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On Numerical Differentiation

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Abstract

In this report the writer investigates a practical method for numerical differentiation of a tabulated function, which does not employ the differences of the tabulated values. Short tables are added for convenience of practical applications.

It is frequently necessary to determine the derivatives of a function from their tabulated values. Up to the present, usual method for such a purpose depends mainely on the interpolation formulas containing the differences of the tabulated values. Since the construction of the tables of differences is very troublesome for the computers, it is rather desirable and convenient to express the derivative of a function approximately by a linear combination of the tabulated values whenever calculating machine is available.

The present writer, as outlined below, proposes a direct method for numerical differentiation which dose not employ the differences of the tabulated values, basing on the Lagrange's interpolation formula.

Now, let $f(x)$ be a polynomial of n-th degree which assumes the values $y_0, y_1, y_2, \cdots\cdots, y_n$ respectively at the points $x_0, x_1, x_2, \cdots\cdots, x_n$; where $x's$ are all distinct. Then the polynomial $f(x)$ can be reduced to the form known as the Lagrange's interpolation formula

$$f(x)=\sum_{j=0}^{n} y_j \frac{(x-x_0)(x-x_1)\cdots(x-x_{j-1})(x-x_{j+1})\cdots(x-x_n)}{(x_j-x_0)(x_j-x)\cdots(x_j-x_{j-1})(x_j-x_{j+1})\cdots(x_j-x_n)}$$

In the important case in which the values of the $x's$ are equally spaced so that

$$x_1-x_0=x_2-x_1=\cdots=x_n-x_{n-1}=h$$

the formula becomes

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$$f(x)=\sum_{j=0}^{n} y_j(-1)^{n+j}[\binom{n}{j}/n!]s(s-1)(s-2)\cdots(s-j-1)(s-j+1)\cdots(s-n),$$

where we set $x=x_0+hs$ and $\binom{n}{j}$ as usual denotes the binomial coefficient.

Differentiation of the above equation gives

$$f'(x)=\frac{1}{h}\frac{df}{ds}=\frac{1}{h}\sum_{j=0}^{n} D_{nj}(s)y_j$$

where

$$D_{nj}(s)=(-1)^{n+j}[\binom{n}{j}/n!]\frac{d}{ds}\left[\prod_{k=0}^{n}(s-k)/(s-j)\right]$$

If the tables of the coefficients $D_{nj}(s)$ are prepared for the possible values of n, j, and s, the computaions of the derivatives of the tabulated functions from their tabulated values will be much facilitated by the aid of calculating machines.

The writer has prepared short tables of the coefficients $D_{nj}(s)$ for $n=2$, 3, 4, 5 and for range from $s=0.00$ to $s=0.50$ at intervals of 0.01.

These tables apply to the problems where equally spaced values of x : x_0, x_1, x_2, ... are given and the point x at which differentiation is to be made is nearer to x_0, than to x_1. If the point x is nearer to x_1 than to x_0, the points are to be numbered ..... x_2, x_1, x_0.

In either case $s=(x-x_0)/(x_1-x_0)$.

(Recieved May 31, 1955)

