On the stress analysis of the plates with multi-crosswise ribs (Part 1)

On the stress analysis of the plates with multi-crosswise ribs (Part 1)

Sumio G. Nomachi*, Kenichi G. Matsuoka**
and Toshiyuki Ohshima***

Abstract

Bending and horizontal deformation of a ribbed plate which is built up with many thin rectangular plates as shown in Fig. 1, is considered here.

Making use of Displacement-Shear-Equations concerning folded plate theory, we can express the equilibrium of shearing forces at the joint line where three or four component strips meet with one another, by simultaneous finite difference equations with respect to five components of displacement, and an analytical method for solving those finite difference equations by means of finite fourier transforms based on finite integration, is discussed.

As numerical examples, the presenting paper deals with the simply supported ribbed plates subjected to lateral and horizontal loads.

1. Introduction

The structure on which we are going to study, is a plate stiffened in two mutually perpendicular directions by a system of longitudinal and transverse ribs connected with it.

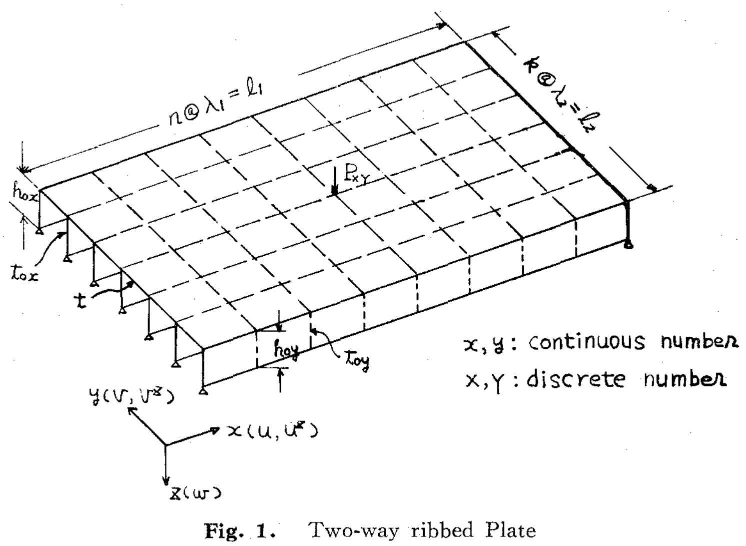

Fig. 1. Two-way ribbed Plate

* Department of Civil Engineering, Hokkaido University.
** Department of Civil Engineering, Muroran Institute of Technology.
*** Department of Development Engineerings, Kitami Institute of Technology.

Let us call it the "Two-way ribbed plate." The structure of this kind has a good design efficiency, and is used for the steel plate deck construction of the bridge structure and a partial reinforcement of main girder, at which the stiffening cable is anchored[1),2)].

Besides then, it can be seen in composite construction bridges, well suited for short and medium range spans, the concrete deck participates in the stresses of the main girder to which it is bonded.

Since the two-way ribbed plate is widely used, its stress behaviour has been extensively investigated by many engineers and reseachers.

The reseaches so far made, can roughly be grouped in three categories. The first one stands on the base emphasizing the nature of the grid work, and the plate is replaced by the grid of perpendicularly intersecting T-beams, which are composed by ribs and platets.

In such modelling, the shearing resistance of the plane stress in the plate, which is supposed to have fairly effect for some cases, is neglected. H. Homberg[3)] and F. Leonhart[4)] did much in this area.

The second one is the bending theory of the orthotropic plate. It is natural that the two-way ribbed plate should be modeled by an orthogonal anisotropic plate, which is defined as a plate which has different elastic properties in two mutually perpendicular directions, in the plane of plate. In this case, the characteristics of the ribs which have discrete properties, may be averaged and the ribbed plate is replaced by a model of continuous media. M. T. Huber, S. P. Timoshenko and W. Cornelius are known as outstanding reseachers in this field.

The third theory is something like the theory of "Schubfeld Theorie" by H. Ebner[6)] who established it on the assumption that the plate might bear only the plane shear, and it is widely used for the design of the thin walled frame work structure. Our discussion will stand from the idea of the third category.

In the bridge structure, the two-way ribbed plate is often adapted as the web plate or the flange plate, and the thickness of the plate is not so thin that we can not neglect the normal stress in the plate any more.

Taking the effect of normal stress and of shearing stress in the plate into account, we use "Displacement and Shear Equation" of the folded plate theory. Considering this three plates of strip meet at a nodal line with one another, around its nodal line we have an equation of equilibrium of shear in which the displacements and their derivertives are included.

Integrating the equation successively, we can get the relations between nodal displacements and nodal forces. Thus the fundemental finite difference equations for the stress problem of the two-way ribbed plate is established, and for solving the finite difference equations, "Finite Integration Transform" is used.

2. Basic formulas and symbolic notations

1) Displacement-Shear-Equations

The shearing forces are expressed with the displacements and the normal stresses at the lower and upper sides of a folded plates element, by the displacement-shear-equations[7].

