On an Analytic Method of the Synchronous Machines

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On an Analytic Method of the Synchronous Machines

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Abstract

The purpose of this paper is to generalize the fundamental equations of synchronous generators as to the standard rectangular coordinates. The analyses dealt here are on three cases, and in one of these the excitation circuit is not considered, in another this is considered without amortisseur, and in another with amortisseur.

I. Introduction

This is the theoretical studies on extending and generalizing the fundamental equations of rotating electrical machinery which have been derived by R. H. PARK[1], G. KRON[2], and others[3], for the purpose of modifying it for the standard rectangular coordinate systems and in particular analizing the synchronous alternators. For the analytic method, the author adopted the tensor analysis, basing on the BLONDEL's two-reaction theory on salient-pole synchronous machines. The tensorial analysis does represent all the physical concepts in problem only by one tensor equation, so that the manifestation is greatly simplified. Lastly, the three-phase turbo-generator was analized as an application of this theorem.

II. Fundamental Equation of Salient-pole Alternator for Standard Rectangular Coordinates

According to the two-reaction theory, current, voltage and flux in each armature phase can be resolved to direct and quadrature components. Now, let the number of the armature phases be three and denote

$$\theta = \omega t + \varphi$$
$$\theta_1 = \theta$$
$$\theta_2 = \theta + 120^\circ$$
$$\theta_3 = \theta - 120^\circ$$

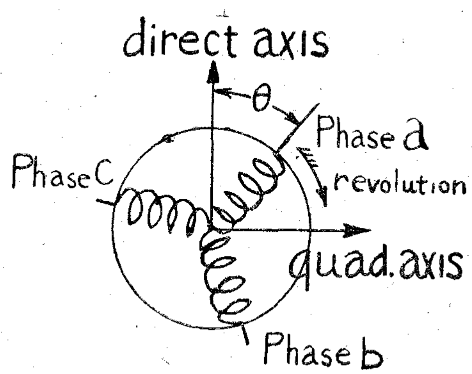

Fig. 1, Three-phase Synchronous Machine.

We have

$$[i] = [C][i'] \qquad \cdots\cdots (1)$$

$$[e] = [C][e'] \qquad \cdots\cdots (2)$$

$$[\phi] = [C][\phi'] \qquad \cdots\cdots (3)$$

where $[i]$, $[e]$ and $[\phi]$ concern real axis a, b and c, and $[i']$, $[e']$ and $[\phi']$ concern direct axis d, quadrature axis q and zero-sequence axis 0. And

$$[C] = \begin{array}{c|c|c|c|} & d & q & 0 \\ \hline a & \cos\theta_1 & \sin\theta_1 & 1 \\ \hline b & \cos\theta_2 & \sin\theta_2 & 1 \\ \hline c & \cos\theta_3 & \sin\theta_3 & 1 \\ \hline \end{array} \qquad \cdots\cdots (4)$$

From Eq. (1), (2) and (3)

$$[i'] = [C]^{-1}[i] \qquad \cdots\cdots (5)$$

$$[e'] = [C]^{-1}[e] \qquad \cdots\cdots (6)$$

$$[\phi'] = [C]^{-1}[\phi] \qquad \cdots\cdots (7)$$

and

$$[C]^{-1} = \frac{2}{3}\begin{array}{c|c|c|c|} & a & b & c \\ \hline d & \cos\theta_1 & \cos\theta_2 & \cos\theta_3 \\ \hline q & \sin\theta_1 & \sin\theta_2 & \sin\theta_3 \\ \hline 0 & 1/2 & 1/2 & 1/2 \\ \hline \end{array} \qquad \cdots\cdots (8)$$

While, from the MAXWELL's equation, for generator action

$$\left.\begin{aligned} -e_a &= r\,i_a + p\,\phi_a \\ -e_b &= r\,i_b + p\,\phi_b \\ -e_c &= r\,i_c + p\,\phi_c \\ -e_0 &= r\,i_0 + p\,\phi_0 \end{aligned}\right\} \qquad \cdots\cdots (9)$$

with

$$p\phi = p(\underset{\cdot}{L} \cdot \underset{\cdot}{i}) = \frac{d}{dt}(\underset{\cdot}{L} \cdot \underset{\cdot}{i})$$