s	D_{20}	D_{21}	D_{22}	s	D_{30}	D_{31}	D_{32}	D_{33}
0.00	—1.50	2.00	—0.50	0.00	—1.83333	3.00000	—1.50000	0.33333
0.01	—1.49	1.98	—0.49	0.01	—1.81338	2.95015	—1.46015	0.32338
0.02	—1.48	1.96	—0.48	0.02	—1.79352	2.90060	—1.42060	0.31353
0.03	—1.47	1.94	—0.47	0.03	—1.77378	2.85135	—1.38135	0.30378
0.04	—1.46	1.92	—0.46	0.04	—1.75413	2.80240	—1.34240	0.29413
0.05	—1.45	1.90	—0.45	0.05	—1.73458	2.75375	—1.30375	0.28458
0.06	—1.44	1.88	—0.44	0.06	—1.71513	2.70540	—1.26540	0.27513
0.07	—1.43	1.86	—0.43	0.07	—1.69578	2.65735	—1.22735	0.26578
0.08	—1.42	1.84	—0.42	0.08	—1.67653	2.60960	—1.18960	0.25653
0.09	—1.41	1.82	—0.41	0.09	—1.65738	2.56215	—1.15215	0.24738
0.10	—1.40	1.80	—0.40	0.10	—1.63833	2.51500	—1.11500	0.23833
0.11	—1.39	1.78	—0.39	0.11	—1.61938	2.46815	—1.07815	0.22938
0.12	—1.38	1.76	—0.38	0.12	—1.60053	2.42160	—1.04160	0.22053
0.13	—1.37	1.74	—0.37	0.13	—1.58178	5.37535	—1.00535	0.21178
0.14	—1.36	1.72	—0.36	0.14	—1.56313	2.32940	—0.96940	0.20313
0.15	—1.35	1.70	—0.35	0.15	—1.54458	2.28375	—0.93375	0.19458
0.16	—1.34	1.68	—0.34	0.16	—1.52613	2.23840	—0.89840	0.18613
0.17	—1.33	1.66	—0.33	0.17	—1.50778	2.19335	—0.86335	0.17778
0.18	—1.32	1.64	—0.32	0.18	—1.48953	2.14860	—0.82860	0.16953
0.19	—1.31	1.62	—0.31	0.19	—1.47138	2.10415	—0.79415	0.16138
0.20	—1.30	1.60	—0.30	0.20	—1.45333	2.06000	—0.76000	0.15333
0.21	—1.29	1.58	—0.29	0.21	—1.43538	2.01615	—0.72615	0.14538
0.22	—1.28	1.56	—0.28	0.22	—1.41753	1.97260	—0.69260	0.13753
0.23	—1.27	1.54	—0.27	0.23	—1.39978	1.92935	—0.65935	0.12978
0.24	—1.26	1.52	—0.26	0.24	—1.38213	1.88640	—0.62640	0.12213
0.25	—1.25	1.50	—0.25	0.25	—1.36458	1.84375	—0.59375	0.11458
0.26	—1.24	1.48	—0.24	0.26	—1.34713	1.80140	—0.56140	0.10713
0.27	—1.23	1.46	—0.23	0.27	—1.32978	1.75935	—0.52935	0.09978
0.28	—1.22	1.44	—0.22	0.28	—1.31253	1.71760	—0.49760	0.09253
0.29	—1.21	1.42	—0.21	0.29	—1.29538	1.67615	—0.46615	0.08538
0.30	—1.20	1.40	—0.20	0.30	—1.27833	1.63500	—0.43500	0.07833
0.31	—1.19	1.38	—0.19	0.31	—1.26138	1.59415	—0.40415	0.07138
0.32	—1.18	1.36	—0.18	0.32	—1.24453	1.55360	—0.37360	0.06453
0.33	—1.17	1.34	—0.17	0.33	—1.22778	1.51335	—0.34335	0.05778
0.34	—1.16	1.32	—0.16	0.34	—1.21113	1.47340	—0.31340	0.05113
0.35	—1.15	1.30	—0.15	0.35	—1.19458	1.43375	—0.28375	0.04458
0.36	—1.14	1.28	—0.14	0.36	—1.17813	1.39440	—0.25440	0.03813
0.37	—1.13	1.26	—0.13	0.37	—1.16178	1.35535	—0.22535	0.03178
0.38	—1.12	1.24	—0.12	0.38	—1.14553	1.31660	—0.19660	0.02553
0.39	—1.11	1.22	—0.11	0.39	—1.12938	1.27815	—0.16815	0.01938
0.40	—1.10	1.20	—0.10	0.40	—1.11333	1.24000	—0.14000	0.01333
0.41	—1.09	1.18	—0.09	0.41	—1.09738	1.20215	—0.11215	0.00738
0.42	—1.08	1.16	—0.08	0.42	—1.08153	1.16460	—0.08460	0.00153
0.43	—1.07	1.14	—0.07	0.43	—1.06578	1.12735	—0.05735	—0.00422
0.44	—1.06	1.12	—0.06	0.44	—1.05013	1.09040	—0.03040	—0.00987
0.45	—1.05	1.10	—0.05	0.45	—1.03458	1.05375	—0.00375	—0.01542
0.46	—1.04	1.08	—0.04	0.46	—1.01913	1.01740	0.02260	—0.02087
0.47	—1.03	1.06	—0.03	0.47	—1.00378	0.98135	0.04865	—0.02622
0.48	—1.02	1.04	—0.02	0.48	—0.98853	0.94560	0.07440	—0.03147
0.49	—1.01	1,02	—0.01	0.49	—0.97338	0.91015	0.09985	—0.03662
0.50	—1.00	1.00	0.00	0.50	—0.95833	0.87500	0.12500	—0.04167

