	3650 0	0.87
0.88	0.00926 46400 0	−0.00069 33437 5	3750 0	0.88
0.89	0.00855 76837 5	−0.00070 69562 5	3850 0	0.89
0.90	0.00783 75000 0	−0.00072 01837 5	3950 0	0.90
0.91	0.00710 44837 5	−0.00073 30162 5	4050 0	0.91
0.92	0.00635 90400 0	−0.00074 54437 5	4150 0	0.92
0.93	0.00560 15837 5	−0.00075 74562 5	4250 0	0.93
0.94	0.00483 25400 0	−0.00076 90437 5	4350 0	0.94
0.95	0.00405 23437 5	−0.00078 01962 5	4450 0	0.95
0.96	0.00326 14400 0	−0.00079 09037 5	4550 0	0.96
0.97	0.00246 02837 5	−0.00080 11562 5	4650 0	0.97
0.98	0.00164 93400 0	−0.00081 09437 5	4750 0	0.98
0.99	0.00082 90837 5	−0.00082 02562 5	4850 0	0.99
1.00	0.00000 00000 0	−.0.00082 90837 5	4950 0	1.00

p	S_2	δ	δ^3	p
0.00	0.00000 00000 0	−0.00082 90837 5	4950 0	0.00
0.01	−0.00083 74162 5	−0.00083 74162 5	5050 0	0.01
0.02	−0.00168 26600 0	−0.00084 52437 5	5150 0	0.02
0.03	−0.00253 52162 5	−0.00085 25562 5	5250 0	0.03
0.04	−0.00339 45600 0	−0.00085 93437 5	5350 0	0.04
0.05	−0.00426 01562 5	−0.00086 55962 5	5450 0	0.05
0.06	−0.00513 14600 0	−0.00087 13037 5	5550 0	0.06
0.07	−0.00600 79162 5	−0.00087 64562 5	5650 0	0.07
0.08	−0.00688 89600 0	−0.00088 10437 5	5750 0	0.08
0.09	−0.00777 40162 5	−0.00088 50562 5	5850 0	0.09
0.10	−0.00866 25000 0	−0.00088 84837 5	5950 0	0.10
0.11	−0.00955 38162 5	−0.00089 13162 5	6050 0	0.11
0.12	−0.01044 73600 0	−0.00089 35437 5	6150 0	0.12
0.13	−0.01134 25162 5	−0.00089 51562 5	6250 0	0.13
0.14	−0.01223 86600 0	−0.00089 61437 5	6350 0	0.14
0.15	−0.01313 51562 5	−0.00089 64962 5	6450 0	0.15
0.16	−0.01403 13600 0	−0.00089 62037 5	6550 0	0.16
0.17	−0.01492 66162 5	−0.00089 52562 5	6650 0	0.17
0.18	−0.01582 02600 0	−0.00089 36437 5	6750 0	0.18
0.19	−0.01671 16162 5	−0.00089 13562 5	6850 0	0.19
0.20	−0.01760 00000 0	−0.00088 83837 5	6950 0	0.20
0.21	−0.01848 47162 5	−0.00088 47162 5	7050 0	0.21
0.22	−0.01936 50600 0	−0.00088 03437 5	7150 0	0.22
0.23	−0.02024 03162 5	−0.00087 52562 5	7250 0	0.23
0.24	−0.02110 97600 0	−0.00086 94437 5	7350 0	0.24
0.25	−0.02197 26562 5	−0.00086 28962 5	7450 0	0.25
0.26	−0.02282 82600 0	−0.00085 56037 5	7550 0	0.26
0.27	−0.02367 58162 5	−0.00084 75562 5	7650 0	0.27
0.28	−0.02451 45600 0	−0.00083 87437 5	7750 0	0.28
0.29	−0.02534 37162 5	−0.00082 91562 5	7850 0	0.29
0.30	−0.02616 25000 0	−0.00081 87837 5	7950 0	0.30
0.31	−0.02697 01162 5	−0.00080 76162 5	8050 0	0.31
0.32	−0.02776 57600 0	−0.00079 56437 5	8150 0	0.32
0.33	−0.02854 86162 5	−0.00078 28562 5	8250 0	0.33
0.34	−0.02931 78600 0	−0.00076 92437 5	8350 0	0.34
0.35	−0.03007 26562 5	−0.00075 47962 5	8450 0	0.35
0.36	−0.03081 21600 0	−0.00073 95037 5	8550 0	0.36
