On the Motion of a Blowing Hammer

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On the Motion of a Blowing Hammer

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Abstract

The author analysed the motion of a blowing hammer which is used to get continous blowing and discussed the condition to keep continuously stable blowing.

§ 1, Introduction

There are various methods for blowing objects continuously. Here I take a certain arrangement of continous blowing, which uses springs as elastic medium and piston-crank mechanism as forced force. I study analyticaly the motion of its blowing hammer, and moreover consider the condition to keep continuously stable vibration by making one blow for one cycle.

§ 2. Arrangement

Fig. 1 shows the arrangement and the relative position of "Spring system" and "Crank mechanism system" at the mid-position of piston stroke. A is a nut, fixed at the top of the rod P. B is a blowing hammer which moves down- and up-wards under the working of spring S_1 and S_2. C is the place which accepts impact. D is the washer to which the one end of S_2 is pushed and through which P can move but A can not.

E and F are the upper and the lower fixed position respectively. S_1 and S_2 are the same springs having high frequency of proper vibration and both always being compressed. S_2 is more strongly compressed than S_1.

P is the reciprocating rod. R is the crank (Radius $R = 10$ mm, rotational speed $n = 1000$ r.p.m.). L is the connecting rod (Length $L = 300$ mm). Q denotes crank angle from the upper dead point.

§ 3. Motion of Nut A

We get Displacement, Velocity and Acceleration of A for each crank angle by following equations.

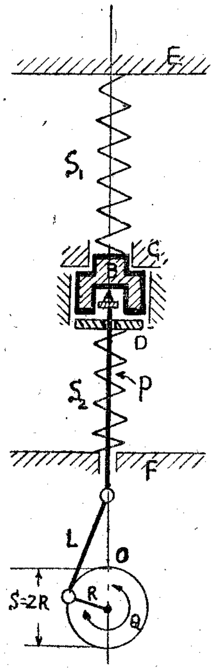


Fig. 1

a) Displacement

$$x = R(1-\cos\theta) + \frac{R^2}{2L}\sin^2\theta \quad \cdots\cdots\cdots (1)$$

Curve calculated by eq. (1) is plotted in Fig. 3 by dotted lines.

b) Velocity

$$V = \frac{\pi Rn}{30}\left(\sin\theta + \frac{R}{2L}\sin^2\theta\right) \quad \cdots\cdots\cdots (2)$$

c) Acceleration

$$\alpha = \frac{\pi^2 Rn^2}{30^2}\left(\cos\theta + \frac{R}{L}\cos 2\theta\right) \quad \cdots\cdots\cdots (3)$$

§ 4. Motion of the Blowing Hammer *B*

At first we consider that blowing hammer vibrates under the action of spring S_1 and S_2 as shown in Fig. 2 (both spring S_1 and S_2 being compressed).

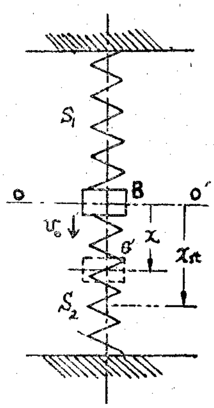


Fig. 2

Where:

x_{st} ······ initial compression of each spring.

x ······ displacement.

k ······ spring constant of both spring.

M ······ mass of the blowing hammer B.

$0-0'$ ······ balancing position of springs.

The hammer B recieves downward force $(x_{st}-x)\,k$ by S_1 and upward force $(x_{st}+x)\,k$ by S_2; taking downward force for positive, resultant force by S_1 and S_2 is $(x_{st}-x-x_{st}-x)\,k = -2xk$.

If we neglect friction of guide walls, we get following differential equation.

$$-M\frac{d^2x}{dt^2} - 2xk = 0$$

putting $a^2 = \dfrac{2k}{M}$, we get

$$\ddot{x} + a^2x = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$

Secondly, we must consider as follows. Eq. (4) is available only from the first strike of B against C to the next strike of B against C (B, after restitution going downwards to the extremity and rising up again).

But from the instance of impact (B on C) till separation (B from C) other equation is required, because B accepts reaction $-M(V_2 - V_1)$ from C. So we get following equation.

$$M\ddot{x} + M\left(1 + \frac{1}{\lambda}\right)\dot{x} + 2kx = 0 \quad \cdots\cdots\cdots\cdots (5)$$

$$x \doteqdot x_q \longrightarrow \frac{1}{\lambda} = -1$$

$$x = x_q \longrightarrow \frac{1}{\lambda} = \frac{1}{\lambda}$$

x_q = displacement when B touches C.

