# Studies in the Properties of Bending Elastic-Plastic Behavior of Some I Steel Beams

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**Abstract**

In this paper the present writer stated that he pursued and found the behaviors and properties of bending elasticity and plasticity of some I steel beams simply and hinge-fixedly supported on both the ends, by giving two concentrated line loads on the span centre of each beam.

## I. Introduction

The present writer made some I steel beams by use of H shaped steel of SS-41 and FCM-41 that are much applied as the members of bridges and architectural structures.

He pursued the extreme bending strength, the stress distribution and the elastic-plastic property within elastic limit and found some interest behaviors[1)~3)] in some I steel beams having two concentrated line loads in the centre of span, simply and hinge-fixedly supported[2)~4)] on both the ends.

Recently with the progress of science and the spread of its applied extent, the experiments[5)~8)] of I steel beams are being actively carried out for the investigation[9)~13)] of their elastic-plastic behaviors and the theoretical analyses are also being performed by using not only the elastic theory but the plastic theory and the elastic-plastic theory of I steel beams.

But it is very difficult to get the reasonable method of the theoretical analysis to consist with the actual elastic-plastic behavior on experiments, because the usual analytical formulae of elastic-plastic theory have the many problematic assumptions and are matched ill with the actual behavior.

In this study, the present writer pursued and discussed the bending elastic-plastic behaviors and many unknown properties of I steel beams simply and hinge-fixedly supported on both the ends.

## II. Experiments of I-Beams on the Bending Elastic-Plastic Behavior

### 1. Experimental Apparatuses and Instruments

Universal testing machine: RH-100 type, max capacity-100 t; X-Y Recorder; Strain meter (Indicator): SM-4J type electric resistance static strain meter, DPM-E type electric resistance dynamic strain meter; Switch box SS-24J type; Strain

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gauge : KL–20A5 type (Gauge length 20 mm, Resistance $\Omega$ 119.5±0.3, Gauge factor 2.06±1%), KP–10–A1 type (Gauge length 10 mm, Resistance $\Omega$ 120.0±0.3, Gauge factor 2.03±1%) ; Glued material : Bond 3,000 ; Dial gauge (Accuracy 1/100 mm).

**2. Kinds of Steel**[14)]

SS–41 : General structural rolled steel (Mechanical quality is the same as JIS standard)

FCM–41 : Atomospheric corrosion resistance steel corresponding with SM–41 (Mechanical quality is the same as welding structural rolled steel and welding quality is the same as JIS standard of SM–41)

The two kinds of above-mentioned steels are standardized goods of Fuji Iron-Manufacturing Company.

**3. Cross-sectional Dimensions of I Steel Beams and Positions of Their Strain Gauges**

The cross section of experimental beam-models is I-shape constructed of H-steel and their cross-sectional dimension and the position of their strain gauges is as Fig. 1.

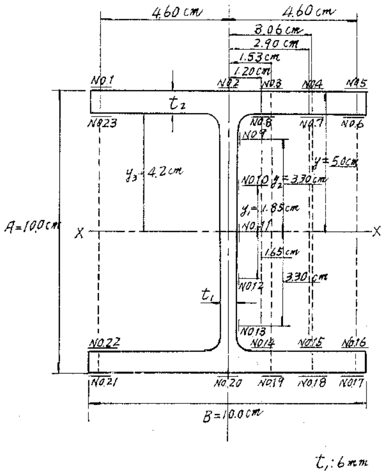



**Fig. 1.** Sectional Dimensions, Gauge Points and Gauge Nos. of a I-Beam Constructed of H-Steel.

**4. Methods of Experiments**

Four I steel beams of SS–41 and four ones of FCM–41 are experimented respectively as follows.

**A. Experiments of Simply Supported Beams**[1)~3)]

Two beams of SS–41 and two ones of FCM–41 are respectively experimented.

**B. Experiments of Hinge-Fixedly Supported Beams**

Two beams of SS–41 and two ones of FCM–41 are respectively experimented. But in the case of hinge-fixedly supported beams, he experimented strictly one by one by making use of two different bearing holders.

Nextly the point, the order and the method of experiments are explained as follows.

Each beam-model one by one is laid on the load pedestal of universal testing machine (RH–100 type) in the state of beams simply and hinge-fixedly supported on both the ends as Fig. 2.

A load is set on each beam-model by the loading method of two concentrated line loads in the centre of span, and increased gradually to the limit of the ultimate load.

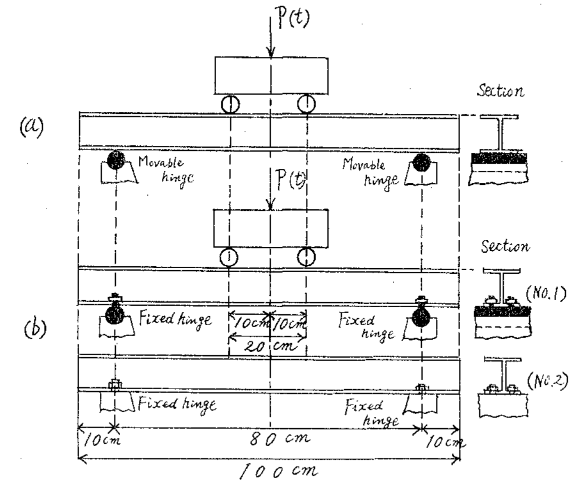



**Fig. 2.** Loaded and Supported States in the Experiments of I Steel Beams (Load: two symmetrically concentrated line-loads on the left and right of the span centre, Support: one kind of both simple supports and two kinds of both hinged and fixed supports No. 1 & No. 2).