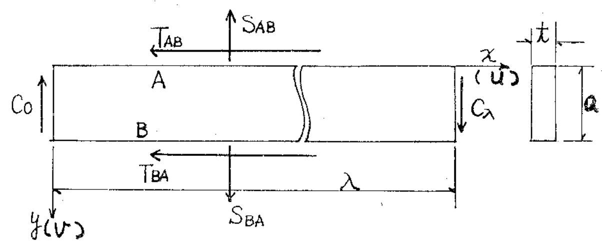

Fig. 2. Folded plate element

$$\dot{T}_{AB}(x)=\frac{N}{6}(2\ddot{u}_A+\ddot{u}_B)+\frac{1}{a}(S_{AB}-S_{BA}) \qquad (1)$$

$$\dot{T}_{BA}(x)=\frac{N}{6}(2\ddot{u}_B+\ddot{u}_A)+\frac{1}{a}(S_{BA}-S_{AB}) \qquad (2)$$

where

$$\dot{u}=\frac{\partial u}{\partial x}, \qquad N \fallingdotseq Eta$$

$$\frac{1}{2}Gt(\dot{v}_A+\dot{v}_B)=\frac{Gt}{a}(u_A-u_B)+\frac{1}{a}(\bar{S}_{AB}-\bar{S}_{BA}) \qquad (3)$$

2) Finite Fourier Integration Transforms and their inverse formulars[8)]

a) Let us introduce the symbolic notation

$$\left.\begin{aligned} \boldsymbol{S}_i\left[f(x)\right]&=\sum_{x=1}^{n-1} f(x)\cdot\sin\frac{i\pi}{n}x \\ \boldsymbol{C}_i\left[f(x)\right]&=\sum_{x=1}^{n-1} f(x)\cdot\cos\frac{i\pi}{n}x \end{aligned}\right\} \qquad (4)$$

which are coupled with

$$\left.\begin{aligned} f(x)&=\frac{2}{n}\sum_{i=1}^{n-1}\boldsymbol{S}_i\left[f(x)\right]\cdot\sin\frac{i\pi}{n}x \\ f(x)&=\frac{2}{n}\sum_{i=1}^{n}\boldsymbol{R}_i\left[f(x)\right]\cdot\cos\frac{i\pi}{n}x \end{aligned}\right\} \qquad (5)$$

where

$$\boldsymbol{R}_0\left[f(x)\right]=\frac{1}{2}\left\{\boldsymbol{C}_0\left[f(x)\right]+\frac{1}{2}f(n)+\frac{1}{2}f(0)\right\}$$

$$\boldsymbol{R}_i\Big[f(x)\Big]=\boldsymbol{C}_i\Big[f(x)\Big]+\frac{1}{2}(-1)^i f(n)+\frac{1}{2}f(0)$$

$$\boldsymbol{R}_n\Big[f(x)\Big]=\frac{1}{2}\Big\{\boldsymbol{C}_n\Big[f(x)\Big]+\frac{1}{2}(-1)^n f(n)+\frac{1}{2}f(0)\Big\}$$

$$i=0,1,\cdots n\,,\qquad x=0,1,\cdots n\,.$$

b) Related formulas

For convenience sake, let us difine the second difference and the modiffied difference as follows,

$$\Delta^2 f(x)=f(x+1)-2\cdot f(x)+f(x-1)$$

$$\Delta f(x)=f(x+1)-f(x-1)$$

Applying the above formulas to the sine and cosine transforms, we have

$$\boldsymbol{S}_i\Big[\Delta^2 f(x)\Big]=-\sin\frac{i\pi}{n}\Big\{(-1)^i f(n)-f(0)\Big\}-D_i\cdot\boldsymbol{S}_i\Big[f(x)\Big] \tag{6}$$

$$\boldsymbol{S}_i\Big[\Delta f(x)\Big]=-2\cdot\sin\frac{i\pi}{n}\boldsymbol{R}_i\Big[f(x)\Big] \tag{7}$$

$$\boldsymbol{C}_i\Big[\Delta^2 f(x)\Big]=(-1)^i\Delta f(n-1)-\Delta f(0)-D_i\cdot\boldsymbol{R}_i\Big[f(x)\Big] \tag{8}$$

$$\boldsymbol{C}_i\Big[\Delta f(x)\Big]=-(-1)^i\Delta f(n-1)-\Delta f(0)$$
$$+\left(1+\cos\frac{i\pi}{n}\right)\Big\{(-1)^i f(n)-f(0)\Big\}+2\cdot\sin\frac{i\pi}{n}\cdot\boldsymbol{S}_i\Big[f(x)\Big] \tag{9}$$

where

$$D_i=2\left(1-\cos\frac{i\pi}{n}\right)$$

3. Analysis of two-way ribbed plate

The four sides of the ribbed plate are parallel to the coordinate axis x and y, whose positive directions are given by the arrowhead, as shown in Fig. 1.

And the three components of displacements in x, y and z directions are denoted by u, v and w.

And also let the letter T be the shear flow and the letter S, the normal forces per unit length.