0.37	−0.03153 55162 5	−0.00072 33562 5	8650 0	0.37
0.38	−0.03224 18600 0	−0.00070 63437 5	8750 0	0.38
0.39	−0.03293 03162 5	−0.00068 84562 5	8850 0	0.39
0.40	−0.03360 00000 0	−0.00066 96837 5	8950 0	0.40
0.41	−0.03425 00162 5	−0.00065 00162 5	9050 0	0.41
0.42	−0.03487 94600 0	−0.00062 94437 5	9150 0	0.42
0.43	−0.03548 74162 5	−0.00060 79562 5	9250 0	0.43
0.44	−0.03607 29600 0	−0.00058 55437 5	9350 0	0.44
0.45	−0.03663 51562 5	−0.00056 21962 5	9450 0	0.45
0.46	−0.03717 30600 0	−0.00053 79037 5	9550 0	0.46
0.47	−0.03768 57162 5	−0.00051 26562 5	9650 0	0.47
0.48	−0.03817 21600 0	−0.00048 64437 5	9750 0	0.48
0.49	−0.03863 14162 5	−0.00045 92562 5	9850 0	0.49
0.50	−0.03906 25000 0	−0.00043 10837 5	9950 0	0.50

p	S_2	δ	δ^3	p
0.50	−0.03906 25000 0	−0.00043 10837 5	9950 0	0.50
0.51	−0.03946 44162 5	−0.00040 19162 5	10050 0	0.51
0.52	−0.03983 61600 0	−0.00037 17437 5	10150 0	0.52
0.53	−0.04017 67162 5	−0.00034 05562 5	10250 0	0.53
0.54	−0.04048 50600 0	−0.00030 83437 5	10350 0	0.54
0.55	−0.04076 01562 5	−0.00027 50962 5	10450 0	0.55
0.56	−0.04100 09600 0	−0.00024 08037 5	10550 0	0.56
0.57	−0.04120 64162 5	−0.00020 54562 5	10650 0	0.57
0.58	−0.04137 54600 0	−0.00016 90437 5	10750 0	0.58
0.59	−0.04150 70162 5	−0.00013 15562 5	10850 0	0.59
0.60	−0.04160 00000 0	−0.00009 29837 5	10950 0	0.60
0.61	−0.04165 33162 5	−0.00005 33162 5	11050 0	0.61
0.62	−0.04166 58600 0	−0.00001 25437 5	11150 0	0.62
0.63	−0.04163 65162 5	0.00002 93437 5	11250 0	0.63
0.64	−0.04156 41600 0	0.00007 23562 5	11350 0	0.64
0.65	−0.04144 76562 5	0.00011 65037 5	11450 0	0.65
0.66	−0.04128 58600 0	0.00016 17962 5	11550 0	0.66
0.67	−0.04107 76162 5	0.00020 82437 5	11650 0	0.67
0.68	−0.04082 17600 0	0.00025 58562 5	11750 0	0.68
0.69	−0.04051 71162 5	0.00030 46437 5	11850 0	0.69
0.70	−0.04016 25000 0	0.00035 46162 5	11950 0	0.70
0.71	−0.03975 67162 5	0.00040 57837 5	12050 0	0.71
0.72	−0.03929 85600 0	0.00045 81562 5	12150 0	0.72
0.73	−0.03878 68162 5	0.00051 17437 5	12250 0	0.73
0.74	−0.03822 02600 0	0.00056 65562 5	12350 0	0.74
0.75	−0.03759 76562 5	0.00062 26037 5	12450 0	0.75
0.76	−0.03691 77600 0	0.00067 98962 5	12550 0	0.76
0.77	−0.03617 93162 5	0.00073 84437 5	12650 0	0.77
0.78	−0.03538 10600 0	0.00079 82562 5	12750 0	0.78
0.79	−0.03452 17162 5	0.00085 93437 5	12850 0	0.79
0.80	−0.03360 00000 0	0.00092 17162 5	12950 0	0.80
0.81	−0.03261 46162 5	0.00098 53837 5	13050 0	0.81
0.82	−0.03156 42600 0	0.00105 03562 5	13150 0	0.82
0.83	−0.03044 76162 5	0.00111 66437 5	13250 0	0.83
0.84	−0.02926 33600 0	0.00118 42562 5	13350 0	0.84