Here, he observed, measured and calculated in detail the strain, the deflection and the stress intensity in the elastic and plastic behaviors of beam-models by making together use of the above-mentioned static and dynamic strain gauge, dial gauge and X−Y recorder.

Still more in this experiment he took specially a serious view of this plastic behavior of the vicinity of ultimate strength and the elastic-plastic behavior in the process from proportional limit to ultimate limit.

## III. Theoretical Bending Solutions of I-Beams

### 1. Theoretical Elastic Solutions

The present writer adopted intact the assumptions of Bernoulli-Navier's law and Hook's one and some else assumptions used as ever in the bending elastic analysis of beams, and he introduced the next formulae.

$$\left.\begin{array}{lll} \varepsilon/y = 1/\rho = \phi\,, & M = \int_A \sigma y dA = EI\phi\,, & \sigma = E\varepsilon = Ey\phi\,, \\ M_y = EI\phi_y = \sigma_y S\,, & 0 < \phi < \phi_y & \end{array}\right\} \quad (1)$$

Hereupon, $\rho$: Radius of curvature, $\varepsilon$: Strain, $y$: Vertical distance from neutral axis to any point, $E$: Modulus of elasticity, $A$: Sectional area, $I$: Geometrical moment of inertia, $M$: Bending moment within elastic limit, $M$: Yield bending moment, $\sigma$: Bending stress intensity within elastic limit, $\sigma_y$: Yield bending stress intensity, $S$: Section modulus, $\phi$: Curvature within elastic limit, $\phi_y$: Curvature in yield bending moment.

## A. Theoretical Solutions of Simple Beams

Referring to Fig. 3, the present writer got the formulae of reaction, bending moment, shearing force, bending stress intensity, shearing stress intensity and deflection of the simple beams having two concentrated line loads in the centre of span as follows.

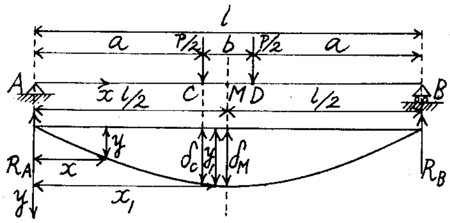



**Fig. 3.** Elastic Bending and Deflection of a simple Beam.

Reaction

$$R_A = R_B = P/2\,;$$

Bending moment

$$M = Px/2 \cdots 0 \leqq x \leqq a\,, \qquad M = P(l-x)/2 \cdots (a+b) \leqq x \leqq l\,,$$

$$M_C = M_D = M_{\max} = Pa/2\,;$$

Shearing force

$$Q = P/2 \cdots 0 \leqq x \leqq a\,, \qquad Q = -P/2 \cdots (a+b) \leqq x \leqq l\,; \qquad (2)$$

Bending stress intensity

$$\sigma = \pm My/I = \pm M/S$$

(+ : Tensile stress intensity, − : Compressive stress intensity)

Shearing stress intensity

$$\tau_{yz} = (Q/2ZI)\int_{y_1}^{y_0} y dF$$

Hereupon, $y$, $z$: Vertical and horizontal distance from a origin of the rectangular coordinate—the centre of sectional figure, $dF$: Area of minute section in the vertical distance $y$ from a origin of the rectangular coordinate, $y_0$: Vertical distance from a origin of the rectangular coordinate to each surface of the upper and lower flanges of section, $y_1$: Vertical distance from a origin of the rectangular coodinate to the any point of $\tau_{yz}$.

Bending deflection in the centre of span

$$\delta_M = (Pa/4EI)(a^3/3 - l^2/4)$$

Shearing deflection in the centre of span

$$\delta_Q = 2Pa(l-a)/lGA_w \qquad (3)$$

Hereupon, $G$: Modulus of rigidity, $A_w$: Sectional area of web, $l$: Span length, $P$: Resultant of two concentrated line loads.

### B. Theoretical Solutions of Hinge-Fixedly Supported Beams on Both the Ends

Hence referring to Fig. 4, the present writer showed the analytical formulae[4] of the reaction, the deflection angle, the deflection, the bending moment, the shearing force, the axial tensile force, the bending stress intensity and the shearing stress intensity in the hinge-fixedly supported beams having two symmetrically concentrated line loads and one side only load in their span centre part as (4)~(18).

In the case of beams having one left-side only load $P/2$ only referring to Fig. 4,