1) Equilibrium of shearing forces at the nodal line in the x direction

The three shearing forces and the outside surface traction along the nodal line parallel to the x axis on which the deck plate is intersected by the rib plate, is expressed by

$$T_{Y.Y+1}(x)+T_{Y.Y-1}(x)+T_Y^{0z}(x)=p(x) \tag{10}$$

which together with Eqs. (1), (2) and (3) yields

$$\left[\frac{N_x}{3}+\frac{N_{0x}}{6}\right]2\cdot\ddot{u}_Y+\frac{N_x}{6}\ddot{u}_{Y+1}+\frac{N_x}{6}\ddot{u}_{Y-1}+\frac{N_{0x}}{6}u_Y^z$$
$$-\left(2\frac{Gt}{\lambda_2}+\frac{Gt_{0x}}{h_{0x}}\right)u_Y+\frac{Gt}{\lambda_2}(u_{Y+1}+u_{Y-1})+\frac{Gt_{0x}}{h_{0x}}u_Y^z$$
$$-\frac{Gt}{2}(\dot{v}_{Y+1}-\dot{v}_{Y-1})+Gt_{0x}\dot{w}_Y=p(x) \tag{11}$$

where

$$N_{0x}\fallingdotseq Et_{0x}h_{0x}, \qquad N_x\fallingdotseq Et\lambda_2 .$$

2) **Equilibrium of shearing forces in the *y*-direction**

Similarly in the y direction,

$$\left[\frac{N_y}{3}+\frac{N_{0y}}{6}\right]\cdot 2\cdot\ddot{v}_X+\frac{N_y}{6}\ddot{v}_{X+1}+\frac{N_y}{6}\ddot{v}_{X-1}+\frac{N_{0y}}{6}\ddot{v}_X^z$$
$$-\left(2\frac{Gt}{\lambda_1}+\frac{Gt_{0y}}{h_{0y}}\right)v_X+\frac{Gt}{\lambda_1}v_{X+1}+\frac{Gt}{\lambda_1}v_{X-1}+\frac{Gt_{0y}}{h_{0y}}v_X^z$$
$$-\frac{Gt}{2}(\dot{u}_{X+1}-\dot{u}_{X-1})+Gt_{0y}\dot{w}_X=p(y) \tag{12}$$

where

$$\dot{v}=\frac{\partial v}{\partial y}, \qquad N_{0y}\fallingdotseq Et_{0y}h_{0y}, \qquad N_y\fallingdotseq Et\lambda_1 .$$

3) **Boundary conditions of the rib plate in the *x* and *y* directions**

$$T_{XY}^z=\frac{N_{0x}}{3}\ddot{u}_Y^z+\frac{N_{0x}}{6}\ddot{u}_Y-Gt_{0x}\dot{w}_Y+\frac{Gt_{0x}}{h_{0x}}(u_Y-u_Y^z) \tag{13}$$

$$T_{X.y}^z=\frac{N_{0y}}{3}\ddot{v}_X^z+\frac{N_{0y}}{6}\ddot{v}_X-Gt_{0y}\dot{w}_X+\frac{Gt_{0y}}{h_{0y}}(v_X-v_X^z) \tag{14}$$

4) **Equilibrium of shearing forces at the node *x*, *y* in the *z* direction**

Look at the rib element, from Eq. (3) we have at once

$$Gt_{0x}h_{0x}\dot{w}_{x.Y}=Gt_{0x}(u_{x.Y}-u_{xY}^z)+(\bar{S}_{xY}-\bar{S}_{xY}^z) \tag{15}$$

$$Gt_{0y}h_{0y}\dot{w}_{Xy}=Gt_{0y}(v_{Xy}-v_{Xy}^z)+(\bar{S}_{Xy}-\bar{S}_{Xy}^z) \tag{16}$$

in which $\bar{S}(x)=\int S(x)\cdot dx$, and it is supposed to be a shearing force inside of the rib, so we can write it as

$$\bar{S}(x)=\boldsymbol{C}_{X.X+1}-(\bar{\boldsymbol{P}}_{X.X+1}-\bar{\boldsymbol{P}}_{X.X+1}^z) \tag{17}$$

where

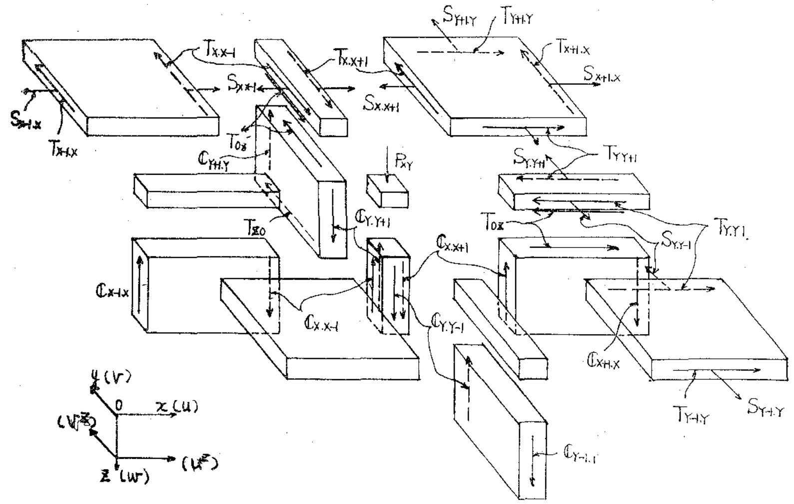

Fig. 3. Geometry of two-way ribbed System

$C_{X,X+1}$ = resultant shearing force acting on the boundary

$$\bar{P}_{X,X+1}=\int_X^{X+1} P(x)\,dx$$

Integrating Eqs. (15) and (16) by x and y, on the assumption that u and v respectively may be linear with respect to x and y, we find that

$$Gt_{0x}h_{0x}\Delta_X w_{XY}=Gt_{0x}\left\{u_{XY}+u_{X+1.Y}-(u^z_{X.Y}+u^z_{X+.1Y})\right\}\frac{\lambda_1}{2}$$
$$+{}_YC_{X.X+1}\cdot\lambda_1-({}_Y\bar{\bar{P}}_{X.X+1}|_0^{\lambda_1}-{}_Y\bar{\bar{P}}^z_{X.X+1}|_0^{\lambda_1}) \qquad (18)$$