0.85	−0.02801 01562 5	0.00125 32037 5	13450 0	0.85
0.86	−0.02668 66600 0	0.00132 34962 5	13550 0	0.86
0.87	−0.02529 15162 5	0.00139 51437 5	13650 0	0.87
0.88	−0.02382 33600 0	0.00146 81562 5	13750 0	0.88
0.89	−0.02228 08162 5	0.00154 25437 5	13850 0	0.89
0.90	−0.02066 25000 0	0.00161 83162 5	13950 0	0.90
0.91	−0.01896 70162 5	0.00169 54837 5	14050 0	0.91
0.92	−0.01719 29600 0	0.00177 40562 5	14150 0	0.92
0.93	−0.01533 89162 5	0.00185 40437 5	14250 0	0.93
0.94	−0.01340 34600 0	0.00193 54562 5	14350 0	0.94
0.95	−0.01138 51562 5	0.00201 83037 5	14450 0	0.95
0.96	−0.00928 25600 0	0.00210 25962 5	14550 0	0.96
0.97	−0.00709 42162 5	0.00218 83437 5	14650 0	0.97
0.98	−0.00481 86600 0	0.00227 55562 5	14750 0	0.98
0.99	−0.00245 44162 5	0.00236 42437 5	14850 0	0.99
1.00	0.00000 00000 0	0.00245 44162 5	14950 0	1.00

p	$\bar{S}_3$	δ	δ^3	δ^5	p
0.00	0.00000 00000 00	− 0.00016 72013 20	1258 21	−51	0.00
0.01	− 0.00016 60903 48	− 0.00016 60903 48	1241 55	−51	0.01
0.02	− 0.00033 09455 69	− 0.00016 48552 21	1224 38	−48	0.02
0.03	− 0.00049 44432 25	− 0.00016 34976 56	1206 73	−46	0.03
0.04	− 0.00065 64626 13	− 0.00016 20194 18	1188 62	−47	0.04
0.05	− 0.00081 68849 61	− 0.00016 04223 18	1170 04	−48	0.05
0.06	− 0.00097 55931 75	− 0.00015 87082 14	1150 98	−40	0.06
0.07	− 0.00113 24721 87	− 0.00015 68790 12	1131 52	−48	0.07
0.08	− 0.00128 74088 45	− 0.00015 49366 58	1111 58	−39	0.08
0.09	− 0.00144 02919 91	− 0.00015 28831 46	1091 25	−45	0.09
0.10	− 0.00159 10125 00	− 0.00015 07205 09	1070 47	−36	0.10
0.11	− 0.00173 94633 25	− 0.00014 84508 25	1049 33	−44	0.11
0.12	− 0.00188 55395 33	− 0.00014 60762 08	1027 75	−35	0.12
0.13	− 0.00202 91383 49	− 0.00014 35988 16	1005 82	−41	0.13
0.14	− 0.00217 01591 91	− 0.00014 10208 42	983 48	−32	0.14
0.15	− 0.00230 85037 11	− 0.00013 83445 20	960 82	−40	0.15
0.16	− 0.00244 40758 27	− 0.00013 55721 16	937 76	−32	0.16
0.17	− 0.00257 67817 63	− 0.00013 27059 36	914 38	−33	0.17
0.18	− 0.00270 65300 81	− 0.00012 97483 18	890 67	−34	0.18
0.19	− 0.00283 32317 14	− 0.00012 67016 33	866 62	−32	0.19
0.20	− 0.00295 68000 00	− 0.00012 35682 86	842 25	−29	0.20
0.21	− 0.00307 71507 14	− 0.00012 03507 14	817 59	−29	0.21
0.22	− 0.00319 42020 97	− 0.00011 70513 83	792 64	−30	0.22
0.23	− 0.00330 78748 85	− 0.00011 36727 88	767 39	−28	0.23
0.24	− 0.00341 80923 39	− 0.00011 02174 54	741 86	−25	0.24
0.25	− 0.00352 47802 73	− 0.00010 66879 34	716 08	−25	0.25
0.26	− 0.00362 78670 79	− 0.00010 30868 06	690 05	−27	0.26
0.27	− 0.00372 72837 52	− 0.00009 94166 73	663 75	−20	0.27
0.28	− 0.00382 29639 17	− 0.00009 56801 65	637 25	−27	0.28