Reaction of the point A and B,

$$R_1=(P/2)(1-\varepsilon), \qquad R_2=(P/2)\varepsilon\,;$$

Deflection angle,

$$\varphi_1=(Pl^2/8KI)f_1, \qquad \varphi_2=(Pl^2/8KI)f_2$$

However,

$$f_1=\omega^{-2}\Big[\big\{\sinh 2\varepsilon\omega+\cosh 2\varepsilon\omega-\sinh 2(2-\varepsilon)\omega-\cosh 2(2-\varepsilon)\omega\big\}\Big/(\sinh 4\omega+\cosh 4\omega-1)+1-\varepsilon\Big]$$

$$f_2=\omega^{-2}\Big[\big\{\sinh 2(1-\varepsilon)\omega+\cosh 2(1-\varepsilon)\omega-\sinh 2(1+\varepsilon)\omega-\cosh 2(1+\varepsilon)\omega\big\}\Big/(\sinh 4\omega+\cosh 4\omega-1)+\varepsilon\Big],$$

$$0<\varepsilon<+1 \qquad (4)$$

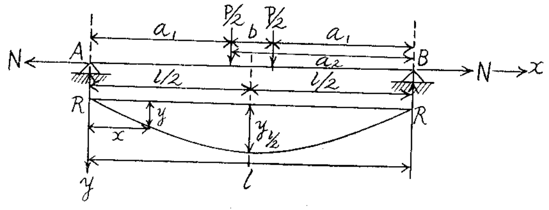



**Fig. 4.** Elastic Bending and Deflection of a Hinge-Fixedly Supported Beam.

In the case of beams having two symmetrically concentrated line loads ($P=P/2+P/2$), the reaction, the deflection angle, the deflection and the bending moment are as follows.

Reaction

$$R=R_1+R_2=P/2$$

Deflection angle

$$\varphi=(\varphi_1+\varphi_2)=(Pl^2/8KI)(f_1+f_2) \qquad (5)$$

Calculating formulae of deflections in the any point,

$$y=(1/\xi)\big\{(P/2N)(\xi x-\sinh \xi x)+(\varphi_1+\varphi_2)\sinh \xi x\big\}\cdots 0\leqq x\leqq a$$

$$\left.\begin{aligned} y &= (1/\xi)\Big[(P\varepsilon/2N)\big\{\xi x - \sinh\xi x + \xi(l-x) - \sinh\xi(l-x)\big\} \\ &\quad + \varphi_2\big\{\sinh\xi(l-x) + \sinh\xi(l-x)\big\}\Big] \cdots a \leqq x \leqq (a+b) \\ y &= (1/\xi)\Big[(P/2N)\big\{\xi(l-x) - \sinh\xi(l-x) + (\varphi_1+\varphi_2)\sinh\xi(l-x) \cdots \\ &\quad (a+b) \leqq x \leqq l \end{aligned}\right\} \quad (6)$$

Calculating formulae of bending moments in the any point,

$$\left.\begin{aligned} M_x &= (1/\xi)\sinh\xi x\big\{P/2 - N(\varphi_1+\varphi_2)\big\} \cdots 0 \leqq x \leqq a \\ M_x &= (1/\xi)\big\{\sinh\xi x + \sinh\xi(l-x)\big\}\big\{(P/2)\varepsilon - N\varphi_2\big\} \cdots a \leqq x \leqq (a+b) \\ M_x &= (1/\xi)\sinh\xi(l-x)\big\{P/2 - N(\varphi_1+\varphi_2)\big\} \cdots (a+b) \leqq x \leqq l \end{aligned}\right\} \quad (7)$$

Deflection in the centre of span,

$$y = (1/\xi)\Big[(P/2N)\big\{\xi(l/2) - \sinh\xi(l/2)\big\} + (\varphi_1+\varphi_2)\sinh\xi(l/2)\Big] \quad (8)$$

Bending moment in the centre of span,

$$M_{l/2} = (1/\xi)\sinh\xi(l/2)\big\{P/2 - N(\varphi_1+\varphi_2)\big\} \quad (9)$$

Shearing force in the any point,

$$\left.\begin{aligned} Q &= P/2 \cdots 0 \leqq x \leqq a \\ Q &= -P/2 \cdots (a+b) \leqq x \leqq l \end{aligned}\right\} \quad (10)$$

In the case of beams having one side only concentrated line load, the calculating equation of the axial tensile force $N$ is as follows.

$$\left.\begin{aligned} &\sum_{n=1}^{2}(1/k_n^4)\,2R_n\varphi_n a_n^5 KI(\sinh 2k_n/2k_n + \sinh k_n/k_n - \cosh k_n - 1) \\ &-\sum_{n=1}^{2}(1/k_n^6)\,R_n^2 a_n^7(\sinh 2k_n/2k_n + 2\sinh k_n/k_n - 2\cosh k_n - 1) \\ &-\sum_{n=1}^{2}(1/k_n^2)\,\varphi_n^2 a_n^3 K^2 I^2(\sinh 2k_n/2k_n - 1) - 4lK^2I^3/4 = 0 \end{aligned}\right\} \quad (11)$$

Moreover, the above-mentioned equation can be shown also by the next equation.

$$\sum_{n=1}^{2}\Big\{2KIR_n\varphi_n a_n^5 f_1(k_n) - R_n^2 a_n^7 f_2(k_n) - K^2I^2\varphi_n^2 a_n^3 f_3(k_n)\Big\} = 4lK^2I^3/A$$

However,

$$\left.\begin{aligned} f_1(k_n) &= (1/k_n^4)(\sinh 2k_n/2k_n - 1 - \cosh k_n + \sinh k_n/k_n) \\ f_2(k_n) &= (1/k_n^6)(\sinh 2k_n/2k_n + 2\sinh k_n/k_n - 2\cosh k_n - 1) \\ f_3(k_n) &= (1/k_n^2)(\sinh 2k_n/2k_n - 1) \end{aligned}\right\} \quad (12)$$