s	D_{40}	D_{41}	D_{42}	D_{43}	D_{44}
0.00	—2.08333 33	4.00000 00	—3.00000 00	1.33333 33	—0.25000 00
0.01	—2.05429 15	3.91378 27	—2.90559 90	1.28701 60	—0.24090 82
0.02	—2.02549 87	3.82846 13	—2.80239 20	1.24139 47	—0.23196 53
0.03	—1.99695 38	3.74403 20	—2.71037 30	1.96465 33	—0.22317 05
0.04	—1.96865 60	3.66049 07	—2.61953 60	1.15222 40	—0.21452 27
0.05	—1.94060 42	3.57783 33	—2.52987 50	1.10866 67	—0.20602 08
0.06	—1.91279 73	3.49605 60	—2.44138 40	1.06578 93	—0.19766 40
0.07	—1.88523 45	3.41515 47	—2.35405 70	1.02358 80	—0.18945 12
0.08	—1.85791 47	3.33512 53	—2.26788 80	0.98205 87	—0.18138 13
0.09	—1.83083 68	3.25596 40	—2.18287 10	0.94119 73	—0.17345 35
0.10	—1.80400 00	3.17766 67	—2.09900 00	0.90100 00	—0.16566 67
0.11	—1.77740 32	3.10022 93	—2.01626 90	0.86146 27	—..15801 98
0.12	—1.75104 53	3.02364 80	—1.93467 20	0.82258 13	—0.15051 20
0.13	—1.72492 55	2.94791 87	—1.85420 30	0.78435 20	—0.14314 22
0.14	—1.69904 27	2.87303 73	—1.77485 60	0.74677 07	—0.13590 93
0.15	—1.67339 58	2.79900 00	—1.69662 50	0.70983 33	—0.12881 25
0.16	—1.64798 40	2.72580 27	—1.61950 40	0.67353 60	—0.12185 07
0.17	—1.62280 62	2.65344 13	—1.54348 70	0.63787 47	—0.11502 28
0.18	—1.59786 13	2.58191 20	—1.46856 80	0.60284 53	—0.10832 80
0.19	—1.57314 85	2.51121 07	—1.39474 10	0.56844 40	—0.10176 52
0.20	—1.54866 67	2.44133 33	—1.32000 00	0.53466 67	—0.09533 33
0.21	—1.52441 48	2.37227 60	—1.25033 90	0.50150 93	—0.08903 15
0.22	—1.50039 20	2.30403 47	—1.17975 20	0.46896 80	—0.08285 87
0.23	—1.47659 72	2.23660 53	—1.11023 33	0.43703 87	—0.07681 38
0.24	—1.45302 93	2.16998 40	—1.04177 60	0.40571 73	—0.07089 60
0.25	—1.42968 75	2.10416 67	—0.97437 50	0.37500 00	—0.06510 42
0.26	—1.40657 07	2.03914 93	—0.90802 40	0.34488 27	—0.05943 73
0.27	—1.38367 78	1.97492 80	—0.84271 70	0.31536 13	—0.05389 45
0.28	—1.36100 80	1.91149 87	—0.77344 80	0.28643 20	—0.04847 47
0.29	—1.33856 02	1.84885 73	—0.71521 10	0.25809 07	—0.04317 68
0.30	—1.31633 33	1.78700 00	—0.65300 00	0.23033 33	—0.03800 00
0.31	—1.29432 65	1.72592 27	—0.59180 90	0.20315 60	—0.03294 32
0.32	—1.27253 87	1.66562 13	—0.53163 20	0.17655 47	—0.02800 53
0.33	—1.25096 88	1.60609 20	—0.47246 30	0.15052 53	—0.02318 55
0.34	—1.22961 60	1.54733 07	—0.41429 60	0.12506 40	—0.01848 27
0.35	—1.20847 92	1.48933 33	—0.35712 50	0.10016 67	—0.01389 58
0.36	—1.18755 73	1.43209 60	—0.30094 40	0.07582 93	—0.00942 40
0.37	—1.16684 95	1.37561 47	—0.24574 70	0.05204 80	—0.00506 62
0.38	—1.14635 47	1.31988 53	—0.19152 80	0.02881 87	—0.00082 13
0.39	—1.12607 18	1.26490 40	—0.13828 10	0.00613 73	0.00331 15
0.40	—1.10600 00	1.21066 67	—0.08600 00	—0.01600 00	0.00733 33
0.41	—1.08613 82	1.15716 93	—0.03467 90	—0.03759 73	0.01124 52
0.42	—1.06648 53	1.10440 80	0.01568 80	—0.05865 87	0.01504 80
0.43	—1.04704 05	1.05237 87	0.06510 70	—0.07918 80	0.01874 28
0.44	—1.02780 27	1.00107 73	0.11358 40	—0.09918 93	0.02233 07
0.45	—1.00877 08	0.95050 00	0.16112 50	—0.11866 67	0.02581 25
0.46	—0.98994 40	0.90064 27	0.20773 60	—0.13762 40	0.02918 93
0.47	—0.97132 12	0.85150 13	0.25342 30	—0.15606 53	0.03246 22
0.48	—0.95290 13	0.80307 20	0.29819 20	—0.17399 47	0.03563 20
0.49	—0.93468 35	0.75535 07	0.34204 90	—0.19141 60	0.03869 98
0.50	—0.91666 67	0.70833 33	0.38500 00	—0.20833 33	0.04166 67

