A Study of the Design and Stress Calculation of the Simple Box Girder Bridge with Varying Sections and Steel Floor Plates

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Sakutaro Nakamura*

Abstract

The present writer tentatively designed the simple box girdr bridge having the span length 50 m, the effective width 6 m and the steel floor board as the test case of the 1st class highway bridge. He treated the economical and structural problems of this bridge.

He considers that the girder bridge of one box is most suitable for the above-mentioned highway bridge, because the effective width of this bridge is comparatively small as the bridge having the steel floor board.

Furthermore he adopted the box girder bridge having the varying sections considering of the economy and the specific character in structures, and selected the superior shape out of many shapes, with reference to the result of the already completed photoelastic experiments.

1. Introduction

Recently the composite box girder bridge and the box girder bridge having the steel floor board have been constructed vigorously for the middle class span 40 m∼100 m that seems to be too unreasonable to adopt the common plate girder[1)~3)] or the composite plate girder[9),18)].

The composite box girder bridge has such a special character that the solid steel structure of thin plate and the floor board of reinforced concrete have been made connectively to bring the reduction of dead loads and united stiffness.

On the contrary, the box girder bridge[4),5)] having the steel floor board has such a special character that the more great reduction of dead loads has been brought and the steel floor board itself has directly cooperated with the action of main girder as one part of the upper flange of box girder bridge.

Moreover, it is reported that the connection of the steel floor board is very firm because the welding and its breaking strength is unexpectedly great.

In the composite box girder bridge, we have to apply the method of prestrest concrete to the structure of the domain bridge having minus bending moments as the case of the Gerbar girder bridge and the continious girder bridge, because the tensile strength of concrete is very little, and we must greatly consider the method of composite construction.

On the other hand, we can expect the more effective cooperative action

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because there is not the above-mentioned restriction in the bridge having the steel floor board, and we can enlarge more and more the span of a bridge by the great reduction of steel materials and dead loads.

The box girder bridge having the steel floor board has progressed rapidly with the composite box girder bridge in Germany since the end of the War, and gradually it has been closely studied by many research workers of each nation, and it is the structural type that has been constructed practically in various places.

The present writer tentatively designed a simple box girder bridge of one box type having the varying sections and the steel floor board as the test case of the 1st class highway bridge of the span length 50 m and the effective width 6 m, and he studied some problems of the economy and the structural mechanics in this box girder bridge.

In the tentative design of this economical type, the present writer referred data of the planning investigation for the Jogashima Large Bridge of the initial type in Japan, and he selected the type of varying sections that seems to be most excellent from a viewpoint of the economy and the structural stability, considering the special structural character of the simple girder bridge.

Still more he used specially the most excellent shape in the structural economics, referring to the results of the experiment of photoelastic models having the varying rectangular sections completed and reported already. (See Fig.-1, Fig.-2)

Next, he calculated the stress of steel floor board, the bending moment, the shearing force and the deflection of the main girder, the stability for buckling of web plate, the stress for bearing compression power of supported ends in the main girder, the stress for torsional and shearing flow of the main girder, the stress of floor system, the stress of shoe on both ends, the stress of the balustrade, the stress of the joining part and the stress for the temperature change of this bridge.

In conclusion, he made the list of materials in the above-mentioned box girder and treated of the decrease of steel materials which was brought by using the varying section type, and compared it with any other type of bridge from the economical viewpoint.

2. Summary of the Design[6)]

Enumerating the condition of the design, the outline plan, the list of materials, ect., these are as follows :

(1) The condition of the design

a. Kind of bridge : 1st class highway bridge (Load : T-20, L-20)
b. Type : Simple box girder bridge with steel floor plates (1-box type)
c. Span : 50 m
d. Effective width : 6 m (Roadway without distinction of footway and carriageway)
e. Pavement : Guss asphalt pavement (Thichness 5 cm)

f. Waterproof layer: Waterproof layer of thickness 1 cm on steel floor board
g. Floor board: Steel plates (Minimum thickness 12 mm)
h. Slope of cross section: Parabolic grade of 1/50
i. High tension steel: Allowable stress of (H.S.J. 50)[7]
(i) Axial tensile stress intensity (per net section) 1,900 kg/cm²
(ii) Axial compressive stress intensity (per gross section)

$l/r \leqq 90 \cdots\cdots 1{,}700 - 0.1032\ (l/r)^2$ kg/cm²

$l/r > 90 \cdots\cdots\cdots\cdots 7{,}000{,}000/(l/r)^2$ kg/cm²

However, l: Length of a member (cm)
r: Section of a member (cm)

(iii) Bending stress intensity
Tension flange (per net section) 1,900 kg/cm²
Compression flange (per gross section) $1{,}700 - 1.0\ (l/b)^2$ kg/cm²
However, l: Distance between two flange fixed points (cm)
b: Width of flange (cm)

(iv) Shearing stress intensity
Web plate of box girder (per net section) 1,300 kg/cm²
Shop rivet (SV 41 A) 1,300 kg/cm²
Field rivet (SV 41 A) 1,200 kg/cm²

(v) Bearing compression stress intensity
Shop rivet (SV 41 A) 2,600 kg/cm²
Field rivet (SV 41 A) 2,300 kg/cm²

j. Applied specifications
(i) Specifications of steel highway bridge[8]
(The committee drew out the same specifications, 1956)
(ii) Specifications of welded steel highway bridge[9]
(The committee drew out the same specifications, 1957)
(iii) Design manual for high strength steels[7]
(Toto steel Manufacture Company, 1955)

k. Designning load
(i) Live load (T–20, L–20)
a) T–load

Table 1.