Then this axial tensile force $N$ of a beam having two symmetrically concentrated line loads ($P = P/2 + P/2$) in the span centre part is twice as large as that of a beam having one side only load ($P/2$) in the same part.

$$
\left.
\begin{array}{l}
\text{Moreover,} \\
\text{Bending stress intensity} \\
\quad \sigma = \pm My/I = \pm M/S \\
\quad\quad (+ : \text{Tensile stress intensity, } - : \text{Compressive stress intensity}) \\
\text{Shearing stress intensity} \\
\quad \tau_{yz} = (Q/2ZI)\int_{y_1}^{y_0} ydF
\end{array}
\right\} \quad (13)
$$

**2. Elastic-Plastic Bending Analytical Solutions**[2),9),12),13)]

**A. Theoretical Assumptions for Elastic-Plastic Bending Analysis of Beams**

The assumptions of analysis in the elastic-plastic bending behavior of beams have been backed up properly by many experiments and may be shown as follows.

i. The strain is proportionate to the distance from the neutral axis.
ii. The physical relation of stress-strain is elastic before the stress intensity $\sigma_y$ in the static yield point and after that it becomes plastically flowing without any restriction under the constant stress intensity $\sigma_y$.
iii. The physical relation of stress-strain in the compressive side is the same as the one in the tensile side.
iv. Assuming that the deformation is microscopic, the curvature of a bended beam is approximately formularized by the twice differentiated expression.

In the plastic bending behavior of beams, the assumption of i. is well applied in the distribution of strain and the ones of ii. and iii. are likewise applied in the distribution of stress.

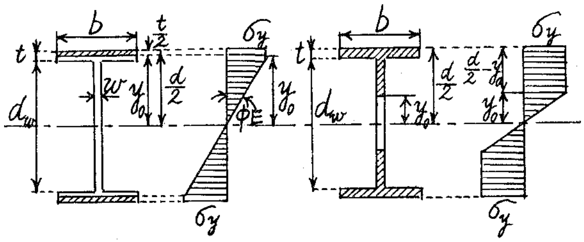
**B. Theoretical Solutions of Simply Bended I-Beams**[12)]

Fig. 5 (a) shows that the yieldings begin with the heads of both flanges and the outside parts of both flanges become the plastic domain.

Then,

$$
\left.
\begin{aligned}
M &= 2\int_{y_0}^{d/2} \sigma_y ybdy + 2\int_{d/2}^{y} (\sigma_y/y_0)\, ybydy \\
&\quad + \int_0^{d_w/2} (\sigma_y/y_0)\, ywydy + 2\int_0^{d_w/2} (\sigma_y/y_0)\, ywydy \\
&= \sigma_y(bd^2/4 - by_0^2/3 - bd_w^3/12y_0 + wd_w^3/12y_0) \\
&= \sigma_y b(d^2/4 - \sigma_y^2/3E^2\phi^2) - (E\phi d/2)(S - bd^2/6)
\end{aligned}
\right\} \quad (14)
$$

Hereupon, $b$: Width of flange, $t$: Thickness of flange, $w$: Thickness of web plate, $d_w$: Depth of web plate, $d$: Depth of I-beam, $I$: Geometrical moment of inertia of I-beam$=bd^3/12 - bd_w^3/12 + wd_w^3/12$, $S$: Section modulus of I-beam$=I/(d/2)$, $y_0$: Depth of the elastic domain$=\sigma_y/E\phi$



(a) Yielding within Flanges (b) Yielding within Web Plate

**Fig. 5.** Elastic-Plastic Bending of a I-Beam.

Moreover,

$$M/M_y = (bd^2/4S)\left\{1-(1/3)(\phi_y/\phi)^2\right\}+(\phi/\phi_y)(1-bd^3/6S) \tag{15}$$

The formula (15) is applied for the extent that the outside parts of both flanges bigin to yield and the total parts of both flanges yield perfectly, namely—the extent of $1<\phi/\phi_y<(d/2)/\{(d/2)-1\}$.

Next, referring to Fig. 5 (b) the present writer considers on the extent that totally both flanges yield perfectly, the web plate begins to yield and the inner parts of a web plate become plastic.

$$\left.\begin{aligned} M &= 2\int_{d_w/2}^{d/2}\sigma_y bydy+2\int_{y_0}^{d_w/2}\sigma_y wydy+2\int_{y}^{y_0}(\sigma_y/y_0)\,ywydy \\ &= \sigma_y(bd^2/4-bd_w^2/4+wd_w^2/4)-\sigma_y wy_0^2/3 \end{aligned}\right\} \tag{16}$$

The plastic section modulus $Z$ of a I-beam are divided into three pieces $Z_1$, $Z_2$ and $Z_3$ as Fig. 6.