s	D_{50}	D_{51}	D_{52}	D_{53}	D_{54}	D_{55}
0.00	—2.28333 33	5.00000 00	—5.00000 00	3.33333 33	—1.25000 00	0.20000 00
0.01	—2.24604 53	4.87255 18	—4.82313 73	3.20455 43	—1.19967 73	0.19175 38
0.02	—2.20917 93	4.74686 47	—4.64919 87	3.07820 14	—1.15036 87	0.18368 07
0.03	—2.17273 24	4.62292 47	—4.47815 83	2.95425 07	—1.10206 32	0.17577 85
0.04	—2.13670 14	4.50071 79	—4.30999 04	2.83267 84	—1.05474 99	0.16804 54
0.05	—2.10103 36	4.38023 05	—4.14466 93	2.71346 09	—1.00841 80	0.16047 94
0.06	—2.05687 59	4.26144 87	—3.98216 94	2.59657 47	—0.96305 67	0.15307 85
0.07	—2.03107 53	4.14435 88	—3.82246 53	2.48199 63	—0.91865 53	0.14584 08
0.08	—1.99667 90	4.02894 72	—3.66553 17	2.36970 24	—0.87520 32	0.13876 44
0.09	—1.96268 41	3.91520 02	—3.51134 33	2.25966 97	—0.83268 97	0.13184 72
0.10	—1.92908 75	3.80310 42	—3.35987 50	2.15187 50	—0.79110 42	0.12508 75
0.11	—1.89588 64	3.69264 57	—3.21110 17	2.04629 53	—0.75043 62	0.11848 33
0.12	—1.86307 80	3.58381 12	—3.06499 84	1.94290 77	—0.71067 52	0.11203 26
0.13	—1.83065 92	3.47658 73	—2.92154 03	1.84168 93	—0.67181 08	0.10573 37
0.14	—1.79862 73	3.37096 07	—2.78070 27	1.74261 74	—0.63383 27	0.09958 47
0.15	—1.76697 94	3.26691 80	—2.64246 09	1.64566 93	—0.59673 05	0.09358 36
0.16	—1.73571 26	3.16444 59	—2.50679 04	1.55082 24	—0.56049 39	0.08772 86
0.17	—1.70482 41	3.06353 12	—2.37366 67	1.45805 43	—0.52511 27	0.08201 80
0.18	—1.67431 11	2.96416 07	—2.24306 54	1.36734 27	—0.49057 67	0.07644 97
0.19	—1.64417 06	2.86632 13	—2.11496 23	1.27866 53	—0.45687 42	0.07102 21
0.20	—1.61440 00	2.77000 00	—1.98933 33	1.19200 00	—0.42400 00	0.06573 33
0.21	—1.58499 64	2.67518 37	—1.86615 43	1.10732 47	—0.39193 92	0.06058 15
0.22	—1.55595 69	2.58185 94	—1.74540 14	1.02461 74	—0.36068 34	0.05556 49
0.23	—1.52727 89	2.49001 42	—1.62705 07	0.94385 63	—0.33022 27	0.05065 18
0.24	—1.49895 96	2.39963 52	—1.51107 84	0.86501 97	—0.30054 72	0.04590 02