Total weight W (t)	Front wheel load 0.1 W (kg)	Hind wheel 0.4 W (kg)	Tire width of front wheel b_1 (cm)	Tire width of front wheel b_2 (cm)	Grounding length of wheel a (cm)
20	2,000	8,000	12.5	50	20

b) L-load

i) Value of α $\quad \alpha = 1 - \dfrac{w-55}{50} = 1.01$

$1 \geqq \alpha \geqq 0.75 \qquad \alpha = 1.0$

ii) Uniform distribution load $\quad p_r = \alpha \times 350 \times 6.0 = 2{,}100$ kg/m

iii) Line load $\quad P = \alpha \times 5{,}000 \times 6.0 = 30{,}000$ kg

iv) Crowd load (including snow load) $\quad p_s = 350 \times 6.0 = 2{,}100$ kg/m^2

v) Coefficient of impact $\quad i = \dfrac{20}{50+l} = 0.20$

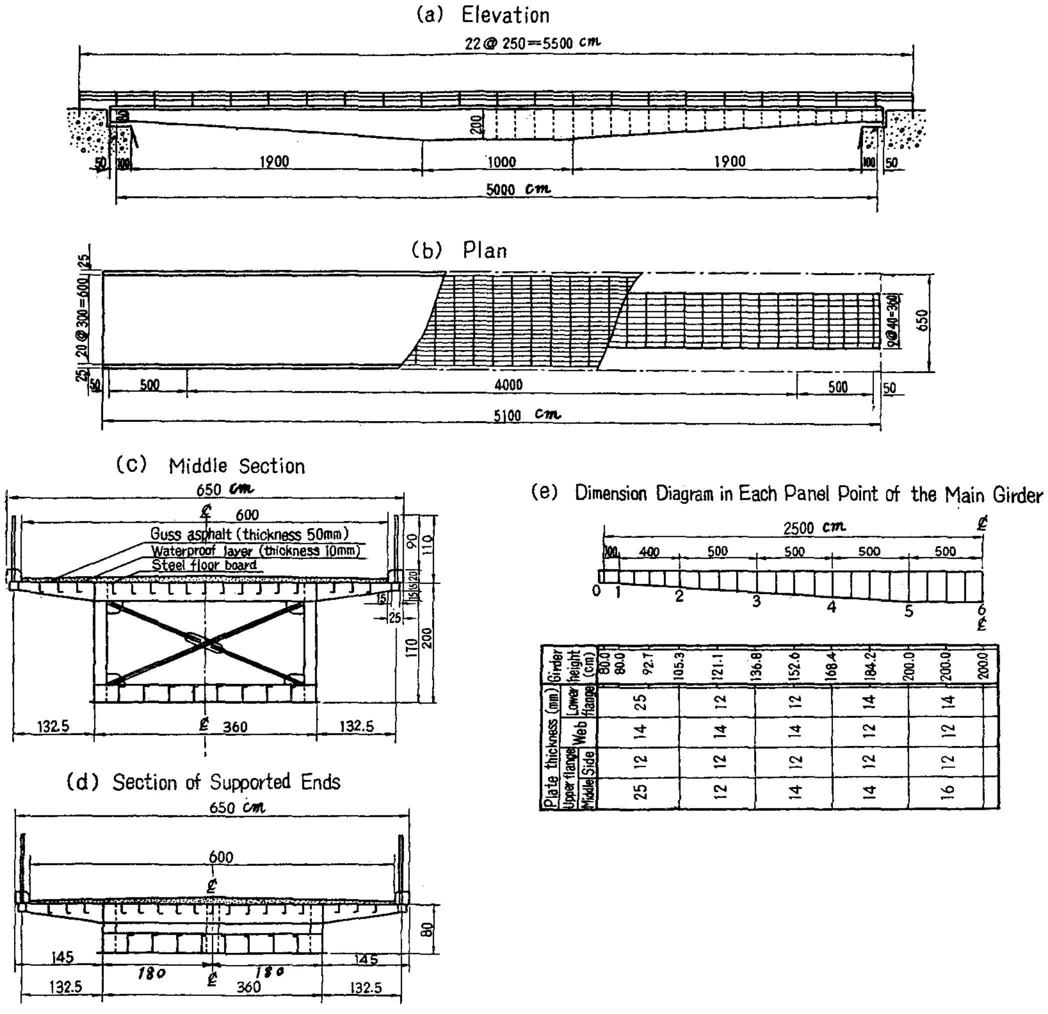

Fig. 1. Out line Plan of a Box Girder Bridge with Varying Sections and Steel Floor Plates.

(ii) Dead load (assumed)

Table 2.

Dead load of box girder bridge with steel floor plates

Pavement	240 kg/m	12 t
Handrail, Footing block	120 kg/m	6 t
Steel floor plates, Ribs	1,240 kg/m	62 t
Steel plates of box giroder	1,000 kg/m	50 t
Steel plates of cantilever part	260 kg/m	13 t
The other part	160 kg/m	8 t
Total weight	3,020 kg/m	151 t

(2) Outline of the designed plan

The present writer made a design and calculation in the above-mentioned conditions, and decided the section, the shape and the connecting arrangement of every part in the bridge as Fig. 1.

(3) List of materials

He made the list of materials from such result of design as in Table 3.