$$Gt_{0x}h_{0x}\Delta_X w_{X-1.Y}=Gt_{0x}\left\{u_{XY}+u_{X-1.Y}-(u^z_{XY}+u^z_{X-1.Y})\right\}\frac{\lambda_1}{2}$$
$$+{}_YC_{X.X-1}\cdot\lambda_1-({}_Y\bar{\bar{P}}_{X.X-1}|_0^{\lambda_1}-{}_Y\bar{\bar{P}}^z_{X.X-1}|_0^{\lambda_1}) \qquad (19)$$

$$Gt_{0y}h_{0y}\Delta_Y w_{XY}=Gt_{0y}\left\{v_{XY}+v_{X.Y+1}-(v^z_{XY}+v^z_{X.Y+2})\right\}\frac{\lambda_2}{2}$$
$$+{}_XC_{Y.Y+1}\cdot\lambda_2-({}_X\bar{\bar{P}}_{Y.Y+1}|_0^{\lambda_2}-{}_X\bar{\bar{P}}^z_{Y.Y+1}|_0^{\lambda_2}) \qquad (20)$$

$$Gt_{0y}h_{0y}\Delta_Y w_{XY-1}=Gt_{0y}\left\{v_{XY}+v_{X.Y-1}-(v^z_{X.Y-1}+v^z_{X.Y})\right\}\frac{\lambda_2}{2}$$
$$+{}_XC_{Y.Y-1}\cdot\lambda_2-({}_X\bar{\bar{P}}_{Y.Y-1}|_0^{\lambda_2}-{}_X\bar{\bar{P}}^z_{Y.Y-1}|_0^{\lambda_2}) \qquad (21)$$

The equilibrium of shearing forces around the cylindrical section centering the node x, y, yields the equation oncerning C as follows:

$${}_YC_{X.X-1}-{}_YC_{X.X+1}+{}_XC_{Y.Y-1}-{}_XC_{Y.Y+1}=P_{XY} \qquad (22)$$

in which all of C can be eliminated by Eqs. (18), (19), (20) and (21), and we come to the expression

$$\frac{Gt_{0x}h_{0x}}{\lambda_1}\Delta_X^2 w_{X.Y}+\frac{Gt_{0y}h_{0y}}{\lambda_2}\Delta_Y^2 w_{XY}$$
$$-\frac{Gt_{0x}}{2}(\Delta_X u_{X.Y}-\Delta_X u_{XY}^z)-\frac{Gt_{0y}}{2}(\Delta_Y v_{XY}-\Delta_Y v_{XY}^z)$$
$$=-P_{XY}+\frac{1}{\lambda_1}\left({}_Y\overline{\overline{\underline{P}}}_{X.X+1}\Big|_0^{\lambda_1}-{}_Y\overline{\overline{\underline{P}}}{}^z_{X.X+1}\Big|_0^{\lambda_1}+{}_Y\overline{\overline{\underline{P}}}_{X.X-1}\Big|_0^{\lambda_1}-{}_Y\overline{\overline{\underline{P}}}{}^z_{X.X-1}\Big|_0^{\lambda_1}\right)$$
$$+\frac{1}{\lambda_2}\left({}_X\overline{\overline{\underline{P}}}_{Y.Y+1}\Big|_0^{\lambda_2}-{}_X\overline{\overline{\underline{P}}}{}^z_{Y.Y+1}\Big|_0^{\lambda_2}+{}_X\overline{\overline{\underline{P}}}_{Y.Y-1}\Big|_0^{\lambda_2}-{}_X\overline{\overline{\underline{P}}}{}^z_{Y.Y-1}\Big|_0^{\lambda_2}\right) \tag{23}$$

By applying the procedures described in Appendix to Eqs. (11), (12), (13) and (14), we can transform them into the equations as follows

$$(A_{11}+2A_{12})\Delta_X^2 u_{XY}+A_{12}\Delta_X^2\Delta_Y^2 u_{XY}+A_{13}\Delta_X^2 u_{XY}^z$$
$$+(B_{11}+2B_{12})u_{XY}+B_{12}\Delta_Y^2 u_{XY}+B_{13}u_{XY}^z$$
$$+C_{12}\Delta_X\Delta_Y v_{XY}+C_{13}\Delta_X w_{XY}=\boldsymbol{P}_1 \tag{24}$$

$$A_{41}\Delta_X^2 u_{XY}^z+A_{42}\Delta_X^2 u_{XY}+B_{41}u_{XY}^z+B_{42}u_{XY}-C_{41}\Delta_X w_{XY}=\boldsymbol{P}_2 \tag{25}$$

$$-(A_{31}\Delta_X^2 w_{XY}+A_{32}\Delta_Y^2 w_{XY})+C_{31}(\Delta_X u_{XY}-\Delta_X u_{XY}^z)$$
$$+C_{32}(\Delta_Y v_{XY}-\Delta_Y v_{XY}^z)=\boldsymbol{P}_3 \tag{26}$$

$$A_{51}\Delta_Y^2 v_{XY}^z+A_{52}\Delta_Y^2 v_{XY}+B_{51}v_{XY}^z+B_{52}v_{XY}-C_{51}\Delta_Y w_{XY}=\boldsymbol{P}_4 \tag{27}$$

$$(A_{21}+2A_{22})\Delta_Y^2 v_{XY}+A_{22}\Delta_X^2\Delta_Y^2 v_{XY}+A_{23}\Delta_Y^2 v_{XY}^z$$
$$+(B_{21}+2B_{22})v_{XY}+B_{22}\Delta_X^2 v_{XY}+B_{23}v_{XY}^z$$
$$+C_{22}\Delta_X\Delta_Y u_{XY}+C_{23}\Delta_Y w_{XY}=\boldsymbol{P}_5 \tag{28}$$