0.29	− 0.00391 48438 49	− 0.00009 18799 32	610 48	−17	0.29
0.30	− 0.00400 28625 00	− 0.00008 80186 51	583 54	−24	0.30
0.31	− 0.00408 69615 16	− 0.00008 40990 16	556 36	−16	0.31
0.32	− 0.00416 70852 61	− 0.00008 01237 45	529 02	−23	0.32
0.33	− 0.00424 31808 33	− 0.00007 60955 72	501 45	−13	0.33
0.34	− 0.00431 51980 87	− 0.00007 20172 54	473 75	−20	0.34
0.35	− 0.00438 30896 48	− 0.00006 78915 61	445 85	−13	0.35
0.36	− 0.00444 68109 31	− 0.00006 37212 83	417 82	−15	0.36
0.37	− 0.00450 63201 54	− 0.00005 95092 23	389 64	−15	0.37
0.38	− 0.00456 15783 53	− 0.00005 52581 99	361 31	−12	0.38
0.39	− 0.00461 25493 97	− 0.00005 09710 44	332 86	−10	0.39
0.40	− 0.00465 92000 00	− 0.00004 66506 03	304 31	−11	0.40
0.41	− 0.00470 14997 31	− 0.00004 22997 31	275 65	−11	0.41
0.42	− 0.00473 94210 25	− 0.00003 79212 94	246 88	− 8	0.42
0.43	− 0.00477 29391 94	− 0.00003 35181 69	218 03	− 6	0.43
0.44	− 0.00480 20324 35	− 0.00002 90932 41	189 12	− 7	0.44
0.45	− 0.00482 66818 36	− 0.00002 46494 01	160 14	− 8	0.45
0.46	− 0.00484 68713 83	− 0.00002 01895 47	131 08	0	0.46
0.47	− 0.00486 25879 68	− 0.00001 57165 85	102 02	− 8	0.47
0.48	− 0.00487 38213 89	− 0.00001 12334 21	72 88	1	0.48
0.49	− 0.00488 05643 58	− 0.00000 67429 69	43 75	− 5	0.49
0.50	− 0.00488 28125 00	− 0.00000 22481 42	14 57	4	0.50

p	$\bar{S}_3$	δ	δ^3	δ^5	p
0.50	− 0.00488 28125 00	− 0.00000 22481 42	14 57	4	0.50
0.51	− 0.00488 05643 58	0.00000 22481 42	− 14 57	− 4	0.51
0.52	− 0.00487 38213 89	0.00000 67429 69	− 43 75	5	0.52
0.53	− 0.00486 25879 68	0.00001 12334 21	− 72 88	− 1	0.53
0.54	− 0.00484 68713 83	0.00001 57165 85	− 102 02	8	0.54
0.55	− 0.00482 66818 36	0.00002 01895 47	− 131 08	0	0.55
0.56	− 0.00480 20324 35	0.00002 46494 01	− 160 14	8	0.56
0.57	− 0.00477 29391 94	0.00002 90932 41	− 189 12	7	0.57
0.58	− 0.00473 94210 25	0.00003 35181 69	− 218 03	6	0.58
0.59	− 0.00470 14997 31	0.00003 79212 94	− 246 88	8	0.59
0.60	− 0.00465 92000 00	0.00004 22997 31	− 275 65	11	0.60
0.61	− 0.00461 25493 97	0.00004 66406 03	− 304 31	11	0.61
0.62	− 0.00456 15783 53	0.00005 09710 44	− 332 86	10	0.62
0.63	− 0.00450 63201 54	0.00005 52581 99	− 361 31	12	0.63
0.64	− 0.00444 68109 31	0.00005 95092 23	− 389 64	15	0.64
0.65	− 0.00438 30896 48	0.00006 37212 83	− 417 82	15	0.65
0.66	− 0.00431 51980 87	0.00006 78915 61	− 445 85	13	0.66
0.67	− 0.00424 31808 33	0.00007 20172 54	− 473 75	20	0.67
0.68	− 0.00416 70852 61	0.00007 60955 72	− 501 45	13	0.68
0.69	− 0.00408 69615 16	0.00008 01237 45	− 529 02	23	0.69
0.70	− 0.00400 28625 00	0.00008 40990 16	− 556 36	16	0.70
0.71	− 0.00391 48438 49	0.00008 80186 51	− 583 54	24	0.71