As every element is the rectangular section, the values of $Z_1$, $Z_2$ and $Z_3$ are shown as follows.

$$Z_1=bd^2/4\,,\qquad Z_2=bd^2/4\,,\qquad Z_3=wd_w^2/4$$

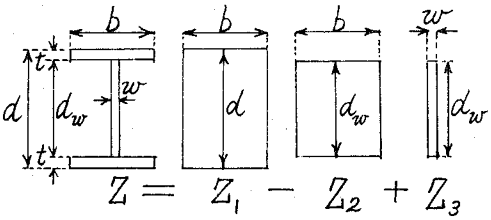



**Fig. 6.** Plastic Section Modulus of a I-Beam.

Then,

$$M=\sigma_y(Z-wy_0^2/3)=\sigma_y\left\{Z-(w/3)(\sigma_y^2/E^2\phi^2)\right\} \tag{17}$$

However,

$$0<y_0<d_w/2$$

$$M/M_y = Z/S-(w/3)(\sigma_y^2/E^2\phi^2 S)$$
$$= Z/S-(wd^2/12S)(\phi_y/\phi)^2 = f-(wd^2/12S) \qquad (18)$$

However,

$$(d/2)/\{(d/2)-1\} < \phi/\phi_y < \infty$$

In the formula (18),

$$M_y = \sigma_y S = EI\phi_y \qquad (19)$$

or,

$$\sigma_y = E\phi_y(d/2)$$

In the case of a I-section, the moment of a beam goes rapidly near the ultimate moment $M_p$ than the case of a rectangular section.

Namely, the excess strength of a I-beam decreases rapidly after the yielding of its total, not only outside surface, flanges.

### C. Theoretical Solutions of a Bended I-Beam with Axial Force[12)]

Referring Fig. 7, the present writer shows explanatively the theoretical formulae of plastic moments and stress intensities of I-beams.

By the value's ratio of the bending moment $M$ and the axial force $N$, the behavior of I-beams is classified in the two cases that the neutral axis exists within the web plate or the flange.

i. The Case of a I-Beam with Neutral Axis Existing within Its Web Plate

From the stress distribution of the ultimate moment of Fig. 7,

$$N = \int_A \sigma dA = \sigma_y(2wy_0)$$
$$N_{yc} = \sigma_y(A_f + A_w) \qquad (20)$$

However, $N_{yc}$: Yielding axial force

Therefore,

$$N/N_{yc} = 2y_0/d_w(1 + A_f/A_w) \qquad (21)$$

And the plastic moment $M_{pc}$ of a beam with the axial force is shown as the next formula.

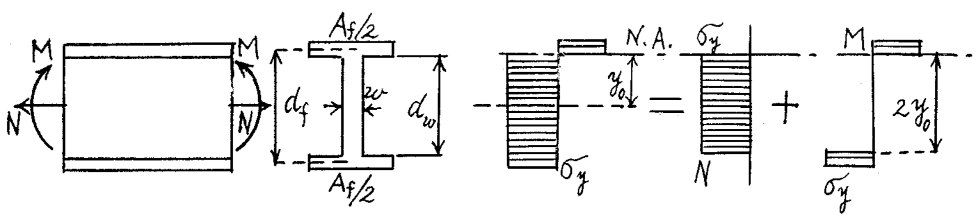



**Fig. 7.** I-Beam with Bending Moment and Axial Force, Case I (Neutral Axis within Web Plate).

$$M_{pc}=\int_A \sigma_y dA=\sigma_y(A_f d_f/2+A_w d_w/4-by_0^2)$$
$$M_p=\sigma_y(A_f d_f/2+A_w d_w/4) \qquad (22)$$

However, $M$: Common plastic moment of simple beam

Thereupon,

$$M_{pc}=M_p-\sigma_y b y_0^2$$

or,

$$M_{pc}/M_p=1-by_0^2/Z \qquad (23)$$

By putting $y_0$ of the formula (21) into (23), the next formula is got.

$$M_{pc}/M_p=1-\left\{(N/N_{pc})(1+A_f/A_w)\right\}^2\left\{1/(1+2A_f d_f/A_w d_w)\right\} \qquad (24)$$

Presuming from the elastic theory, the elastic-plastic stress intensity of a I-beam with the plastic section modulus $Z$ is shown as the next formula.

$$\sigma=N_p/(A_f+A_w)\pm M_{pc}/Z \qquad (25)$$

ii. The Case of a I-Beam with Neutral Axis Existing within Its Flange

The neutral axis movement of a I-beam grows larger in proportion as the magnitude of its axial force and its neutral axis goes on moving within its flange at last.

The formula (24) or (25) may be applied when its neutral axis exists within its web plate, but the special relation of the bending moment and the axial force is induced from the stress distribution of Fig. 8.

Then, from Fig. 8,

$$N=\sigma_y\left[A_w+A_f\left\{1-2\varDelta/(d-d_w)\right\}\right], N_{yc}=\sigma_y(A_w+A_f) \qquad (26)$$

Therfore,

$$N/N_{yc}=\left[1+(A_f/A_w)\left\{1-2\varDelta/(d-d_w)\right\}\right]/(1+A_f/A_w) \qquad (27)$$

$$M_{pc}=(\sigma_y A_f/2)\,2\varDelta(d-\varDelta)/(d-d_w), \qquad M_p=\sigma_y(A_f d_f/2+A_w d_w/4)$$

Therefore,

$$M_{pc}/M_p=\left\{2\varDelta(d-\varDelta)/(d-d_w)\right\}/(d_f+A_w d_w/2A_f) \qquad (28)$$