Table 1. Coefficient of Eq. (24), (25), (26), (27) and (28)

Aij

i	j		
	1	2	3
1	$\frac{2}{\lambda_1}\left[\frac{N_x}{3}+\frac{N_{0x}}{6}\right]-\frac{\lambda_1}{6}\left(2\frac{Gt}{\lambda_2}+\frac{Gt_{0x}}{h_{0x}}\right)$	$\frac{1}{6}\left(\frac{N_x}{\lambda_1}+\frac{Gt\lambda_1}{\lambda_2}\right)$	$\frac{1}{6}\left(\frac{N_{0x}}{\lambda_1}+\frac{Gt_{0x}\lambda_1}{h_{0x}}\right)$
2	$\frac{2}{\lambda_2}\left[\frac{N_y}{3}+\frac{N_{0y}}{6}\right]-\frac{\lambda_2}{6}\left(2\frac{Gt}{\lambda_1}+\frac{Gt_{0y}}{h_{0y}}\right)$	$\frac{1}{6}\left(\frac{N_y}{\lambda_2}+\frac{Gt\lambda_2}{\lambda_1}\right)$	$\frac{1}{6}\left(\frac{N_{0y}}{\lambda_2}+\frac{Gt_{0y}\lambda_2}{\eta_{0y}}\right)$
3	$\frac{Gt_{0x}h_{0x}}{\lambda_1}$	$\frac{Gt_{0y}h_{0y}}{\lambda_2}$	—
4	$\frac{N_{0x}}{3\lambda_1}-\frac{Gt_{0x}\lambda_1}{6h_{0x}}$	$\frac{N_{0x}}{6\lambda_1}+\frac{Gt_{0x}\lambda_1}{6h_{0x}}$	—
5	$\frac{N_{0y}}{3\lambda_2}-\frac{Gt_{0y}\lambda_2}{6h_{0y}}$	$\frac{N_{0y}}{6\lambda_2}+\frac{Gt_{0y}\lambda_2}{6h_{0y}}$	—

Table 1. Continue

Bij

i	j		
	1	2	3
1	$-\lambda_1\left(2\frac{Gt}{\lambda_2}+\frac{Gt_{0x}}{h_{0x}}\right)$	$\frac{Gt\,\lambda_1}{\lambda_2}$	$\frac{Gt_{0x}\,\lambda_1}{h_{0x}}$
2	$-\lambda_2\left(2\frac{Gt}{\lambda_1}+\frac{Gt_{0y}}{h_{0y}}\right)$	$\frac{Gt\,\lambda_2}{\lambda_1}$	$\frac{Gt_{0y}\,\lambda_2}{h_{0y}}$
3	—	—	—
4	$-\frac{Gt_{0x}\,\lambda_1}{h_{0x}}$	$\frac{Gt_{0x}\,\lambda_1}{h_{0x}}$	—
5	$-\frac{Gt_{0y}\,\lambda_2}{h_{0y}}$	$\frac{Gt_{0x}\,\lambda_1}{h_{0y}}$	—

Cij

i	j		
	1	2	3
1	—	$\frac{Gt}{4}$	$\frac{Gt_{0x}}{2}$
2	—	$\frac{Gt}{4}$	$\frac{Gt_{0x}}{2}$
3	$\frac{Gt_{0x}}{2}$	$\frac{Gt_{0y}}{2}$	—
4	$\frac{Gt_{0x}}{2}$	—	—
5	$\frac{Gt_{0y}}{2}$	—	—

which are the fundermental difference equations for this case. Using formulas (6), (7), (8) and (9), we can perform finite fourier integration transform to these equations.

When we take the condition that the deflections and the stress components are zero along the four edges, the boundary values in the equations vanish, and they become

$$\begin{aligned}&-(A_{11}+2A_{12})D_m\cdot \boldsymbol{R}_m\boldsymbol{S}_i[u_{XY}]+A_{12}D_mD_i\boldsymbol{R}_m\boldsymbol{S}_i[u_{XY}]\\&-A_{13}D_m\boldsymbol{R}_m\boldsymbol{S}_i[u^z_{XY}]+(B_{11}+2B_{12})\boldsymbol{R}_m\boldsymbol{S}_i[u_{XY}]\\&-B_{12}D_i\boldsymbol{R}_m\boldsymbol{S}_i[u_{XY}]+B_{13}\boldsymbol{R}_m\boldsymbol{S}_i[u^z_{XY}]\\&-4C_{12}\cdot\sin\frac{i\pi}{k}\sin\frac{m\pi}{n}\boldsymbol{S}_m\boldsymbol{R}_i[v_{XY}]+2C_{13}\cdot\sin\frac{m\pi}{n}\boldsymbol{S}_m\boldsymbol{S}_i[w_{XY}]=\widetilde{\boldsymbol{P}}_1\end{aligned}\quad(29)$$