Table 3. List of materials of box girder bridge with varying sections and steel floor plates (Super structure only)

Materials used	Place of using	Demension of cross section	Total length	Number	Total weight	Remarks
Water proof stuff	Water proof layer of floor board	10 mm×6.00 m	51.00 m		3,366 kg	Thickness 10 mm
Guss asphalt	Using in pavement	5 cm×6.00 m	51.00 m		7,920 kg	" 50 mm
Sum total					11,286 kg	Weight of pavement and water proof layer
Steel plate	Handrail, Footing block	6 mm×90 cm	51.00 m	2	4,324 kg	Thickness 6.0 mm
Steel pipe	Handrail	ϕ 76.3 mm	1.10 m	42	266 kg	" 3.2 mm
"	"	"	55.00 m	2	635 kg	" 3.2 mm
"	"	ϕ 48.6 mm	2.50 m	88	526 kg	" 2.4 mm
Total					5,751 kg	
Steel plate	Floor board (Middle part)	25 mm×3.60 m	5.50 m	2	7,771 kg	Thickness 25 mm
"	" (")	14 mm×3.00 m	20.00 m	2	1,319 kg	" 14 mm
"	" (Side part)	12 mm×1.45 m	51.00 m	2	13,932 kg	" 12 mm
Total					23,022 kg	

Materials used	Place of using	Demension of cross section	Total length	Number	Total weight	Remarks
Steel plate	Longitudinal rib (Upper part)	9 mm×0.25 m	51.00 m	17	15,313 kg	Thickness 9 mm
〃	〃 (Lower part)	9 mm×0.34 m	51.00 m	8	9,800 kg	〃 〃
〃	Lateral rib (Upper part)	9 mm×0.45 m	3.60 m	49	5,608 kg	〃 〃
〃	〃 (Lower part)	9 mm×0.50 m	3.60 m	49	6,231 kg	〃 〃
〃	〃 (Cantilever part)	9 mm×0.23 m	1.20 m	98	1,869 kg	〃 〃
Total					38,821 kg	
Steel plate	Web plate of box girder	25 mm×3.60 m	5.50 m	4	4,002 kg	Thickness 25 mm
〃	〃	14 mm×1.52 m	15.00 m	4	10,023 kg	〃 14 mm
〃	〃	14 mm×2.00 m	5.00 m	4	4,396 kg	〃 〃
Total					18,421 kg	
Steel plate	Lower plate of box girder	25 mm×3.70 m	5.50 m	2	7,987 kg	Thickness 25 mm
〃	〃	14 mm×3.70 m	20.00 m	2	16,265 kg	〃 14 mm
Total					24,252 kg	
Steel plate	Vertical stiffner	9 mm×0.20 m	1.50 m	43	908 kg	Average girder height 1.496 m
〃	Frame stiffner				1,000 kg	
〃	Sway bracing				1,000 kg	
Total					2,908 kg	
Steel plate	Cantilever part	8 mm×0.15 m	51.00 m	8	3,843 kg	End box
〃	〃	9 mm×1.23 m	51.00 m	2	8,863 kg	Bed plate
Total					12,706 kg	
Steel plate	Box girder, Cantilever part				7,000 kg	Used for processing
Sum total					132,881 kg	Weight of steel materials
Sum total weight					144,167 kg	

3. Investigations from a Viewpoint of Structural Mechanics[4),5),14)]

(1) Stress calculations of steel floor board

There are two methods applying the grid theory and the orthotropic plane theory in the stress solution of steel floor board.

The present writer applied the orthotropic plane theory to calculate the stress of those briefly, and solved the equation assuming that the floor plate of box girder is simply supported in the direction of x and it forms a infinitely long plate in the direction of y.

The equation by K. Girkmann is as follows.

$$\left.\begin{aligned} m_x &= -k\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \\ m_y &= -k\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \\ w &= \frac{pl^3}{k\pi^4}\sum_m \frac{1}{m^4}\left(1+\frac{m\pi y}{l}\right)e^{\frac{-m\pi y}{l}}\sin\frac{m\pi x}{l} \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} m_x &= \frac{2pl^2}{\pi^3}\sum(-1)^{\frac{m-1}{2}}\frac{1}{m^3}\left[2-\left\{2+(1-\nu)\,m\beta\right\}e^{-m\beta}\right] \\ m_y &= \frac{2pl^2}{\pi^3}\sum(-1)^{\frac{m-1}{2}}\frac{1}{m^3}\left[2\nu-\left\{2\nu-(1-\nu)\,m\beta\right\}e^{-m\beta}\right] \\ &\text{However,}\quad m=1,3,5,\quad \nu=\text{Poisson's ratio} \\ &\qquad a=\text{Load width},\quad \beta=\pi a/2l \end{aligned}\right\} \qquad (2)$$

From these equations (1) and (2), he calculated the bending moment and the stress intensity of cross section.

The uniform distribution load p is calculated by applying the hind wheel load P as the following equation.

$$p_r = \frac{P(1+i)}{(50+2b)(20+2b)} \quad (\text{kg/cm}^2) \qquad (3)$$

The present writer calculated bending moments m_x, m_y and stress intensities σ_x, σ_y in the floor plate assuming that the axis of y and x shows the direction of bridge axis and the orthogonal direction crossing it each other.

Then, from $t=12$ mm, $a=33.2$ cm, $l=30$ cm, $p_r=3.81$ kg/cm², $\beta=1.67$ and $\nu=0.3$, he got $m_x=303.6$ kg-cm/cm, $m_y=104.9$ kg-cm/cm

And from

$$\left.\begin{aligned} \sigma_x &= 0.8\, m_x\cdot 6\,(1+0.4)/t^2 \\ \sigma_y &= 0.8^2\, m_y\cdot 6\,(1+0.4)/t^2 \end{aligned}\right\} \qquad (4)$$

He got

$$\sigma_x = \pm 1{,}417\ \text{kg/cm}^2 < 1{,}900\ \text{kg/cm}^2$$

$$\sigma_y = \pm 392\ \text{kg/cm}^2 < 1{,}900\ \text{kg/cm}^2$$

Sufficiently safe.