λ is not function of time but of x, and is integral form. Accordingly eq. (5) is non-linear and hardly soluble.

Energy being absorbed by impact, it is irregular damped vibration, and as understood from the second term of eq. (5) it is positive damping. Therefore other energy must be supplied from the outside to continue the vibration constantly. For this purpose the crank mechanism is used. Where upon by washer D and by nut A working of spring S_2 is taken off during downward stroke. Accordingly, balance of spring S_1 and S_2 is broken, and it can be considered that the following force is added to eq. (5).

$$-(x_{st} + x_1 + x_p)\,k$$

x_p ······ distance between midposition of reciprocating motion of A and spring blance position.

Where x_1 is written as following form

$$x_1 = R\left(1 - \cos\theta\right) + \frac{R^2}{2L}\sin^2\theta$$

$$\pi < \theta < \frac{3\pi}{2} \qquad \left(\theta = \frac{2\pi n t}{60}\right)$$

After all we get following equation.

$$M\ddot{x} + M\left(1 + \frac{1}{\lambda}\right)\dot{x} + \left\{2x - \left(x_1 + x_{st} + x_p\right)\right\}k = 0 \quad \cdots\cdots\cdots (6)$$

But it is complicated and difficult to slove. So we investigate partially and calculate about a practical example under given conditions. By the design of the arrangement the position where B strikes against C is 2 mm before midposition of stroke. By crank angle it corresponds to $\theta = 260°$. The velocity is from eq. (2)

$$V_{260} = -1.022 \text{ m/s}$$

Assuming the mass of C is infinitely large for B and taking $\lambda = \frac{5}{9}$, the velocity after restitution is

$$V_{01} = \lambda V_{260} = 0.568 \text{ m/s} \quad \cdots\cdots (7)$$

Having this initial velocity B starts downward motion under the action of S_1 and S_2.

Details of the motion are given as follows.

1) Motion of B from crank angle $\theta = 260°$

The solution of eq. (4) is

$$x = A \cos at + B \sin at$$

at $t = 0 \longrightarrow x = x_0, \quad V = V_0$

$$\therefore \quad x = x_0 \cos at + \frac{V_0}{a} \sin at \quad \cdots\cdots (8)$$

$$\dot{x} = -x_0 a \sin at + V_0 \cos at \quad \cdots\cdots (9)$$

The given conditions are:

Compressed length of springs ········ $S_1 = 166$ mm, $S_2 = 162$ mm. (natural length $S_1 = S_2 = 176$ mm).

$$x_q = \frac{166-162}{2} = 2 \text{ mm}, \quad k = 10 \text{ kg/cm},$$

$$W \doteq 0.78 \text{ kg} \quad \text{(contained washer } D)$$

$$\therefore \quad a = \sqrt{2k \Big/ \frac{W}{g}} = 158.5 \left[\frac{1}{\text{sec}}\right]$$

$$V_0 = V_{01} = 568 \text{ [mm/s]}$$

$$x_0 = x_q = 2.0 \text{ [mm]}.$$

Inserting these numerical value into eq. (8) we get displacement curve of B as plotted in Fig. 3. As understood from the diagram, after restitution going down and rising up B strikes against C again at $\theta = 340°$. Then the velocity is from eq. (9)

$$V_1 = -565 \text{ mm/s}$$

2) Motion of B after restitution (at $\theta = 340°$)

The velocity after restitution is likewise

$$V_{02} = -\frac{5}{9} V_1 = 314 \text{ mm/s}$$

$$V_{02}/a = 1.98 \text{ mm}$$

Like as before B strikes C after 55° ($\theta = 35°$).

Velocity of this position is

$$V_2 = -277 \text{ mm/s}.$$

3) Motion of B after $\theta = 35°$

Velocity after restitution

$$V_{03} = -\frac{5}{9} V_2 = 155 \text{ mm/s}$$

B strikes C after 35° ($\theta = 70°$)

Velocity of this position

$$V_3 = -160 \text{ mm/s}$$

Velocity after restitution

$$V_{04} = -\frac{5}{9} V_3 = 89 \text{ mm/s.}$$

Thus blowing hammer B, repeating restitution and being damped gradually, comes to a standstill. While nut A completing its upward stroke and going down again, from crank angle $\theta = 101°$ pressing spring S_2 through washer D goes down. At the same time the blowing hammer which was at a standstill goes under the action of spring S_1 (S_2 does not work this time).

