# Harmonic Resonance on Unbalanced Transmission Lines with a Salient-pole Synchronous Generator

# Harmonic Resonance on Unbalanced Transmission Lines with a Salient-pole Synchronous Generator

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**Abstract**

On account of the saliency of poles of an alternating current generator, a lot of harmonics can be emerged on transmission lines when some kinds of unbalanced faults occur. This paper deals with the mathematical development of a fundamental equation with the case of one line-fault, and clarifies critical conditions of the occurrence of this harmonic resonance.

The author has lately a chance to assist and cooperate Mr. Ogushi, Professor in Hokkaido University, with his research for the problem of harmonic resonance of a transmission line caused by an unbalanced line-fault. As a part of the theoretical developement of this research was already published[1], the treatment of mathematics is one of considerably complicated and it would be liable to obstruct the clear understanding of the physical nature of the phenomena.

Soon after, the author succeeded to manage it in the more systematic and more direct procedure by using "tensor"—a powerful mathematical tool in the engineering—and obtained the same results as before; of which process of the treatise is felt valuable and so published here.

As all the winding-axes of a balanced three-phase salient-pole synchronous generator will revolve with synchronous speed because of these holonomic natures, its impedance tensor is represented by a single equation, $[Z]=[R]+p[L]$, according to the Maxwell's equation. If it is assumed $[R]=0$ as a justifiable approximation,

$[Z]$[2] $= p$

| | $f$ | $d_a$ | $d_b$ | $d_c$ | $q_c$ | $q_b$ | $q_a$ |
|---|---|---|---|---|---|---|---|
| $f$ | $x_f$ | $x_{af}$ | $x_{af}$ | $x_{af}$ | 0 | 0 | 0 |
| $d_a$ | $x_{af}$ | $x_d$ | $x_{rd}$ | $x_{rd}$ | 0 | 0 | 0 |
| $d_b$ | $x_{af}$ | $x_{rd}$ | $x_d$ | $x_{rd}$ | 0 | 0 | 0 |
| $d_c$ | $x_{af}$ | $x_{rd}$ | $x_{rd}$ | $x_d$ | 0 | 0 | 0 |
| $q_c$ | 0 | 0 | 0 | 0 | $x_q$ | $x_{rq}$ | $x_{rq}$ |
| $q_b$ | 0 | 0 | 0 | 0 | $x_{rq}$ | $x_q$ | $x_{rq}$ |
| $q_a$ | 0 | 0 | 0 | 0 | $x_{rq}$ | $x_{rq}$ | $x_q$ |

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(1). G. Miura: A Prompt Memoir of The Muroran College of Technology Vol. I, No. 2.

(2). The nomenclature owes to: G. Miura; An Analytic Method of the Synchronous Machines, Memoirs of The Muroran College of Technology, Vol. 1, No. 1.

On the other hand, the current transformation tensor is

$[C]=$

| | $f$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $f$ | 1 | | | |
| $d_a$ | | $\cos\theta_1$ | | |
| $d_b$ | | | $\cos\theta_2$ | |
| $d_c$ | | | | $\cos\theta_3$ |
| $q_c$ | | | | $\sin\theta_3$ |
| $q_b$ | | | $\sin\theta_2$ | |
| $q_a$ | | $\sin\theta_1$ | | |

Then, a new impedance tensor is calculated from

$$[Z']=[C]_t[Z][C]=p[L']$$

In the tensor $[Z']$, the following simplification shall be made:

$$A=\frac{x_d+x_q}{2}, \qquad B=\frac{x_d-x_q}{2}.$$

Then, the above machine constants shall be replaced by the three-phase constants; namely $x_{rd}$ by $\frac{2}{3}(x_d-x_o)$, $x_d$ by $\frac{2}{3}(x_d+\frac{x_o}{2})$,

$x_{rq}$ by $\frac{2}{3}(x_q-x_o)$, $x_q$ by $\frac{2}{3}(x_q+\frac{x_o}{2})$.