(2) Calculations of bending stress intensities in the section of main girder

a. Bending moment of main girder

(i) Bending moment by live load

$$M_{l+i} = (P.\eta_M + p_r \cdot A)(1+i)$$

However, η_M: Vertical intervals in the influence diagram of bending moments
A: The Influence area of bending moments
i: Coefficients of impact
P: Line load
p_r: Uniform distribution load (5)

(ii) Bending moment by dead load

$$M_d = g_0 \cdot A$$

However, A: The influence area of bending moments
g_0: Total dead load (6)

b. Maximum fibre stress intensity by simple bending

$$\sigma_0 = \frac{M}{J_x} \cdot y_0 = \frac{M}{W_0}$$

$$\sigma_u = \frac{M}{J_x} \cdot y_u = \frac{M}{W_0}$$

W_0, W_u: Section modulus of upper or lower flange of main girder
σ_0, σ_u: Fibre stress intensity of upper or lower flange of main girder (7)

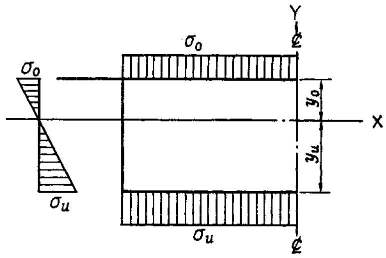

Fig. 2.

From the equation (7) he got σ_0 and σ_u as Table 4.

Table 4.

Classification	Panel point						
	0	1	2	3	4	5	6
M_{max} (t-m)	0	178.85	821.25	1,460.00	1,916.25	2,190.00	2,281.25
W_0 (cm^3)	7.25×10^4	7.25×10^4	10.14×10^4	15.45×10^4	19.70×10^4	26.35×10^4	26.35×10^4
W_u (cm^3)	4.84×10^4	4.84×10^4	6.78×10^4	10.48×10^4	13.50×10^4	18.28×10^4	18.28×10^4
$\sigma_{0\ max}$ (kg/cm^2)	0	−247	−810	−945	−973	−831	−866
$\sigma_{u\ max}$ (kg/cm^2)	0	370	1,210	1,393	1,419	1,198	1,248

As the result of Table 4, he found that values of $\sigma_{0\ max}$ and $\sigma_{u\ max}$ are within the limit of allowable stress intensity and sufficiently safe.

(3) Calculations of shearing stress intensities in the section of main girder

a. Shear of main girder

(i) Shear by live load

$$S_{l+i} = (P\cdot\eta_s + p_r\cdot A)\cdot(1+i) \tag{8}$$

However, η_s: Vertical intervals in the influence diagram of shear
A: The influence area of shears
i: Coefficients of impact
P: Line load
p_r: Uniform distribution load

(ii) Shear by Dead load

$$S_d = g_0\cdot A \tag{9}$$

However, A: The influence area of shears
g_0: Total dead load

b. Maximum shearing stress intensity

Maximum shearing stress intensity by simple bending is caused in the neutral axis of web plate, because the web plate is thinner than the flange plate in the ordinary place.

Then, the maximum shearing stress intensity is shown by the following formula :

$$\tau_{\max} = \frac{S}{2J_x t_w}\int_0^s yt\, ds = \frac{SG}{2J_x t_w} \tag{10}$$

However, S: Maximum shear of the section
t_w: Thickness of web plate
G: Geometrical moment of area on one side section around the neutral axis $x-x$

He calculated $\tau_{\max}$ by following the formula (10) and got values of Table 5.

These resuls are sufficiently safe because every value of panel points is within the limit of allowable stress intensity.

Table 5.

Classification	Panel point						
	0	1	2	3	4	5	6
$S_{\max}$ (t)	182.50	175.94	150.24	119.22	89.48	60.98	33.76
G (cm³)	3.37×10^4	3.37×10^4	4.63×10^4	7.00×10^4	8.92×10^4	11.85×10^4	11.85×10^4
J (cm⁴)	2.42×10^6	2.42×10^6	4.42×10^6	8.79×10^6	13.81×10^6	22.08×10^6	22.08×10^6
G/J (cm⁻¹)	13.93×10^{-3}	13.93×10^{-3}	10.48×10^{-3}	7.96×10^{-3}	6.46×10^{-3}	5.37×10^{-3}	5.37×10^{-3}
$S\cdot G/J$ (t-cm⁻¹)	$2{,}542\times10^{-3}$	$2{,}451\times10^{-3}$	$1{,}358\times10^{-3}$	949×10^{-3}	578×10^{-3}	327×10^{-3}	181×10^{-3}
t_w (cm)	1.6	1.6	1.4	1.4	1.4	1.2	1.2
$\tau_{\max}$ (kg/cm²)	794	766	549	339	206	136	75

(4) Calculations of deflection[4)]

Maximum deflection in the centre of simple beam with a constant geometrical moment of inertia I is shown by the following formulae:

$$\left.\begin{array}{l} y_{\max}=\dfrac{5\cdot ql^4}{384\,EI} \\ y_{\max}=\dfrac{P\cdot l^3}{48\,EI} \\ \text{However, } l:\ \text{Span} \\ \quad E:\ \text{Elasticity} \\ \quad I:\ \text{Geometrical moment of inertia} \\ \quad q:\ \text{Uniform distribution load} \\ \quad P:\ \text{Concentrated load} \end{array}\right\} \quad (11)$$

In deflections of this bridge calculating that the geometrical moment of inertia of the main girder varies in every panel, he applied the following equation (12):

$$\left.\begin{array}{l} y=\sum (M\lambda\eta/EI) \\ \text{However, } M:\ \text{Bending moment of every panel} \\ \quad \lambda:\ \text{Panel length} \\ \quad \eta:\ \text{Increased distance from the supported point} \\ \quad E:\ \text{Elasticity} \\ \quad I:\ \text{Geometrical moment of inertia} \end{array}\right\} \quad (12)$$

a. Deflection by live load in the centre of span

Table 6.