4) Motion of B after crank angle $\theta = 101°$

In this case working spring is S_1 only. Taking the position of start as origin we get following equation.

$$-M \frac{d^2x}{dt^2} + \left(x_{st} - x\right) k = 0 \quad \cdots\cdots\cdots\cdots\cdots (10)$$

Solution of eq. (10) is

$$x = x_{st}(1 - \cos a't)$$
$$\dot{x} = a' x_{st} \sin a't$$

where $a'^2 = \frac{k}{M}$ and initial condition

$$t = 0\ , \quad x = x_0 = 0\ , \quad V = V_0 = 0$$

given condition

$$W = 0.73 \text{ kg (excluded washer } D)$$

$$a' = \sqrt{k \Big/ \frac{W}{g}} = 116 \qquad \left[\frac{1}{\text{sec}}\right]$$

$$x_{st} = 10 \text{ mm}$$

$$V_{01} = 0\ , \quad x_0 = 0\ .$$

The result of calculation is plotted in Fig. 3. 70° after, at $\theta = 171°$, blowing hammer B overtakes P. Then the velocity is from eq. (12)

$$V_1 = 1130 \text{ mm/s}$$

From this position B moves with this initial velocity under the action of

S_1 and S_2 (while washer D goes parting from A).

5) Motion of B after 171°
In this case we use eq. (8).

$$x_0 = 9.85\,\text{mm} \qquad \text{(from diagram)}$$
$$V_{02} = 1130\,\text{mm/s}$$

Seeing the plotted curve we know that about 30° after blowing hammer B (and D) reaches to the lower extrimity and rising up again, at $\theta = 235°$ strikes against A.

Velocity of this position is

$$V_2 = -1685\,\text{mm/s}$$

Here after having this velocity as initial velocity, blowing hammer B rises up resisting compressive force of S_1.

6) Motion of B after $\theta = 235°$ (working S_1 only)

$$V_{03} = V_2 = -1685\,\text{mm/s}$$
$$x_{st} = 6.1\,\text{mm (from diagram)}$$

Using eq. (11), (12) in this case 15° after (at $\theta = 250°$) blowing hammer B strikes on C. Accordingly, as the starting position was $\theta = 260°$, so at the end of one cycle the blow takes place by enough 10° before the starting position.