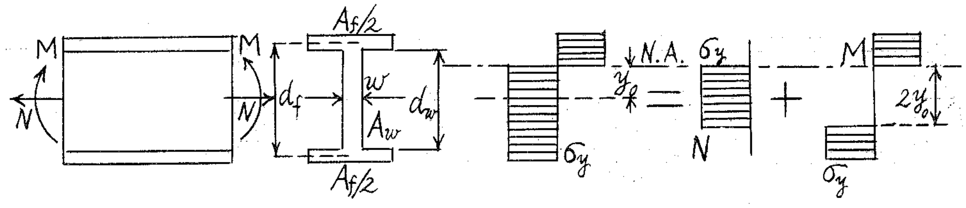



**Fig. 8.** I-Beam with Bending Moment and Axial Force, Case II (Neutral Axis within Flange).

As $\Delta$ is smaller than $d$,

$$d-\Delta \cong d$$

Therefore,

$$M_{pc}/M_p=(2\Delta/d-d_w)\{2/(1+d_w/d+A_w d_w/A_f d)\} \tag{29}$$

Eliminating $\Delta$ from the formulae (27) and (29),

$$M_{pc}/M_p=\left[(2d/d_w)/\{1+(1+d/d_w)(A_f/A_w)\}\right] \times\left[A_f/A_w-\{(N/N_{yc})(1+A_f/A_w)-1\}\right] \tag{30}$$

Or, applying the plastic section modulus $Z$ and the total sectional area $A$,

$$M_{pc}/M_p=(A/2Z)(1-N/N_{yc})\{d-(A/2b)(1-N/N_{yc})\} \tag{31}$$

Generally it may be certified that the average value of $d_f/d_w$ or $d/d_w$ in I steel beams is nearly 1.05 or 1.10 and these values are applied in the calculation of I steel beams.

Then when the axial force $N$ is zero, $M_{pc}$ becomes equal to $M_p$, and when the bending moment $M_p$ is zero, $N$ becomes properly equal to $N_p$.

## IV. Comparison of Experimental Results and Theoretical Values

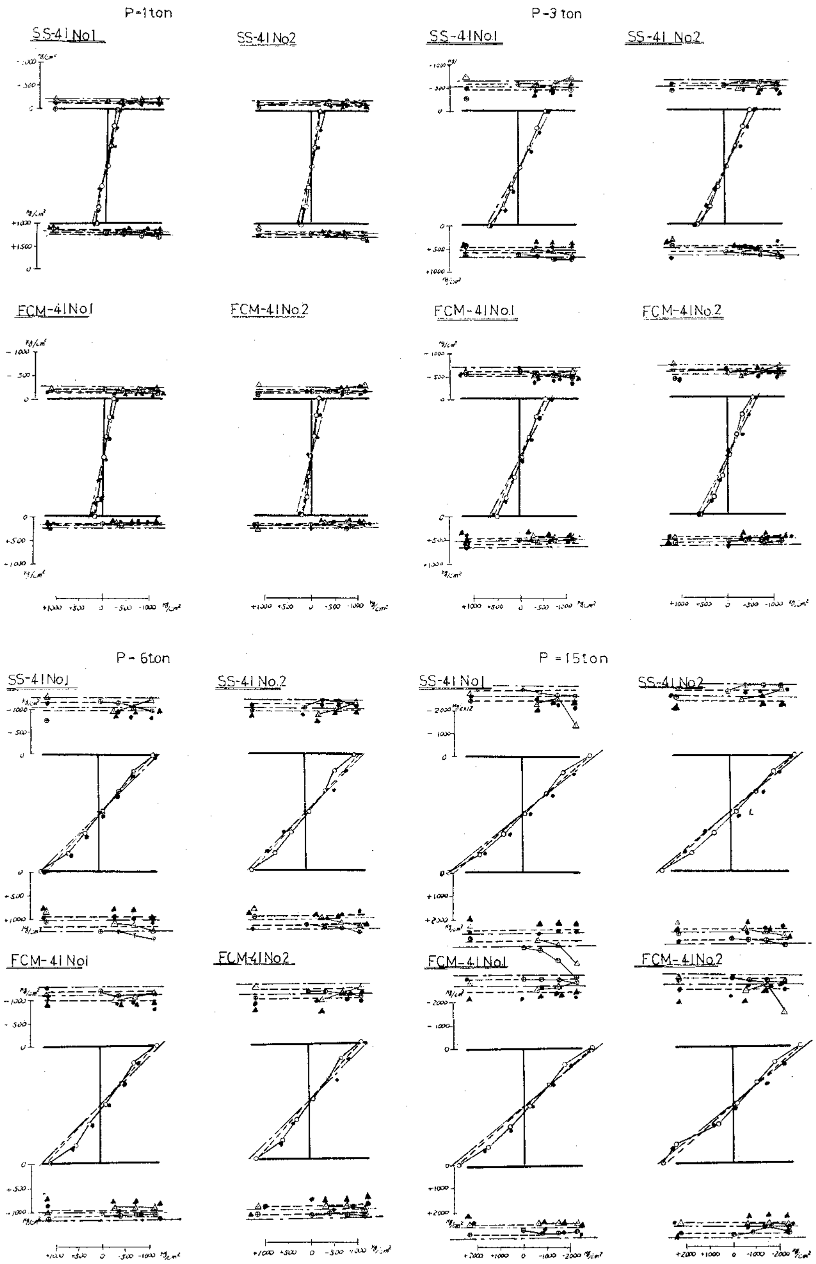
Fig. 9, Fig. 10, Fig. 11 and Fig. 12 show severally the distribution curves of sectional stress intensities within a elastic limit, the Load-Strain Curves of the elastic and plastic behaviors at the principal points of the upper and lower flanges and the web plate and the Load-Deflection Curves of the elastic and plastic behaviors at the centre of span.