Panel division	M_p (t-m)	M_P (t-m)	$\sum M$ (t-m)	I (m⁴)	η (m)	$M\lambda\eta/EI$ (m)
0 ~ 1	7.5	25.99	33.49	0.0242	0.5	0.329×10^{-4}
1 ~ 2	45.0	148.05	193.05	0.0337	3.0	32.734×10^{-4}
2 ~ 3	112.5	334.69	447.19	0.0642	7.5	124.385×10^{-4}
3 ~ 4	187.5	492.19	679.69	0.1114	12.5	181.588×10^{-4}
4 ~ 5	262.5	597.19	859.69	0.1773	17.5	202.033×10^{-4}
5 ~ 6	337.5	649.69	987.19	0.2208	22.5	239.516×10^{-4}
		$P=30.0$t, $p=2.10$ t/m, λ: panel length			$\sum$	780.585×10^{-4}

Deflection in the centre of span $y_{\max}=7.806$ cm
$y/l=1/641<1/600$ Safe

b. Deflection by dead load in the centre of span

Table 7.

Panel division	M_q (t-m)	I (m⁴)	η (m)	λ (m)	E (t/m²)	$M\lambda\eta/EI$ (m)
0 ~ 1	40.92	0.0240	0.5	1.0	2.1×10^7	0.403×10^{-4}
1 ~ 2	228.79	0.0337	3.0	4.0	"	38.794×10^{-4}
2 ~ 3	522.27	0.0642	7.5	5.0	"	145.268×10^{-4}
3 ~ 4	772.77	0.1114	10.5	5.0	"	206.455×10^{-4}
4 ~ 5	939.37	0.1773	17.5	5.0	"	220.758×10^{-4}
5 ~ 6	1,022.87	0.2208	22.5	5.0	"	248.173×10^{-4}
			q = 3.34 t/m			Σ 859.851×10^{-4}

y_{max} = 0.086 m = 8.6 cm

(5) **Calculations of Stability for buckling**[4),10),11)] (See Fig. 3)

The whole buckling strength of web plate stiffened by stiffners can be calculated by the energy method in "Theory of Elastic Stability" of S. Timoshenko, assuming that the deflection surface of buckling deformation on the plane is shown in the following equation :

$$w = w_0 \sin \frac{\pi (x-ky)}{a} \sin \frac{\pi y}{b} \tag{13}$$

The eqution of strain energy by the deformation of plane is given as follows :

$$V_{Bl} = \frac{1}{2} D \iint \left[\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w^2}{\partial y^2} \right) -2(1-\nu)\left\{ \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} - \left(\frac{2^2 w}{2x2y} \right)^2 \right\} \right] dx \cdot dy \tag{14}$$

The equation of bending strain energy of the rib member with bending stiffness EJ_i in the place of $y=y_i$ is given by

$$V_i = \frac{EJ_i}{2} \int_0^a \left(\frac{\partial^2 w}{\partial x^2} \right)_{y=y_i} \cdot dx = \frac{\pi^4 EJ_i}{4a^3} w_0^3 \left(\sin \frac{\pi y_i}{b} \right)^2 \tag{15}$$

Then, the present writer showed the work equation of bending stress σ acting on the plane as follows :

$$T_{Bl} \cdot \sigma = \frac{1}{2} \iint \left(\frac{\partial w}{\partial x} \right)^2 (\sigma_0 + qy)\, dx \cdot dy = \frac{bt\pi^2}{8a} w_0^2 \left(\sigma_0 + \frac{b}{2} q \right) \tag{16}$$

The work equation acting on the stiffner is shown by

$$T_i = \frac{\sigma_i A_i}{2} \int_0^a \left(\frac{\partial w}{\partial x} \right)_{y=y_i} \cdot dx = \frac{\sigma_i A_i}{4a} \pi^2 w_0^2 \left(\sin \frac{\pi y_i}{b} \right)^2 \tag{17}$$

The work caused by shearing force acting along the plane is shown by

$$T_{Bl}\cdot\tau = -\tau\cdot t\iint\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)dx\cdot dy = \frac{\pi^2 kb}{4a}\tau t w_0^2 \tag{18}$$

The buckling condition obtained from this energy amount is indicated as follows:

$$V_{Bl}+\sum_i V_i = T_{Bl}\cdot\sigma+\sum_i T_i + T_{Bl}\cdot\tau \tag{19}$$

Therefore, the safety factor for buckling is given by the following formula:

$$\nu_k = \frac{V_{Bl}+\sum_i V_i}{T_{Bl}\cdot\sigma+\sum_i T_i+T_{Bl}\cdot\tau} \tag{20}$$

Then, the safety factor for the buckling of web plate in any panel point is obtained as Table 8 by applying the equation (20).

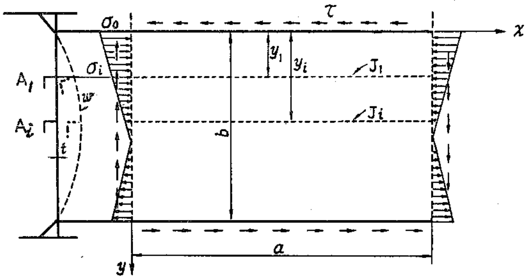

Fig. 3.

Table 8.

Panel point	Thickness of web plate (cm)	Safety factor
0	1.6	11.58
3	1.4	14.5
5	1.2	24.1

The present writer calculated on the buckling and its safety factor following the specifications DIN 4114 in Germany.

(6) Calculations for compressive bearing stress intensity of main girder[6),15),16),17)]

For the investigation of compressive bearing stress intensities of supported ends, he calculated the fibre stress intensity caused by the bending moment of every plate in the box rahmen uniformly loaded as Fig. 4.