If the unnecessary term $f$ is eliminated (assuming the excitation voltage $E_f=0$) by a short-circuit matrix, the final impedance tnsor is

$[Z']=\frac{p}{3}$

| | $a$ | $b$ | $c$ |
|---|---|---|---|
| $a$ | $x_o+2A+2B\cos 2\theta_1$ | $x_o-A+2B\cos\overline{\theta_1+\theta_2}$ | $x_o-A+2B\cos\overline{\theta_1+\theta_3}$ |
| $b$ | $x_o-A+2B\cos\overline{\theta_1+\theta_2}$ | $x_o+2A+2B\cos 2\theta_2$ | $x_o-A+2B\cos\overline{\theta_2+\theta_3}$ |
| $c$ | $x_o-A+2B\cos\overline{\theta_1+\theta_3}$ | $x_o-A+2B\cos\overline{\theta_2+\theta_3}$ | $x_o+2A+2B\cos 2\theta_3$ |

where[(2)] $A=\frac{x_d(p)+x_q(p)}{2}$, $B=\frac{x_d(p)-x_q(p)}{2}$.

$x_d(p)$ and $x_q(p)$ in this case should be equal to $x_d'$ and $x_q$ respectively from the approximation $[R]=0$. Now, since the steady state phenomena will be discussed here, $p=j$ should be permitted in all above equations (using the per-unit-method, $\omega=1$).

Nextly, it is assumed that the transmission line which is treated here has no resistance, inductance and leakance but has only capacitance between line-to-earth and line-to-line; of which capacitance can be transformed to the equivalent balanced star circuit capacities, one side of capacity being $C$.

So, $\dfrac{1}{pwC}=-j\dfrac{1}{C}=-jx_c$ is taken.

Then, the phenomena with a line-earth fault can be represented by a next equivalent circuit, Fig. 1.

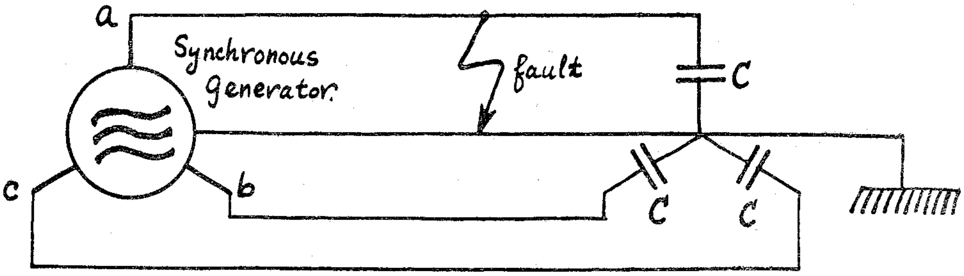


Fig. 1. Equivalent circuit of a line-fault.

The circuit, Fig. 1. may be considered as equal to a circuit Fig. 2,

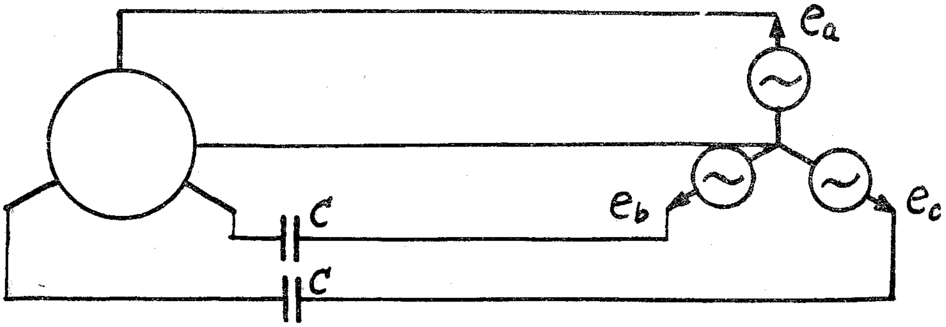


Fig. 2. Equivalent circuit of Fig. 1.

where $e_a=e\sin\theta_1$ , $e_b=e\sin\theta_2$ , $e_c=e\sin\theta_3$ , and exitation D.C. source in a generator is assumed to be absence.

Accordingly, the differential equation will be

$$-\underset{\cdot}{e} = j(\underset{\cdot}{Z} - \underset{\cdot}{x_c})\,\underset{\cdot}{i}\,.$$

Since $j$ means differentiation, the both sides of the equation should be integrated, resulting

$$\begin{bmatrix} 3\,e\cos\theta_1 \\ 3\,e\cos\theta_2 \\ 3\,e\cos\theta_3 \end{bmatrix} = \begin{bmatrix} x_o+2A+2B\cos 2\theta, & x_o-A+2B\cos\overline{2\theta+120}, & x_o-A+2B\cos\overline{2\theta-120} \\ x_o-A+2B\cos\overline{2\theta+120}, & x_o+2A+2B\cos\overline{2\theta-120}-3x_c, & x_o-A+2B\cos 2\theta \\ x_o-A+2B\cos\overline{2\theta-120}, & x_o-A+2B\cos 2\theta, & x_o+2A+2B\cos\overline{2\theta+120}-3x_c \end{bmatrix}[i].$$