## V. Conclusion

1. The present writer could certify that from the view point of the distribution curves of sectional stress intensities, the Load-Strain Curves of the principal points and the Load-Deflection Curves of the span centre, every experimental result agrees almost to its corresponding theoretical value and that the elastic beam theory is generally right.

2. He could also confirm that in the plastic domain and the special extent of the elastic-plastic shifting process the experimental Load-Strain Curves of the principal points and the experimental Load-Deflection Curves of the span centre get both approximately near their corresponding theoretical ones, but he felt keenly that the elastic-plastic beam theory are considerably imperfect judging from the comparison of theoretical rough values and experimental scrupulous results.

3. It was confirmed that the experimental process from the yielding load to




Theoretical values of simply supported beam : -------- (web plate, outside and inside of flanges)
Experimental values of simply supported beam : ○ (web plate) , ⊕ (outside of flanges) , △ (inside of flanges)
Theoretical values of hinged-fixedly supported beam : —-—-— (web plate, outside and inside of flanges)
Experimental values of hinged-fixedly supported beam : ● (web plate, outside of flanges), ▲ (inside of flanges)

**Fig. 9.** Distributions of Bending Stress Intensities $\sigma_x$ in the Elastic Domain of I Steel Beams (Measured Points $x=l/2$, Loads $P=1$ t, 3 t, 6 t, 15 t).

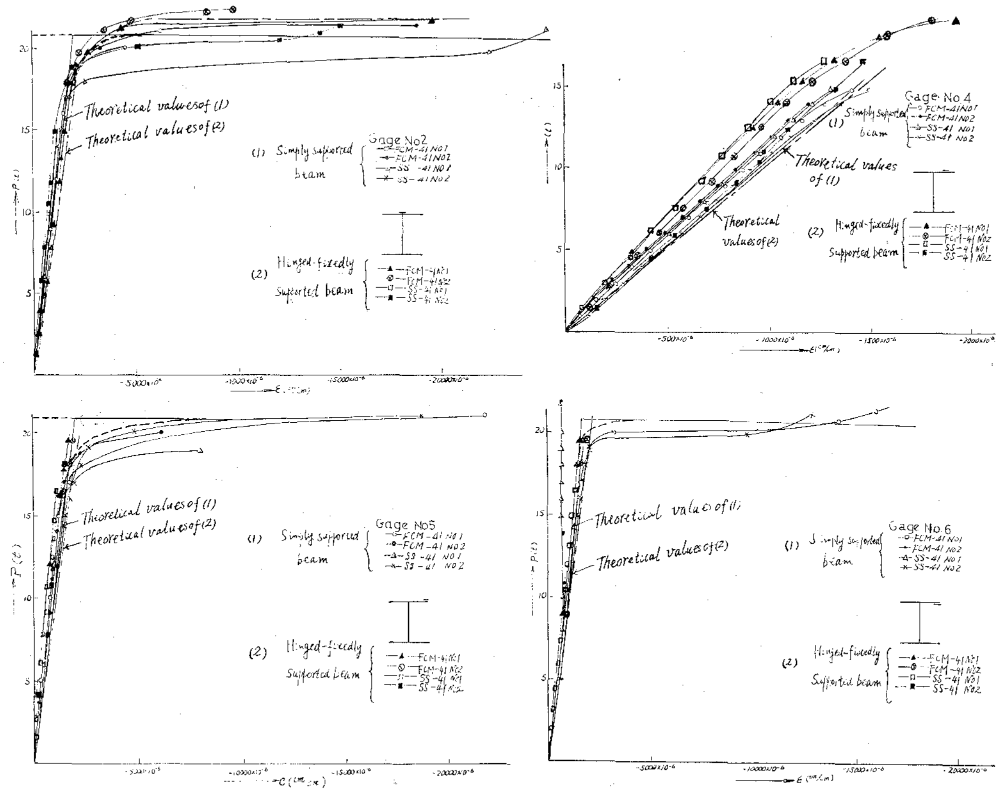



**Fig. 10.** Load-Strain Curves on the Principal Points (No. 2, 4, 5, 6) of Upper Flanges in the Elastic and Plastic Behaviors of I Steel Beams (Measured Points $x=l/2$).

the ultimate load approaches indicatively to the corresponding theoretical process and the present writer could also confirm that the ultimate strengths of I steel beams agree nearly to the ones of steel materials in the standard test[14].

**4.** It was also certified that the distribution curves of sectional stress intensities and the Load-Strain Curves of the principal points in the FCM–41 steel-mounted I-beams are more excellent than those in the SS–41 steel-mounted ones.

**5.** The present writer could lastly confirm that the magnitude of an axial tensile force $N$ in the I steel beams hinge-fixedly supported on both the ends changes remarkably by the ratio of a sectional area $A$ to a span $l$—$A/l$, namely—$N$ decreases so much more as $A/l$ increases.