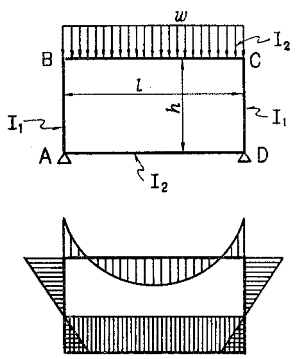

Fig. 4.

$$\left.\begin{aligned} M_{AB} &= M_{DA} = \frac{wl^2}{12} \cdot \frac{1}{5+3k} \\ M_{BC} &= M_{CD} = -\frac{wl^2}{12} \cdot \frac{1}{5+3k} \\ \text{However,}\quad k &= \frac{I_2}{I_1} \cdot \frac{h}{l} \end{aligned}\right\} \tag{21}$$

$$\left.\begin{aligned} &\sigma = \frac{M}{W} \\ &\quad W:\ \text{Section modulus of member} \\ &\quad\ = \frac{bh^2}{6} \end{aligned}\right\} \tag{22}$$

We can calculate the bending stress intensity σ of every member by equations (21) and (22), but we must handle the problem of effective length x' in the distribution of a reaction.

The present writer calculated this length x applying the distributing influence line of a load, assuming that this box girder having each reaction on every supported point is a beam on elastic foundation loaded concentrially turning upside down on each support.

Refering to Fig. 4, y is shown in the following equation.

$$y = \frac{P}{2\alpha m} f_1(\xi) \tag{23}$$

From the point of intersection that the curved line $f_1(\xi)$ intersected the axis ξ, he obtained $\xi = 3\pi/4$.

$$\left.\begin{aligned} &\text{Therefore,}\quad p = \alpha y, \quad m = \sqrt[4]{\frac{4EI}{\alpha}}, \quad \xi = \frac{x}{m} \\ &\qquad p:\ \text{Distribution of load} \\ &\qquad E:\ \text{Elasticity} \\ &\qquad y:\ \text{Deflection} \\ &\qquad \alpha:\ \text{Sinking factor} \\ &\qquad I:\ \text{Geometrical moment of inertia} \end{aligned}\right\} \tag{24}$$

By equations (23) and (24), he got the value of x and decided the effective length x'.

Then, applying the value of uniform load $w = 246$ kg/cm calculated from the total sum of live load and dead load, he got $k = 0.09375$, $M_{AB} = M_{DA} = 503{,}180$ kg-cm, $M_{BC} = M_{CD} = -503{,}180$ kg-cm, $\alpha = \frac{p}{y} = 17{,}307$ kg/cm, $m = \sqrt[4]{\frac{4EI}{\alpha}} = 162.$

At $\xi = \frac{3\pi}{4} = 2.355, \quad p = 0$

$$\therefore \quad x = m\xi = 381.5 \text{ cm}$$

Therefore, he decided the effective length $x' = 3$ m, and assuming the thickness of plate is 2.5 cm, he obtained

$$\sigma_B = \frac{M}{W} = 1{,}610\ \text{kg/cm}^2 < 1{,}900\ \text{kg/cm}^2, \qquad \text{Safe.}$$

As these calculating results, he used steel plates having the thickness of 2.5 cm in the box section of effective length 3 m at the vicinity of each supported end.

(7) Calculations for torsion and shearing flow of main girder[4),12),13)]

We must inquire into the state in which the stress of three kinds, namely the bending, the shearing or the torsional stress is acting severally on the each part of box girder, to find in detail the stress phenomenon of the main girder constructed with the closed section of thin steel plates.

The present writer selected the panel point 0 and 6 where it seemed to be most dangerous in the calculation of torsional and shearing flows.

He calculated mainly on the theory of shearing flow at the ideal section shown as Fig. 5 because the practical section is much complicated.

He affixed each number to its every point along the section with thin plates.

For example, each point of 4, 6 and 7 is the same point but each point of 4 and 7 indicates severally the right or left point of rib in the upper flange plate and the point of 6 indicates the crossing point of rib connected with the flange plate.

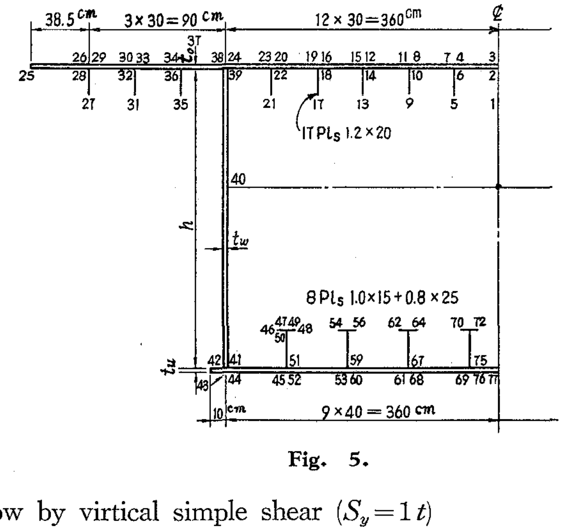

Fig. 5.

a. Shearing flow by virtical simple shear ($S_y = 1\,t$)

We call the product of shearing stress intensity τ and the thickness of steel plate t the shearing flow and can show it by a symbol of q.

$$q = \tau \cdot t \tag{25}$$

When the axial stress intensity σ_z caused by the simple bending moment M_x varies along the direction of z, the shearing stress intensity corresponding to it is always kept equilibratedly in the section.