$$\begin{aligned}&-A_{41}D_m\boldsymbol{R}_m\boldsymbol{S}_i[u^z_{XY}]-A_{42}D_m\boldsymbol{R}_m\boldsymbol{S}_i[u_{XY}]+B_{41}\boldsymbol{R}_m\boldsymbol{S}_i[u^z_{XY}]\\&+B_{42}\boldsymbol{R}_m\boldsymbol{S}_i[u_{XY}]-2C_{41}\sin\frac{m\pi}{n}\boldsymbol{S}_m\boldsymbol{S}_i[w_{XY}]=\widetilde{\boldsymbol{P}}_2\end{aligned}\quad(30)$$

$$(A_{31}D_m + A_{32}D_i)\boldsymbol{S}_m\boldsymbol{S}_i[w_{XY}] - 2C_{31}\cdot\sin\frac{m\pi}{n}\Big(\boldsymbol{R}_m\boldsymbol{S}_i[u_{XY}] - \boldsymbol{R}_m\boldsymbol{S}_i[u^z_{XY}]\Big)$$
$$-2C_{32}\cdot\sin\frac{i\pi}{k}\Big(\boldsymbol{S}_m\boldsymbol{R}_i[v_{XY}] - \boldsymbol{S}_m\boldsymbol{R}_i[v^z_{XY}]\Big) = \widetilde{\boldsymbol{P}}_3 \tag{31}$$

$$-A_{51}D_i\boldsymbol{S}_m\boldsymbol{R}_i[v^z_{XY}] - A_{52}D_i\boldsymbol{S}_m\boldsymbol{R}_i[v_{XY}] + B_{51}\boldsymbol{S}_m\boldsymbol{R}_i[v^z_{XY}]$$
$$+B_{52}\boldsymbol{S}_m\boldsymbol{R}_i[v_{XY}] - 2C_{51}\cdot\sin\frac{i\pi}{k}\boldsymbol{S}_m\boldsymbol{S}_i[w_{XY}] = \widetilde{\boldsymbol{P}}_4 \tag{32}$$

$$-(A_{21}+2A_{22})D_i\boldsymbol{S}_m\boldsymbol{R}_i[v_{XY}] + A_{22}D_iD_m\boldsymbol{S}_m\boldsymbol{S}_i[v_{XY}]$$
$$-A_{23}D_i\boldsymbol{S}_m\boldsymbol{R}_i[v^z_{XY}] + (B_{21}+2B_{22})\boldsymbol{S}_m\boldsymbol{R}_i[v_{XY}]$$
$$-B_{22}D_m\boldsymbol{S}_m\boldsymbol{R}_i[v_{XY}] + B_{23}\boldsymbol{S}_m\boldsymbol{R}_i[v^z_{XY}]$$
$$-4C_{22}\cdot\sin\frac{i\pi}{k}\sin\frac{m\pi}{n}\boldsymbol{R}_m\boldsymbol{S}_i[u_{XY}] + 2C_{23}\sin\frac{i\pi}{k}\boldsymbol{S}_m\boldsymbol{S}_i[w_{XY}] = \widetilde{\boldsymbol{P}}_5 \tag{33}$$

which can be written in

$$\boldsymbol{K}\cdot\boldsymbol{U} = \boldsymbol{P} \tag{34}$$

where

$$\boldsymbol{K} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \quad \boldsymbol{U} = \begin{bmatrix} \boldsymbol{R}_m\boldsymbol{S}_i[u_{XY}] \\ \boldsymbol{R}_m\boldsymbol{S}_i[u^z_{XY}] \\ \boldsymbol{S}_m\boldsymbol{S}_i[w_{XY}] \\ \boldsymbol{S}_m\boldsymbol{R}_i[v_{XY}] \\ \boldsymbol{S}_m\boldsymbol{R}_i[v^z_{XY}] \end{bmatrix} \quad \boldsymbol{P} = \begin{bmatrix} \widetilde{\boldsymbol{P}}_1 \\ \widetilde{\boldsymbol{P}}_2 \\ \widetilde{\boldsymbol{P}}_3 \\ \widetilde{\boldsymbol{P}}_4 \\ \widetilde{\boldsymbol{P}}_5 \end{bmatrix}$$

$$a_{11} = -2\left(\frac{Et\lambda_2}{\lambda_1} + \frac{Et_{0x}h_{0x}}{3\lambda_1} - \frac{Gt_{0x}\lambda_1}{6h_{0x}}\right)\left(1-\cos\frac{m\pi}{n}\right)$$
$$+\frac{2}{3}\left(\frac{Et\lambda_2}{\lambda_1} + \frac{Gt\lambda_1}{\lambda_2}\right)\left(1-\cos\frac{m\pi}{n}\right)\left(1-\cos\frac{i\pi}{k}\right) - \frac{Gt_{0x}\lambda_1}{h_{0x}} - 2\frac{Gt\lambda_1}{\lambda_2}\left(1-\cos\frac{i\pi}{k}\right)$$

$$a_{12} = \frac{Gt_{0x}\lambda_1}{h_{0x}} - \frac{1}{3}\left(\frac{Et_{0x}h_{0x}}{\lambda_1} + \frac{Gt_{0x}\lambda_1}{h_{0x}}\right)\left(1-\cos\frac{m\pi}{n}\right)$$

$$a_{13} = Gt_{0x}\cdot\sin\frac{m\pi}{n}, \quad a_{14} = 0, \quad a_{15} = -Gt\cdot\sin\frac{m\pi}{n}\sin\frac{i\pi}{k}$$