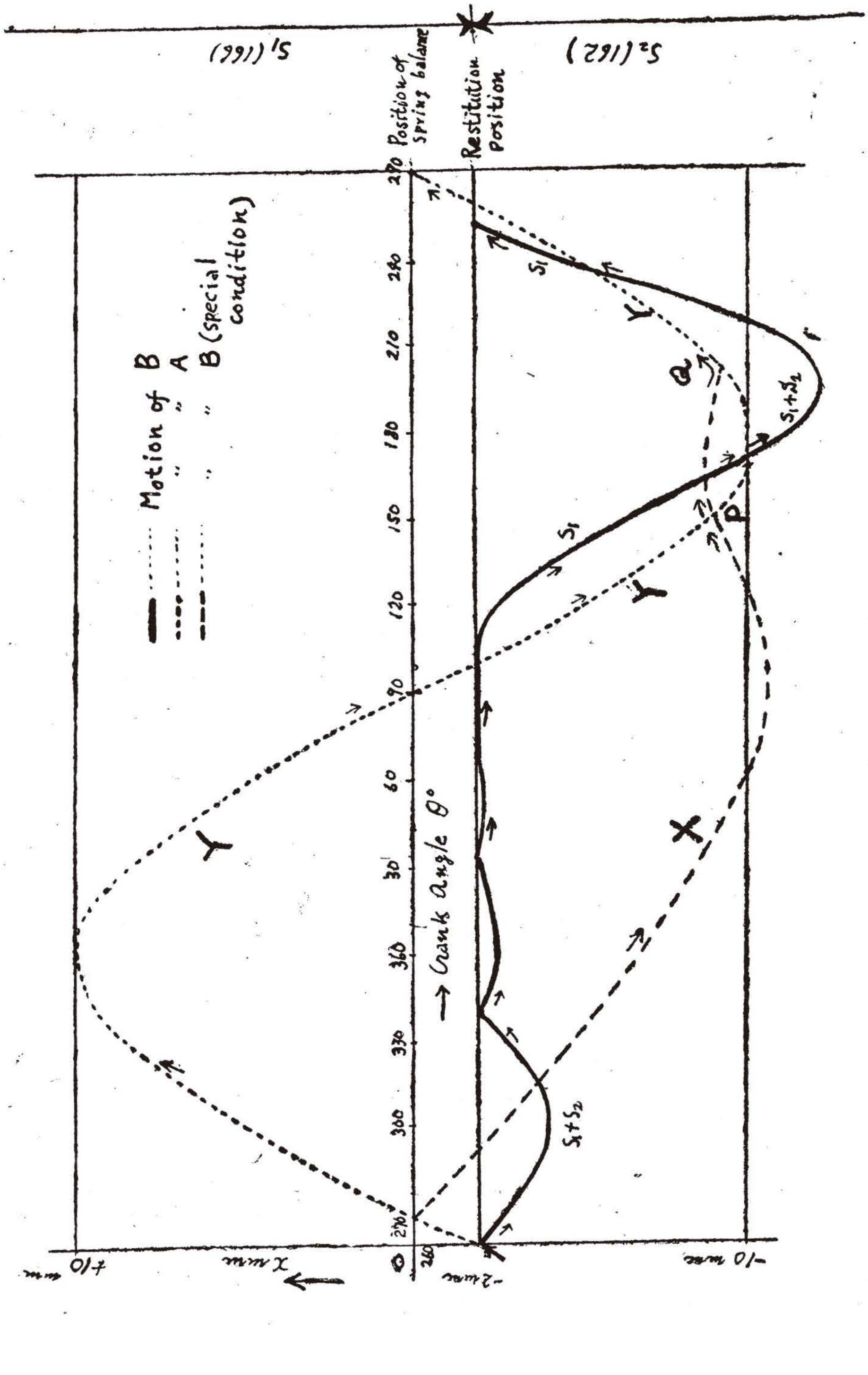
Then velocity is

$$V = -1412\,\text{mm/s}.$$

Thus B does not return to the initial state. As mentioned above, in the arrangement it is far from the condition one cycle for one blow and there is a grat deal of complexity about the state of motion.

§ 5. Condition of Stability

Now we must consider the condition to get continous blowing and stable vibration. Firstly, restitution must be done instantaneously. This condition is satisfied by using elastic body (metatal). Secondly, one blow for one cycle must be done and each displacement-time curve ($x-t$ curve) for one cycle must be equal. To satisfy this condition, blowing hammer B, after restitution going down and rising up again, must strike against C just after 360°; and the velocity of this position must be equal to V_1. As equation (8) shows simple harmonic motion, however, the velocity when B strikes against C rising up again is equal to the initial velocity. While by restitution the next initial velocity is multiplied by λ, where λ is smaller



S1(166)
S2(162)
Position of spring balance
Restitution position
Motion of B
" A
" B (special condition)
→ Crank Angle θ°
x mm
+10 mm
-10 mm
-2 mm

Fig. 3

than 1. So, even if the period of spring side and crankside is equal, equalty of velocity can not be obtained. Namely, exactly fitted condition can not be obtained in this arrangement. So by graphical methods we get to the purpose approximately. We denote by X and Y the displacement curve of blowing hammer B and nut A respectively. Broken line in Fig. 3 shows an example of the case $x_0 = 0$.

We take $a = \sqrt{\frac{2M}{k}}$ properly so that upward part of X touches on the lower part of Y (point P in the Figure) with an easy grade. The position of point P must be taken to take off the action of spring S_2. After this point B goes down with an easy grade by the action of S_1 only. While washer D, pressed on nut A by spring S_2, rises up again and encounter B at point Q. From this point B rises up along Y curve. Thus the motion accords with initial state.

Even if the conditions change more or less, in the above mentioned range the motion is properly regurated, or stability of phase can be obtained.

§ 6. Summary

The state of motion of the blowing hammer under given condition was found and obtained the way how to get the condition of one cycle for one blow.

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