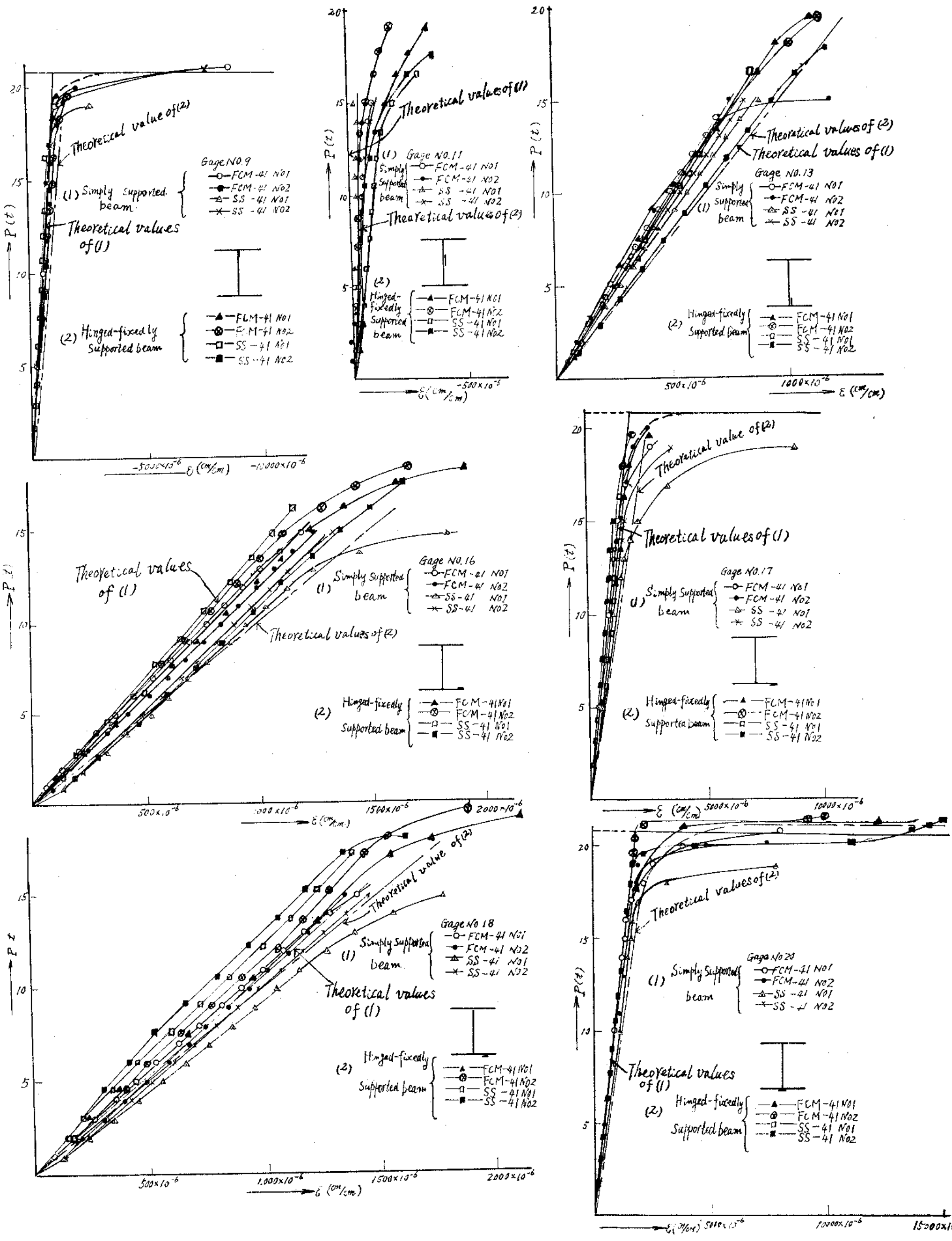


**Fig. 11.** Load-Strain Curves on the Principal Points (No. 9, 11, 13, 16, 17, 18, 20) of Lower Flanges and Web Plates in the Elastic and Plastic Behaviors of I Steel Beams (Measured Points $x=l/2$).

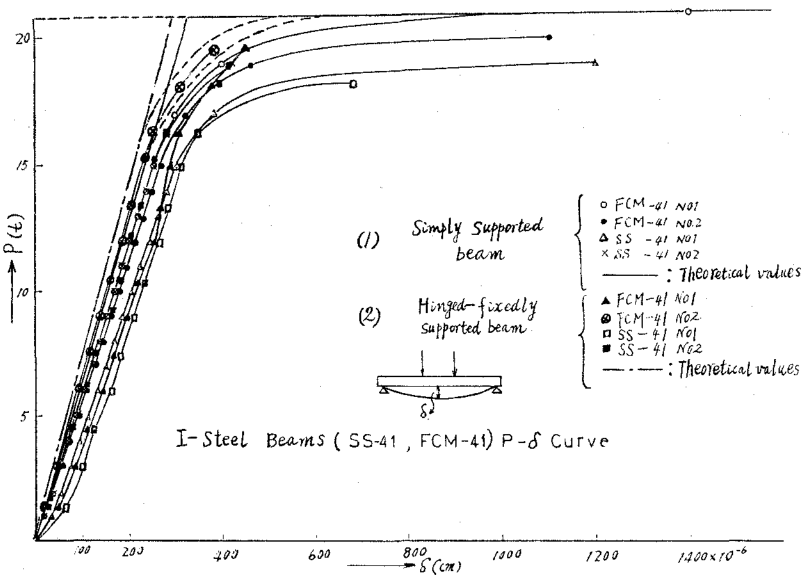



**Fig. 12.** Load-Deflection Curves of the Span Centre in the Elastic and Plastic Bending-Behaviors of I Steel Beams.

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