Therefore, we obtain the following fundamental equations.

$$\left.\begin{aligned} -e_d &= p\,\phi_d + \phi_q\,p\theta + r\,i_d \\ -e_q &= p\,\phi_q - \phi_d\,p\theta + r\,i_q \\ -e_0 &= p\,\phi_0 + r\,i_0 \end{aligned}\right\} \qquad \cdots\cdots (10)$$

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1) PARK; T.A.I.E.E., Vol. 48, 1929.
2) KRON; G. E. Rev., Apr. 1935 & Feb. 1936.
3) DOHERTY & NICKLE; T.A.I.E.E. Vol. 45, 1926.

On steady synchronous speed running,

$$p\theta = \frac{d}{dt}\theta = \omega = 1 \qquad \text{(by the per-unit method)}$$

Then Eq. (10) becomes in this case

$$\left.\begin{aligned} -e_d &= p\,\phi_d + \phi_q + r\,i_d \\ -e_q &= p\,\phi_q - \phi_d + r\,i_q \\ -e_0 &= p\,\phi_0 + r\,i_0 \end{aligned}\right\} \qquad \cdots\cdots (11)$$

where r is the resistance of armature circuit per phase. Therefore, the fundamental equations of synchronous generators for the standard systems are obtained. In the case of motor action, the direction of armature current reverses, so the negative sign of e_d, e_q and e_0 in Eq. (10) and (11) is dropped off.

III. Fundamental Equation of Salient-pole Alternator Without Amortisseur

In the preceding section the writer studied the case where an excitation circuit is not considered. Let us now consider the effect of this circuit. Since generally Eq. (11) can be put into as

$$-[e] = [Z][i] \qquad \cdots\cdots (12)$$

we will find the impedance tensor $[Z]$.

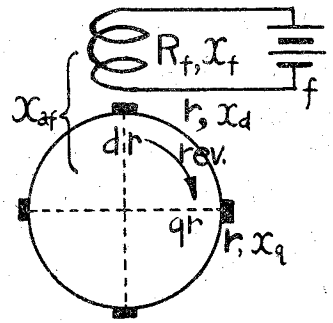

Fig. 2, Generalized machine without amortisseur.

Inverstigating on Fig. 2, the speed emf and the transformer emf induced by flux cutting and interlinking laws, we have the following tensor $[Z]$, taking the per-unit system[4].

$[Z] =$ ·································· (13)

	f	d	q
f	$R_f + x_f p$	$x_{af}\,p$	0
d	$x_{af}\,p$	$r + x_d\,p$	$x_q\,p\theta$
q	$-x_{af}\,p\theta$	$-x_d\,p\theta$	$r + x_q\,p$

In order to take off the excitation circuit term f from Eq. (13) with considering still the effect of this existence, we use the so-called short-circuit matrix $[S]$.

4) T.A.I.E.E. 1937.

$[S] =$

	f	d	q
f	1	$-\dfrac{x_{af}\,p}{R_f+x_f\,p}$	0
d	0	1	0
q	0	0	1

From the fact that $[Z'] = [Z][S]$

$[Z'] =$

	f	d	q
f	$R_f+x_f\,p$	0	0
d	$x_{af}\,p$	$r+x_d(p)p$	$x_q\,p\theta$
q	$-x_{af}\,p\theta$	$-x_d(p)p\theta$	$r+x_q\,p$

.... (14)

where

$$x_d(p) = x_d - \frac{x_{af}^2\,p}{R_f + x_f p} \qquad \text{.................... (15)}$$

$$x_d(\infty) = \rho\, x_d = x_d'$$

$$\rho = \frac{x_f x_d - x_{af}^2}{x_f x_d} = \text{leakage coefficient.}$$

x_d' corresponds to the so-called "direct transient reactance".