Then, the fundamental equation is shown as follows.

$$\frac{d\sigma_z}{dz}+\frac{d\tau}{ds}=0 \tag{26}$$

However, s: Length measured along the section

$\sigma_z=-M_x\,y/J_x$ and $dM/dz=S_y$

$$\therefore \quad \tau=\int\frac{S_y\cdot y}{J_x}\,ds \tag{27}$$

When the thickness of plate varies and the steel plate with different thickness is intersecting or branching off, the continous state of shearing flow is put into the boundary condition and the equation of shearing flow is shown as follows.

$$\tau\cdot t=\frac{S_y}{J_x}\int ty\,ds \tag{28}$$

However, t: Thickness of plate
y: Vertical distance measured from a centre of gravity in the section

He gave the equation $\Delta I=\int ty\,dS$ for integrals of $t\cdot y$ between two neighboring points on the steel plate, and got the shearing flow of box section by adding one after another ΔI from a starting point having the known value of τ ($\tau=0$ in the point 3) and by this equation calculated the value of shearing stress intensity τ of each point in the case of $S_y=1$ t.

Then, he mentioned each value of τ at some points in number order as follows:

At the panel point 0

No. 40…7.354 kg/cm², No. 39…6.970 kg/cm², No. 41…6.560 kg/cm²
No. 44…6.356 kg/cm², No. 45…5.538 kg/cm², No. 52…4.985 kg/cm²
No. 24…4.222 kg/cm², No. 53…4.167 kg/cm², No. 60…3.615 kg/cm²
No. 20…3.518 kg/cm²

At the panel point 6

No. 40…2.701 kg/cm², No. 39…2.518 kg/cm², No. 41…2.314 kg/cm²
No. 44…2.238 kg/cm², No. 45…1.933 kg/cm², No. 52…1.712 kg/cm²
No. 24…1.523 kg/cm², No. 53…1.407 kg/cm², No. 9…1.340 kg/cm²
No. 10…1.269 kg/cm²

b. Shearing flow by simple torsion ($M_t=1$ t-m)

The simple torsion of closed section with thin steel plates is caused by the shearing flow streaming constantly in the closed section part, but it is not related to the rib or the other open section part.

The torsional rigidity k is given as follows:

Then,

For the closed section part, $k_1 = (\oint r\,ds)^2/\oint (1/t)\,ds$

For the total section inclosed open section part, $k_2 = \sum \int (t^3/3)\,ds$

But we can usually neglect the term of k_2 because k_2 is very small compared with k_1.

He calculated the values of k in panel points 0 and 6 and next calculated at the shearing flow for a torsion of $M_t = 1$ t-m in each same point, and also got the shearing stress intensity in every point for each of the upper flange, the web plate and the lower flange in the above-mentioned point of the main girder.

(i) Panel point 0

Shearing flow $q_0 = M_t/\sum \oint r\,ds = 1.724$ kg/cm

Table 9.

Section	Thickness of plate t (cm)	τ (kg/cm²)
Upper flange 2	1.2	1.437
Web plate 40	1.6	1.078
Lower flange 77	1.0	1.724

(ii) Panel point 6

Shearing flow $q_0 = M_t/\sum \oint r\,ds = 0.692$ kg/cm

Table 10.

Section	Thickness of plate t (cm)	τ (kg/cm²)
Upper flage 2	1.6	0.606
Web plate 40	1.2	0.807
Lower flange 77	1.4	0.692

c. Shearing flow by horizontal simple shear ($S_x = 1$ t)

We can calculate by the following equation (29) at the distribution of shearing stress intensity in the box girder section caused by the horizontal shear as the case of vertical shear.

$$\tau \cdot t = \int \frac{S_x}{J_y}\, tx\, ds \qquad (29)$$

In this case the value of shearing stress intensity is only known in the open section part—for example, the flange of deck plate etc., and the shearing stress intensity in the closed section part is indeterminate.

Then, he first calculated the value of shearing flow by taking the upper end of web plate to the starting point of integrals and next he let the indeterminate shearing flow act on the closed section and got the value of indeterminate shearing flow by the following equation (30) on condition that the shearing strain brought about the closed section returns again to the former value in the case of making a round of the section.

$$\oint \tau\, ds = \oint \frac{1}{t}(\tau \cdot t)\, ds = 0 \tag{30}$$

The present writer previously calculated shearing flows of the upper flange plate and the lower one and calculated also indeterminate shearing flows of closed sections.

He showed the indeterminate shearing force by the symbol of I_x and decided the value of I_x on the basis of the following equation $\oint I/t\, ds = 0$.

Lastly, he calculated the shearing stress intensity τ in every point of each section at selected panel points 0 and 6 where the horizontal shear ($S_x = 1\,\text{t}$) has acted on each section of those, and he mentioned these calculated values in number order as follows:

At the panel point 0

No. 3…1.665 kg/cm², No. 4…1.654 kg/cm², No. 7…1.638 kg/cm²
No. 8…1.604 kg/cm², No. 11…1.574 kg/cm², No. 12…1.516 kg/cm²
No. 15…1.469 kg/cm², No. 16…1.389 kg/cm², No. 19…1.327 kg/cm²
No. 20…1.224 kg/cm²

At the panel point 6

No. 3…1.963 kg/cm², No. 4…1.951 kg/cm², No. 7…1.940 kg/cm²
No. 8…1.905 kg/cm², No. 11…1.882 kg/cm², No. 12…1.825 kg/cm²
No. 15…1.788 kg/cm², No. 16…1.708 kg/cm², No. 19…1.665 kg/cm²
No. 20…1.562 kg/cm²

d. Shearing flow by bending torsion ($T_{bd} = 1$ t-m)

If the acting direction of S_x or S_y is out of the shearing centre, the torsional moment is added in the girder.

This shearing centre can be called also the torsional centre and first we must calculate a distance of this torsional centre, because this distance is indispensable for calculating the shearing flow caused by a torsional moment.

He got in regular calculating order the resultant of shearing forces in each part of box girder sections and calculated the resultant of moments $A \cdot r$ caused by the component of shearing force that is shown as the area of shearing flow $A = \int I\, ds$ gotten by applying the equation $\Delta I = \int tx\, ds$ or $I = \sum \Delta I$, and calculated it assuming that this resultant of moments is equall to the product of external force S_x and shearing centre distance r_0.