$$a_{21} = a_{12}, \quad a_{22} = -\frac{Gt_{0x}\lambda_1}{h_{0x}} - 2\left(\frac{Eh_{0x}t_{0x}}{3\lambda_1} - \frac{Gt_{0x}\lambda_1}{6h_{0x}}\right)\left(1-\cos\frac{m\pi}{n}\right)$$

$$a_{23} = -Gt_{0x}\cdot\sin\frac{m\pi}{n}, \quad a_{24} = a_{25} = 0, \quad a_{31} = a_{13}, \quad a_{32} = a_{23}$$

$$a_{33} = -2\left[\frac{Gt_{0x}h_{0x}}{x_1}\left(1-\cos\frac{m\pi}{n}\right) + \frac{Gt_{0y}h_{0y}}{\lambda_2}\left(1-\cos\frac{i\pi}{k}\right)\right]$$

$$a_{34} = -Gt_{0y}\sin\frac{i\pi}{k}, \quad a_{35} = Gt_{0y}\sin\frac{i\pi}{k}, \quad a_{41} = a_{42} = 0, \quad a_{43} = a_{34}$$

$$a_{44}=-\frac{Gt_{0y}\lambda_2}{h_{0y}}-2\left(\frac{Et_{0y}h_{0y}}{3\lambda_2}-\frac{Gt_{0y}\lambda_2}{6h_{0y}}\right)\left(1-\cos\frac{i\pi}{k}\right)$$

$$a_{45}=\frac{Gt_{0y}\lambda_2}{h_{0y}}-\frac{1}{3}\left(\frac{Et_{0y}h_{0y}}{\lambda_2}+\frac{Gt_{0y}\lambda_2}{h_{0y}}\right)\left(1-\cos\frac{i\pi}{k}\right)$$

$$a_{51}=a_{15},\quad a_{52}=a_{25},\quad a_{53}=a_{35},\quad a_{54}=a_{45}$$

$$a_{55}=-2\left(\frac{Et\lambda_1}{\lambda_2}+\frac{Et_{0y}h_{0y}}{3\lambda_2}-\frac{Gt_{0y}\lambda_2}{6h_{0y}}\right)\left(1-\cos\frac{i\pi}{k}\right)$$
$$+\frac{2}{3}\left(\frac{Et\lambda_1}{\lambda_2}+\frac{Gt\lambda_2}{\lambda_1}\right)\left(1-\cos\frac{m\pi}{n}\right)\left(1-\cos\frac{i\pi}{k}\right)-\frac{Gt_{0y}\lambda_2}{h_{0y}}-2\frac{Gt\lambda_2}{\lambda_1}\left(1-\cos\frac{m\pi}{n}\right)$$

And stresses are obtained as follows,

$$\frac{N_{0x}}{3}\boldsymbol{S}_i[\dot{u}^z_{XY}]=\frac{N_{0x}}{3\lambda_1}\boldsymbol{S}_i[\Delta_X u^z_{XY}]+\frac{N_{0x}}{6\lambda_1}\boldsymbol{S}_i[\Delta_X u_{XY}]$$
$$+\frac{Gt_{0x}\lambda_1}{6h_{0x}}\left(2\cdot\boldsymbol{S}_i[u_{X.Y}]+\boldsymbol{S}_i[u_{X+1.Y}]-2\cdot\boldsymbol{S}_i[u^z_{XY}]-\boldsymbol{S}_i[u^z_{X+1.Y}]\right)$$
$$-\frac{Gt_{0x}}{2}\left(\boldsymbol{S}_i[w_{X+.1Y}]-\boldsymbol{S}_i[w_{X.Y}]\right)-\frac{N_{0x}}{6}\boldsymbol{S}_i[\dot{u}_{XY}] \tag{35}$$

$$\left\{\left(1-\frac{D_i}{6}\right)N_x+\frac{N_{0x}}{4}\right\}\boldsymbol{S}_i[\dot{u}_{XY}]$$
$$=\left\{\left(1-\frac{D_i}{6}\right)\frac{N_x}{\lambda_1}+\frac{N_{0x}}{4\lambda_1}\right\}\boldsymbol{S}_i[\Delta_X u_{X.Y}]+\frac{Gt_{0x}\lambda_1}{4h_{0x}}\left(2\cdot\boldsymbol{S}_i[u^z_{XY}]+\boldsymbol{S}_i[u^z_{X+.1Y}]\right)$$
$$-G\lambda_1\left(\frac{t_{0x}}{4h_{0x}}-\frac{t}{6\lambda_2}D_i\right)\left(2\cdot\boldsymbol{S}_i[u_{XY}]+\boldsymbol{S}_i[u_{X+.1Y}]\right)$$
$$-\frac{Gt}{2}\sin\frac{i\pi}{k}\left(\boldsymbol{R}_i[v_{X+.1Y}]-\boldsymbol{R}_i[v_{X.Y}]\right)+\frac{3Gt_{0x}}{4}\left(\boldsymbol{S}_i[w_{X+.1Y}]-\boldsymbol{S}_i[w_{X.Y}]\right) \tag{36}$$