Now if the fundamental equation (10) is applied on the above relations, we obtain

$$\left.\begin{aligned} -e_d &= p\,\phi_d + \phi_q p\theta + r\,i_d \\ -e_q &= p\,\phi_q - \phi_d p\theta + r\,i_q \\ \phi_d &= F(p)\,E_f + x_d(p)\,i_d \\ \phi_q &= x_q\,i_q \\ F(p) &= -\frac{x_{af}}{R_f + x_f\,p} \end{aligned}\right\} \qquad \text{.................... (16)}$$

in which E_f is applied $d-c$ voltage of the excitation circuit.

And since the machine has not amortisseur windings, the relations between the transient reactances and the subtransient reactances are:

$$\left.\begin{aligned} x_d' &= x_d'' \\ x_q &= x_q' = x_q'' \end{aligned}\right\} \qquad \text{............... (17)}$$

IV. Fundamental Equation of Salient-pole Alternator With Amortisseur

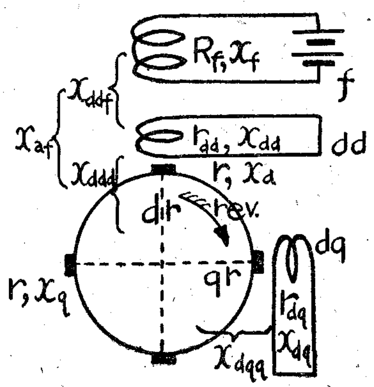

Fig. 3, Generalized machine with amortisseur.

In the fundamental Eq. (12), the impedance tensor $[Z]$ of this case becomes as follows:

$[Z] =$ .... (18)

	f	dd	d	q	dq
f	$R_f+x_f p$	$x_{ddf} p$	$x_{af} p$	0	0
dd	$x_{ddf} p$	$r_{dd}+x_{dd} p$	$x_{ddd} p$	0	0
d	$x_{af} p$	$x_{ddd} p$	$r+x_d p$	$x_q p\theta$	$x_{dqq} p\theta$
q	$-x_{af} p\theta$	$-x_{ddd} p\theta$	$-x_d p\theta$	$r+x_q p$	$x_{dqq} p$
dq	0	0	0	$x_{dqq} p$	$r_{dq}+x_{dq} p$

Since in general the amortisseur terms are not necessary, they should be taken off from Eq. (18) as before by using the short-circuit matrix $[S]$ and the lower-class unit-matrix $[I]$.

$[S] =$

	f	d	q
f	1	0	0
dd	$-\dfrac{x_{ddf} p}{r_{dd}+x_{dd} p}$	$-\dfrac{x_{ddd} p}{r_{dd}+x_{dd} p}$	0
d	0	1	0
q	0	0	1
dq	0	0	$-\dfrac{x_{dqq} p}{r_{dq}+x_{dq} p}$

$[I] =$

	f	dd	d	q	dq
f	1	0	0	0	0
d	0	0	1	0	0
q	0	0	0	1	0

From $[Z'] = [I][Z][S]$

$[Z'] =$ .... (19)

	f	d	q
f	$R_f + x_f(p)p$	$x_{af}(p)p$	0
d	$x_{af}(p)p$	$r + x_d'(p)p$	$x_q(p)p\theta$
q	$-x_{af}(p)p\theta$	$-x_d'(p)p\theta$	$r + x_q(p)p$

where

$$\left.\begin{aligned} x_f(p) &= x_f - \frac{x^2_{ddf}\,p}{r_{dd}+x_{dd}\,p}, & x_d'(p) &= x_d - \frac{x^2_{ddd}\,p}{r_{dd}+x_{dd}\,p}, \\ x_q(p) &= x_q - \frac{x^2_{dqq}\,p}{r_{dq}+x_{dq}\,p}, & x_{af}(p) &= x_{af} - \frac{x_{ddf}\,x_{ddd}\,p}{r_{dd}+x_{dd}\,p}. \end{aligned}\right\} \quad \cdots\cdots (20)$$

$x_d'(p)$, $x_q(p)$, $x_f(p)$ and $x_{af}(p)$ are short-circuited impedances due to the existence of amortisseur windings. If this windings are lacked, these become respectively x_d, x_q, x_f and x_{af} since $x_{ddf} = x_{ddd} = 0$ in this case, and Eq. (19) just coincides with Eq. (13).