0.72	− 0.00382 29639 17	0.00009 18799 32	− 610 48	17	0.72
0.73	− 0.00372 72837 52	0.00009 56801 65	− 637 25	27	0.73
0.74	− 0.00362 78670 79	0.00009 94166 73	− 663 75	20	0.74
0.75	− 0.00352 47802 73	0.00010 30868 06	− 690 05	27	0.75
0.76	− 0.00341 80923 39	0.00010 66879 34	− 716 08	25	0.76
0.77	− 0.00330 78748 85	0.00011 02174 54	− 741 86	25	0.77
0.78	− 0.00319 42020 97	0.00011 36727 88	− 767 39	28	0.78
0.79	− 0.00307 71507 14	0.00011 70513 83	− 792 64	30	0.79
0.80	− 0.00295 68000 00	0.00012 03507 14	− 817 59	29	0.80
0.81	− 0.00283 32317 14	0.00012 35682 86	− 842 25	29	0.81
0.82	− 0.00270 65300 81	0.00012 67016 33	− 866 62	32	0.82
0.83	− 0.00257 67817 63	0.00012 97483 18	− 890 67	34	0.83
0.84	− 0.00244 40758 27	0.00013 27059 36	− 914 38	33	0.84
0.85	− 0.00230 85037 11	0.00013 55721 16	− 937 76	32	0.85
0.86	− 0.00217 01591 91	0.00013 83445 20	− 960 82	40	0.86
0.87	− 0.00202 91383 49	0.00014 10208 42	− 983 48	32	0.87
0.88	− 0.00188 55395 33	0.00014 35988 16	−1005 82	41	0.88
0.89	− 0.00173 94633 25	0.00014 60762 08	−1027 75	35	0.89
0.90	− 0.00159 10125 00	0.00014 84508 25	−1049 33	44	0.90
0.91	− 0.00144 02919 91	0.00015 07205 09	−1070 47	36	0.91
0.92	− 0.00128 74088 45	0.00015 28831 46	−1091 25	45	0.92
0.93	− 0.00113 24721 87	0.00015 49366 58	−1111 58	39	0.93
0.94	− 0.00097 55931 75	0.00015 68790 12	−1131 52	48	0.94
0.95	− 0.00081 68849 61	0.00015 87082 14	−1150 98	40	0.95
0.96	− 0.00065 64626 43	0.00016 04223 18	−1170 04	48	0.96
0.97	− 0.00049 44432 25	0.00016 20194 18	−1188 62	47	0.97
0.98	− 0.00033 09455 69	0.00016 34976 56	−1206 73	46	0.98
0.99	− 0.00016 60903 48	0.00016 48552 21	−1224 38	48	0.99
1.00	0.00000 00000 00	0.00016 60903 48	−1241 55	51	1.00

p	S_3	δ	δ^3	δ^5	p
0.00	0.00000 00000 00	0.00016 60903 48	−1241 55	51	0.00
0.01	0.00016 72013 20	0.00016 72013 20	−1258 21	51	0.01
0.02	0.00033 53877 91	0.00016 81864 71	−1274 36	50	0.02
0.03	0.00050 44319 77	0.00016 90441 86	−1290 01	52	0.03
0.04	0.00067 42048 77	0.00016 97729 00	−1305 14	55	0.04
0.05	0.00084 45759 77	0.00017 03711 00	−1319 72	55	0.05
0.06	0.00101 54133 05	0.00017 08373 28	−1333 75	53	0.06
0.07	0.00118 65834 86	0.00017 11701 81	−1347 25	60	0.07
0.08	0.00135 79517 95	0.00017 13683 09	−1360 15	53	0.08
0.09	0.00152 93822 17	0.00017 14304 22	−1372 52	63	0.09
0.10	0.00170 07375 00	0.00017 13552 83	−1384 26	56	0.10
0.11	0.00187 18792 18	0.00017 11417 18	−1395 44	64	0.11
0.12	0.00204 26678 27	0.00017 07886 09	−1405 98	57	0.12
0.13	0.00221 29627 29	0.00017 02949 02	−1415 95	67	0.13
0.14	0.00238 26223 29	0.00016 96596 00	−1425 25	60	0.14