Next, we must calculate the function of warp. This function of warp is indicated by the equation (31), and this function expresses the adding axial stress caused by the warp of figure that comes to pass naturally in the section because the shearing flow by simple torsion does not always act perpendicularly on the shearing centre in every point of box girder sections.

$$\left.\begin{aligned} W &= \int (q_0/t)\, ds - \int r\, ds \\ q_0 &= \oint r\, ds / \oint \frac{ds}{t} \end{aligned}\right\} \tag{31}$$

He calculated this function assuming that the centre of upper flange plate is the most suitable starting point of integral calculations and next calculated also the values of bending torsional rigidity that is equal to the total sum of elastic energy wrought by axial stress intensity in the total section.

Assuming that the influence of plate thickness is very little, he got the following equation:

$$C_{bd} = \int W^2\, t\, ds \tag{32}$$

In the calculation of two points a and b, he assumed that the value of W varies straightly and got the following equation:

$$\int_a^b W^2\, t\, ds = \frac{1}{3}\left[W_b^2 + W_b \cdot W_a + W_a^2\right] \cdot l \tag{33}$$

Then, the torsional moment caused by bending torsions bringing with various axial stresses is shown as follows:

$$T_{bd} = -EC_{bd} \frac{d^3\varphi}{dz^3} \tag{34}$$

The shearing stress intensity of a section caused by $T_{bd}=1$ is shown by shearing flow $\tau \cdot t$ as follows:

$$\tau \cdot t = \frac{1}{C_{bd}} \int t\, W\, ds + (\tau \cdot t)_0 \tag{35}$$

We can calculate the value of $\int t\, W ds$ for every section by applying the function of warp W and can get the total value by adding one after another from a standard point.

The present writer assumed the starting point of integrals is most suitable on the centre of upper flange plate and got the value of indeterminate shearing flow considering the condition of integrals $\oint \tau\, ds = 0$ made around the closed box section, because the shearing flow $(\tau \cdot t)_0$ of starting point of integrals is indeterminate.

Then, he calculated the value of $\int W t\, ds$, the adding sum and the area of shearing flows and the indeterminate shearing flow Q_x, and last got the shearing stress intensity τ of every point by $T_{bd}=1$ t-m.

He mentioned some calculated values of τ at the panel point 0 in number order as follows:

At the panel point 0 (the supported point)

No. 24…3.164 kg/cm², No. 44…3.087 kg/cm², No. 39…2.546 kg/cm²
No. 23…2.291 kg/cm², No. 41…2.135 kg/cm², No. 77…−1.972 kg/cm²

No. 76⋯ $-1.926\ kg/cm^2$, No. 20⋯ $1.918\ kg/cm^2$, No. 69⋯ $-1.861\ kg/cm^2$
No. 45⋯ $1.524\ kg/cm^2$

e. Investigative discussions in the stress intensity of sections from the viewpoint of the shearing flow theory

From the above-mentioned calculated results, the present writer found that the virtical simple shear give the most influence to the stress intensity of sections, and that the shearing stress intensity by torsional moments, the bending stress intensity, and the shearing stress intensity by lateral loads are negligibly slight as compared with the influence of vertical simple shear, and so he omitted those calculated results having the little influence in this paper and mentioned only the stress intensity by vertical simple shear.

(i) Stress intensities by the maximum shearing force at the panel point 0

Table 11. Values of τ ($S_{y\ max}=182.50$ t)

Section	6	24	38	39	40	41	44
τ (kg/cm²)	19	386	251	636	672	600	580

(ii) Stress intensities by the maximum shearing force at the panel point 6

Tabe 12. Value of τ ($S_{y\ max}=33.76$t)

Section	6	24	38	39	40	41	42
τ (kg/cm²)	9	51	34	85	91	78	76

4. Consideration

The present writer could secure positive evidence of the accurate design, because the total designed dead weight was 151 t in comparison with the initial assumed dead weight 151 t and its difference was merely about 4%.

The total steel weight was 132.9 t and steel weight per square meter was 443 kg/m² and its value was not very different from ones of Jogashima Large Bridge aud accordingly it may well be said that this bridge is economical from the viewpoint of designning the simple girder bridge.

But it is considered that we can design the more economical bridge by increasing the stress of every section more nearly to the allowable stress.

This box girder bridge having varying sections is more profitable in two special points of the decrease of steel weight (about 10%) and the good lateral stability at supported ends in comparison with the same type bridge having constant sections.

At the start of design he considered that the most required subjects are investigative problems on the calculation of max shearing stress intensity and compress bearing or buckling stress at supported ends, but he could secure the positive evidence of having the sufficient safety factor as the result of calculations.

5. Conclusion

This tentatively designed bridge with one box girder has an excellent point that its steel weight decrease about 20~50% in comparision with two-box or three-box girder bridges and is more surpassing than any other type of bridge from a view-point of the structural economy.

Moreover, this bridge has the special character in a point of the rationality in the structural mechanics by using the type of varying sections and also can serve two purposes in the point of keeping the lateral stability of supported ends.

The present writer made sure that the safety factor of deflections was larger than that of the Jogashima Large Bridge and also that of vibrations calculated for caution's sake by the Rayleigh-Ritz method[1),4)] was fairly large.

Still more, by way of precaution he experimented on some small steel beam-models with varying and constant rectangular sections and for reference made sure of one more that the beam having varying sections excelled in the structural economy in comsarison with the ones having constant sections with the same height as the middle depth of the former.

There are some more problems left behind, namely—for example, the experimental study on the breaking strength of steel floor board, the study on the connection of main girder and steel floor board, and the study on welding engineering[9)].

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