Let next the excitation current to be taken off too, namely by using matrix $[S']$

$[S'] =$

	f	d	q
f	1	$-\dfrac{x_{af}(p)\,p}{R_f + x_f(p)p}$	0
d	0	1	0
q	0	0	1

From $[Z''] = [Z'][S']$

$[Z''] =$ .... (21)

	f	d	q
f	$R_f + x_f(p)p$	0	0
d	$x_{af}(p)p$	$r + x_d(p)p$	$x_q(p)p\theta$
q	$-x_{af}(p)p\theta$	$-x_d(p)p\theta$	$r + x_q(p)p$

where

$$x_d(p) = x_d'(p) - \frac{x_{af}^2(p)\,p}{R_f + x_f(p)\,p}$$

$$= x_d - \frac{p^2(x_f x_{ddd}^2 - 2x_{ddd}x_{ddf}x_{af} + x_{dd}x_{af}^2) + p(R_f x_{ddd}^2 + r_{dd}x_{af}^2)}{p^2(x_{dd}x_f - x_{ddf}^2) + p(r_{dd}x_f + R_f x_{dd}) + r_{dd}R_f}. \quad (22)$$

and then

$$x_d(\infty) = x_d - \frac{x_d\,x_{ddd}^2 - 2x_{ddd}\,x_{ddf}\,x_{af} + x_{dd}\,x_{af}^2}{x_f\,x_{dd} - x_{ddf}^2} = x_d''$$

$$x_q(\infty) = x_q - \frac{x_{dqq}^2}{x_{dq}} = x_q''$$

x_d'' and x_q'' correspond to the so-called "direct and quadrature sub-transient reactance" respectively.

Then if the fundamental equation (10) is applied on the above relations, we obtain

$$\left.\begin{aligned}
-e_d &= p\,\phi_d + \phi_q\,p\theta + r\,i_d \\
-e_q &= p\,\phi_q - \phi_d\,p\theta + r\,i_q \\
\phi_d &= G(p)\,E_f + x_d(p)\,i_d \\
\phi_q &= x_q(p)\,i_q \\
G(p) &= -\frac{x_{af}(p)}{R_f + x_f(p)\,p} = -\frac{x_{af}(r_{dd} + x_{dd}p) - x_{ddf}\,x_{ddd}\,p}{(R_f + x_f\,p)(r_{dd} + x_{dd}\,p) - x_{ddf}^2\,p^2}
\end{aligned}\right\} \quad (23)$$

The transient reactances x_d' and x_q' of the machine can be found from x_d'' and x_q'' when it is put $x_{ddf} = x_{ddd} = 0$, then

$$\left.\begin{aligned} x_d' &= \frac{x_f x_d - x_{af}^2}{x_f} \\ x_q' &= x_q \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (24)$$

V. An Analysis of Three-phase Non-salient-pole Alternator

In the preceding sections several fundamental equations which are applicable to several cases are derived. Next, as an application of this theorem, the three-phase turbo-generator will be analized and its fundamental equation shall be induced.

The generalized circuit of Fig. 1 can be expressed as Fig. 4. Now let the nomenclature of machine constants be represented as shown in Fig. 4.

Current $[i]$, namely i_a, i_b and i_c is transformed into d and q axis by the transformation tensor $[C]$.

$$[i] = [C][i']$$

in which

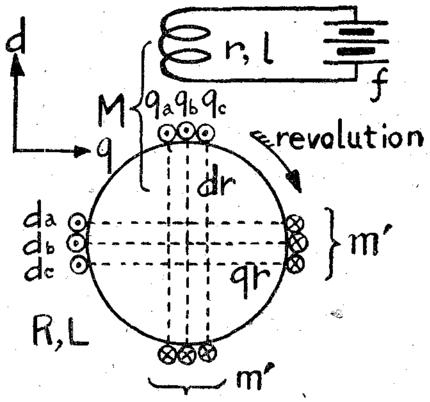

Fig. 4, Generalized 3-phase Cylindrical-rotor machine.