0.25	—1.47099 61	2.31070 96	—1.39746 09	0.78808 59	—0.27164 71	0.04127 86
0.26	—1.44338 57	2.22322 47	—1.28617 47	0.71303 34	—0.24351 27	0.03678 51
0.27	—1.41612 58	2.13716 77	—1.17719 63	0.63984 07	—0.21613 42	0.03241 79
0.28	—1.38921 34	2.05252 59	—1.07050 24	0.56848 64	—0.18950 19	0.02817 54
0.29	—1.36264 60	1.96928 67	—0.96606 97	0.49894 93	—0.16360 62	0.02405 59
0.30	—1.33642 08	1.88743 75	—0.86387 50	0.43120 83	—0.13843 75	0.02005 75
0.31	—1.31053 51	1.80696 58	—0.76389 53	0.36524 23	—0.11398 63	0.01617 86
0.32	—1.28498 62	1.72785 92	—0.66610 77	0.30103 04	—0.09024 32	0.01241 76
0.33	—1.25977 15	1.65010 52	—0.57048 93	0.23855 17	—0.06719 87	0.00877 26
0.34	—1.23488 81	1.57369 14	—0.47701 74	0.17778 54	—0.04484 34	0.00524 21
0.35	—1.21033 36	1.49860 55	—0.38566 93	0.11871 09	—0.02316 80	0.00182 44
0.36	—1.18610 52	1.42483 52	—0.29642 24	0.06130 77	—0.00216 32	—0.00148 22
0.37	—1.16220 02	1.35236 83	—0.20925 43	0.00555 53	0.01818 02	—0.00467 93
0.38	—1.13861 61	1.28119 27	—0.12414 27	—0.04856 66	0.03787 13	—0.00776 85
0.39	—1.11535 03	1.21129 62	—0.04106 53	—0.10107 83	0.05691 93	—0.01075 16
0.40	—1.09240 00	1.14266 67	0.04000 00	—0.15200 00	0.07533 33	—0.01363 00
0.41	—1.06976 27	1.07529 22	0.11907 53	—0.20135 17	0.09312 23	—0.01640 54
0.42	—1.04743 59	1.00916 07	0.19618 26	—0.24915 33	0.10295 30	—0.01907 95
0.43	—1.02541 68	0.94426 03	0.27134 37	—0.29542 47	0.12686 12	—0.02165 37
0.44	—1.00370 30	0.88057 92	0.34458 03	—0.34018 56	0.14282 88	—0.02412 96
0.45	—0.98229 19	0.81810 55	0.41591 41	—0.38345 57	0.15820 70	—0.02650 89
0.46	—0.96118 09	0.75682 74	0.48536 66	—0.42525 46	0.17300 46	—0.02879 31
0.47	—0.94036 75	0.69673 32	0.55295 93	—0.46560 17	0.18723 03	—0.03098 36
0.48	—0.91984 92	0.63781 12	0.61871 36	—0.50451 63	0.20089 28	—0.03308 22
0.49	—0.89962 33	0.58004 98	0.68265 07	—0.54201 77	0.21400 07	—0.03509 02
0.50	—0.87968 75	0.52343 75	0.74479 17	—0.57812 50	0.22656 25	—0.03700 92