0.15	0.00255 15041 02	0.00016 88817 73	−1433 95	67	0.15
0.16	0.00271 94646 53	0.00016 79605 51	−1441 98	65	0.16
0.17	0.00288 63597 84	0.00016 68951 31	−1449 36	65	0.17
0.18	0.00305 20445 59	0.00016 56847 75	−1456 09	68	0.18
0.19	0.00321 63733 69	0.00016 43288 10	−1462 14	70	0.19
0.20	0.00337 92000 00	0.00016 28266 31	−1467 49	69	0.20
0.21	0.00354 03777 03	0.00016 11777 03	−1472 15	69	0.21
0.22	0.00369 97592 63	0.00015 93815 60	−1476 12	72	0.22
0.23	0.00385 71970 68	0.00015 74378 05	−1479 37	74	0.23
0.24	0.00401 25431 81	0.00015 53461 13	−1481 88	73	0.24
0.25	0.00416 56494 14	0.00015 31062 33	−1483 66	72	0.25
0.26	0.00431 63674 01	0.00015 07179 87	−1484 72	80	0.26
0.27	0.00446 45486 70	0.00014 81812 69	−1484 98	72	0.27
0.28	0.00461 00447 23	0.00014 54960 53	−1484 52	81	0.28
0.29	0.00475 27071 08	0.00014 26623 85	−1483 25	75	0.29
0.30	0.00489 23875 00	0.00013 96803 92	−1481 23	84	0.30
0.31	0.00502 89377 76	0.00013 65502 76	−1478 37	76	0.31
0.32	0.00516 22100 99	0.00013 32723 23	−1474 75	85	0.32
0.33	0.00529 20569 94	0.00012 98468 95	−1470 28	79	0.33
0.34	0.00541 83314 33	0.00012 62744 39	−1465 02	88	0.34
0.35	0.00554 08869 14	0.00012 25554 81	−1458 88	80	0.35
0.36	0.00565 95775 49	0.00011 86906 35	−1451 94	88	0.36
0.37	0.00577 42581 44	0.00011 46805 95	−1444 12	87	0.37
0.38	0.00588 47842 87	0.00011 05261 43	−1435 43	86	0.38
0.39	0.00599 10124 35	0.00010 62281 48	−1425 88	88	0.39
0.40	0.00609 28000 00	0.00010 17875 65	−1415 45	91	0.40
0.41	0.00619 00054 37	0.00009 72054 37	−1404 11	91	0.41
0.42	0.00628 24883 35	0.00009 24828 98	−1391 86	90	0.42
0.43	0.00637 01095 08	0.00008 76211 73	−1378 71	92	0.43
0.44	0.00645 27310 85	0.00008 26215 77	−1364 64	95	0.44
0.45	0.00653 02166 02	0.00007 74855 17	−1349 62	95	0.45
0.46	0.00660 24310 97	0.00007 22144 95	−1333 65	93	0.46
0.47	0.00666 92412 05	0.00006 68101 08	−1316 75	1 00	0.47
0.48	0.00673 05152 51	0.00006 12740 46	−1298 85	93	0.48
0.49	0.00678 61233 50	0.00005 56080 99	−1280 02	1 03	0.49
0.50	0.00683 59375 00	0.00004 98141 50	−1260 16	96	0.50

p	S_3	δ	δ^3	δ^5	p
0.50	0.00683 59375 00	0.00004 98141 50	−1260 16	96	0.50
0.51	0.00687 98316 85	0.00004 38941 85	−1239 34	1 04	0.51
0.52	0.00691 76819 71	0.00003 78502 86	−1217 48	97	0.52
0.53	0.00694 93666 10	0.00003 16846 39	−1194 65	1 07	0.53
0.54	0.00697 47661 37	0.00002 53995 27	−1170 75	1 00	0 54
0.55	0.00699 37634 77	0.00001 89973 40	−1145 85	1 07	0.55
0.56	0.00700 62440 45	0.00001 24805 68	−1119 88	1 05	0.56
0.57	0.00701 20958 53	0.00000 58518 08	−1092 86	1 05	0.57
0.58	0.00701 12096 15	− 0.00000 08862 33	−1064 79	1 08	0.58