$[C] =$

	f	a	b	c
f	1	0	0	0
dc	0	0	0	$\cos\theta_3$
db	0	0	$\cos\theta_2$	0
da	0	$\cos\theta_1$	0	0
qa	0	$\sin\theta_1$	0	0
qb	0	0	$\sin\theta_2$	0
qc	0	0	0	$\sin\theta_3$

The impedance matrix of primitive circuit is

$[Z] =$

	f	dc	db	da	qa	qb	qc
f	$r+lp$	Mp	Mp	Mp	0	0	0
dc	Mp	$R+Lp$	mp	mp	$mp\theta$	$mp\theta$	$mp\theta$
db	Mp	mp	$R+Lp$	mp	$mp\theta$	$mp\theta$	$mp\theta$
da	Mp	mp	mp	$R+Lp$	$mp\theta$	$mp\theta$	$mp\theta$
qa	$-Mp\theta$	$-mp\theta$	$-mp\theta$	$-mp\theta$	$R+Lp$	mp	mp
qb	$-Mp\theta$	$-mp\theta$	$-mp\theta$	$-mp\theta$	mp	$R+Lp$	mp
qc	$-Mp\theta$	$-mp\theta$	$-mp\theta$	$-mp\theta$	mp	mp	$R+Lp$

Using the transformation tensor $[C]$ and its transposed matrix $[C]_t$, the above impedance $[Z]$ concerning d, q axis is transformed into $[Z']$ concerning a, b, c axis components.

$$[Z'] = [C]_t [Z] [C]$$

$=$

	f	a	b	c
f	$r+lp$	$Mp\cos\theta_1$	$Mp\cos\theta_2$	$Mp\cos\theta_3$
a	$Mp\cos\theta_1$	$R+Lp$	$-\frac{m}{2}p$	$-\frac{m}{2}p$
b	$Mp\cos\theta_2$	$-\frac{m}{2}p$	$R+Lp$	$-\frac{m}{2}p$
c	$Mp\cos\theta_3$	$-\frac{m}{2}p$	$-\frac{m}{2}p$	$R+Lp$

Again, $[Z']$ is transformed into the symmetrical phase axis components by the symmetrical method using $[C']$.

$[C'] =$

	f	a	b	c
f	1	0	0	0
a	0	1	1	1
b	0	1	a^2	a
c	0	1	a	a^2

in which $a = \varepsilon^{j120^\circ}$, and suffix 0, 1 and 2 represent zero-phase, positive-phase and negative-phase sequence respectively, From the fact $[Z''] = [C']^{-1} [Z'] \cdot [C']$

$[Z''] =$

	f	a	b	c
f	$r+lp$	0	$\frac{3}{2}Mp\varepsilon^{-j\theta}$	$\frac{3}{2}Mp\varepsilon^{j\theta}$
a	0	$R+L_0p$	0	0
b	$\frac{1}{2}Mp\varepsilon^{j\theta}$	0	$R+L_1p$	0
c	$\frac{1}{2}Mp\varepsilon^{-j\theta}$	0	0	$R+L_1p$

in which $L_0 = L - m$, and $L_1 = L + \frac{m}{2}$.

Accordingly, the fundamental equation is represented by $-[e''] = [Z''] \cdot [i'']$, where every concept is expressed in sequence phase quantities.

VI. Conclusion

In this paper the author has generally discussed the old problems concerning the fundamental equations of salient-pole machines, and generalized them for the standard rectangular coordinates. Of course, all sorts of transient phenomena, such as sudden short-circuited current or transient stabilities, can be solved and detailed from those fundamental equations. During the analysis of steady phenomena, we will only put $p = j$ and $p\theta = 1$, the solution being quite easy.

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