The inverse calculation may be made as follows:

$$[i]=\frac{e}{D}\begin{bmatrix} a & b & c \\ g & h & k \\ l & m & n \end{bmatrix}$$

where

$$D=2x_c(x_c-x_o)B\cos 2\theta+(A^2-B^2)(3x_o-2x_c)+x_c^2(x_o+2A)-4x_cx_oA.$$
$$a=A^2-B^2+2x_oA+3x_c^2-2x_c(x_o+2A)+2B(x_c-x_o)\cos 2\theta.$$
$$b=A^2-B^2-x_oA-x_c(A-x_o)+2B(x_c-x_o)\cos\overline{2\theta+120}.$$
$$c=A^2-B^2-x_oA-x_c(A-x_o)+2B(x_c-x_o)\cos\overline{2\theta-120}.$$
$$h=A^2-B^2+2x_oA-x_c(x_o+2A)-2B(x_c\cos 2\theta+x_o\cos\overline{2\theta-120}).$$
$$k=A^2-B^2-x_oA-2x_oB\cos 2\theta.$$
$$n=A^2-B^2+2x_oA-x_c(x_o+2A)-2B(x_c\cos 2\theta+x_o\cos\overline{2\theta+120}).$$

$$g=b, \qquad l=c, \qquad m=k.$$

Accordingly,

$$-i_a=\frac{3e}{D}(x_c-x_o)(A-B-x_c)\cos\theta.$$

$$-i_b=\frac{e}{D}\{-3x_o(A-B)\cos\theta_2+x_c(A-B)(\cos\theta_2-\cos\theta_3)-x_cx_o(\cos\theta_1-\cos\theta_2)\}.$$

$$-i_c=\frac{e}{D}\{-3x_o(A-B)\cos\theta_3+x_c(A-B)(\cos\theta_3-\cos\theta_2)-x_cx_o(\cos\theta_1-\cos\theta_3)\}.$$

From the equation of $i_a$, the resonance condition can be obtained. Namely, if

$$\alpha=(A^2-B^2)(3x_o-2x_c)+x_c^2(x_o+2A)-4x_cx_oA,$$
$$\beta=-x_c(x_c-x_o)B,$$
$$\gamma=3e(x_c-x_o)(A-B-x_c),$$

then

$$i_a=\frac{-\gamma\cos\theta}{\alpha-2\beta\cos 2\theta}=\frac{-2\gamma}{\alpha-2\beta+\sqrt{\alpha^2-4\beta^2}}\sum_{n=1,3,\dots}^{\infty}\eta^{\frac{n-1}{2}}\cos n\theta.$$

$$\eta=\frac{1}{2\beta}\{\alpha-\sqrt{\alpha^2-4\beta^2}\}<1.$$

When $\alpha-2\beta=0$, $i_a$ diverts to $\infty$. The resonance condition is then:

$$(A^2-B^2)(3x_o-2x_c)+x_c^2(x_c+2A)-4x_cx_oA+2x_c(x_c-x_o)B=0.$$

Introducing $\quad A=\dfrac{x_d'+x_q}{2}\quad$ and $\quad B=\dfrac{x_d'+x_q}{2}$,

$$\therefore\; x_c=\frac{2x_d'x_q+x_o(3x_d'+x_q)\pm\sqrt{\{2x_d'x_q+x_o(3x_d'+x_q)\}^2-12x_d'x_qx_o(2x_d'+x_o)}}{2(2x_d'+x_o)}.$$

Assuming $x_o \ll x_d'$, $x_o \ll x_q$, the expression inside the root can be approximated. And the final result will be

$$x_c = x_q \qquad \text{or} \qquad \frac{3x_o x_d'}{2x_d' + x_o} \, .$$

However, as $x_c = x_q$ will made also, $\gamma$, the numerator of $i_a$, equal to zero, it should be excluded. Accordingly,

$$x_c \Big/ \frac{3x_o x_d'}{2x_d' + x_o} = n^2$$

where $n$ is the dimension of the harmonics considered. This coincides with the result in the former publication as stated before.

As already stated in the former publication, this research had been made during two months of the last summer vacation at Hokkaido University. The author wishes to express his gratitude for much valuable guidance of Prof. Ogushi of the university, and for much assistance recieved from Dr. Iguchi, the president, and proffesors of Electrical Department, of the Muroran University of Engineering.

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