0.59	0.00700 34788 52	− 0.00000 77307 63	−1035 64	1 10	0.59
0.60	0.00698 88000 00	− 0.00001 46788 52	−1005 39	1 09	0.60
0.61	0.00696 70725 20	− 0.00002 17274 80	− 974 05	1 09	0.61
0.62	0.00693 81990 07	− 0.00002 88735 13	− 941 62	1 12	0.62
0.63	0.00690 20852 99	− 0.00003 61137 08	− 908 07	1 14	0.63
0.64	0.00685 86405 89	− 0.00004 34447 10	− 873 38	1 13	0.64
0.65	0.00680 77775 39	− 0.00005 08630 50	− 837 56	1 12	0.65
0.66	0.00674 94123 93	− 0.00005 83651 46	− 800 62	1 20	0.66
0.67	0.00668 34650 89	− 0.00006 59473 04	− 762 48	1 12	0.67
0.68	0.00660 98593 79	− 0.00007 36057 10	− 723 22	1 21	0.68
0.69	0.00652 85229 41	− 0.00008 13364 38	− 682 75	1 15	0.69
0.70	0.00643 93875 00	− 0.00008 91354 41	− 641 13	1 24	0.70
0.71	0.00634 23889 43	− 0.00009 69985 57	− 598 27	1 16	0.71
0.72	0.00623 74674 43	− 0.00010 49215 00	− 554 25	1 25	0.72
0.73	0.00612 45675 75	− 0.00011 28998 68	− 508 98	1 19	0.73
0.74	0.00600 36384 41	− 0.00012 09291 34	− 462 52	1 28	0.74
0.75	0.00587 46337 89	− 0.00012 90046 52	− 414 78	1 20	0.75
0.76	0.00573 75121 41	− 0.00013 71216 48	− 365 84	1 28	0.76
0.77	0.00559 22369 13	− 0.00014 52752 28	− 315 62	1 27	0.77
0.78	0.00543 87765 43	− 0.00015 34603 70	− 264 13	1 26	0.78
0.79	0.00527 71046 18	− 0.00016 16719 25	− 211 33	1 28	0.79
0.80	0.00510 72000 00	− 0.00016 99046 18	− 157 35	1 31	0.80
0.81	0.00492 90469 54	− 0.00017 81530 46	− 102 01	1 31	0.81
0.82	0.00474 26352 79	− 0.00018 64116 75	− 45 36	1 30	0.82
0.83	0.00454 79604 39	− 0.00019 46748 40	12 59	1 32	0.83
0.84	0.00434 50236 93	− 0.00020 29367 46	71 86	1 35	0.84
0.85	0.00413 38322 27	− 0.00021 11914 66	132 48	1 35	0.85
0.86	0.00391 43992 89	− 0.00021 94329 38	194 45	1 33	0.86
0.87	0.00368 67443 24	− 0.00022 76549 65	257 75	1 40	0.87
0.88	0.00345 08931 07	− 0.00023 58512 17	322 45	1 33	0.88
0.89	0.00320 68778 83	− 0.00024 40152 24	388 48	1 43	0.89
0.90	0.00295 47375 00	− 0.00025 21403 83	455 94	1 36	0.90
0.91	0.00269 45175 52	− 0.00026 02199 48	524 76	1 44	0.91
0.92	0.00242 62705 15	− 0.00026 82470 37	595 02	1 37	0.92
0.93	0.00215 00553 91	− 0.00027 62146 24	666 65	1 47	0.93
0.94	0.00186 59403 45	− 0.00028 41155 46	739 75	1 40	0.94
0.95	0.00157 39978 52	− 0.00029 19424 93	814 25	1 47	0.95
0.96	0.00127 43098 37	− 0.00029 96880 15	890 22	1 45	0.96
0.97	0.00096 69653 22	− 0.00030 73445 15	967 64	1 45	0.97
0.98	0.00065 20610 71	− 0.00031 49042 51	1046 51	1 48	0.98
0.99	0.00032 97017 35	− 0.00032 23593 36	1126 86	1 50	0.99
1.00	0.00000 00000 00	− 0.00032 97017 35	1208 71	1 49	1.00

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