$$\frac{N_{0y}}{3}\boldsymbol{S}_m[\dot{v}^z_{XY}]=\frac{N_{0y}}{3\lambda_2}\boldsymbol{S}_m[\Delta_Y v^z_{XY}]+\frac{N_{0y}}{6\lambda_2}\boldsymbol{S}_m[\Delta_Y v_{XY}]$$
$$+\frac{Gt_{0y}\lambda_2}{6h_{0y}}\left(2\cdot\boldsymbol{S}_m[v_{XY}]+\boldsymbol{S}_m[v_{X.Y+1}]-2\cdot\boldsymbol{S}_m[v^z_{X.Y}]-\boldsymbol{S}_m[v^z_{X.Y+1}]\right)$$
$$-\frac{Gt_{0y}}{2}\left(\boldsymbol{S}_m[w_{X.Y+1}]-\boldsymbol{S}_m[w_{XY}]\right)-\frac{N_{0y}}{6}\boldsymbol{S}_m[\dot{v}_{XY}] \tag{37}$$

$$\left\{\left(1-\frac{D_m}{6}\right)N_y+\frac{N_{0y}}{4}\right\}\boldsymbol{S}_m[\dot{v}_{XY}]$$
$$=\left\{\left(1-\frac{D_m}{6}\right)\frac{N_y}{\lambda_2}+\frac{N_{0y}}{4\lambda_2}\right\}\boldsymbol{S}_m[\Delta_Y v_{XY}]+\frac{Gt_{0y}\lambda_2}{4h_{0y}}\left(2\cdot\boldsymbol{S}_m[v^z_{XY}]+\boldsymbol{S}_m[v^z_{X.Y+1}]\right)$$
$$-G\lambda_2\left(\frac{t_{0y}}{4h_{0y}}-\frac{t}{6\lambda_1}D_m\right)\left(2\cdot\boldsymbol{S}_m[v_{XY}]+\boldsymbol{S}_m[v_{X.Y+1}]\right)$$
$$-\frac{Gt}{2}\cdot\sin\frac{m\pi}{n}\left(\boldsymbol{R}_m[u_{X.Y+1}]-\boldsymbol{R}_m[u_{XY}]\right)+\frac{3Gt_{0y}}{4}\left(\boldsymbol{S}_m[w_{X.Y+1}]-\boldsymbol{S}_m[w_{XY}]\right) \tag{38}$$

4. Numerical examples

In order to illustrate the numerical results obtained by the method presented in this paper, some simple cases are taken.

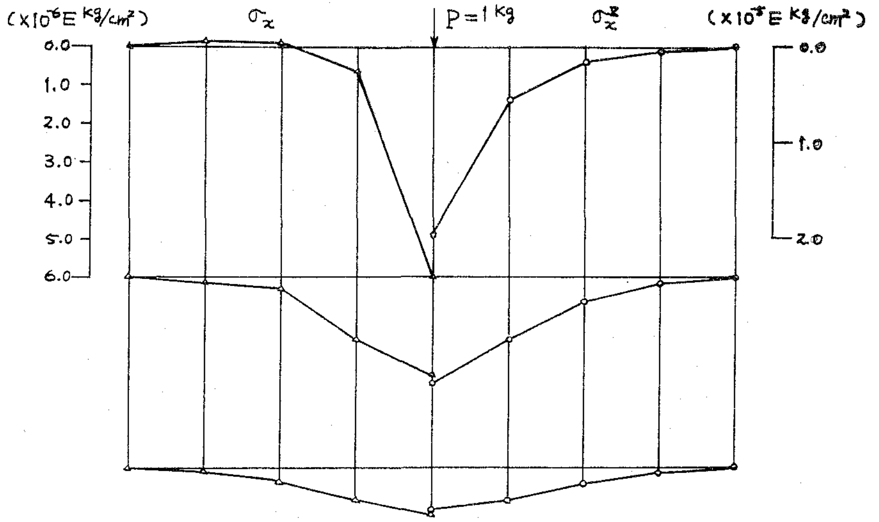

Fig. 4. σ_x^0 and $\sigma_x^{\bar{x}}$ Diagraw

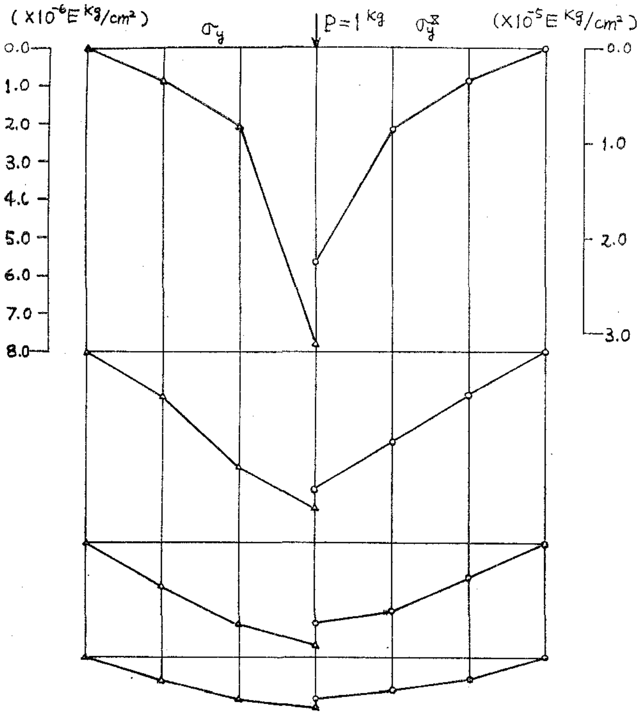

Fig. 5. σ_y^0 and $\sigma_y^{\bar{y}}$ Diagraw (E=34800 kg/cm², v=0)

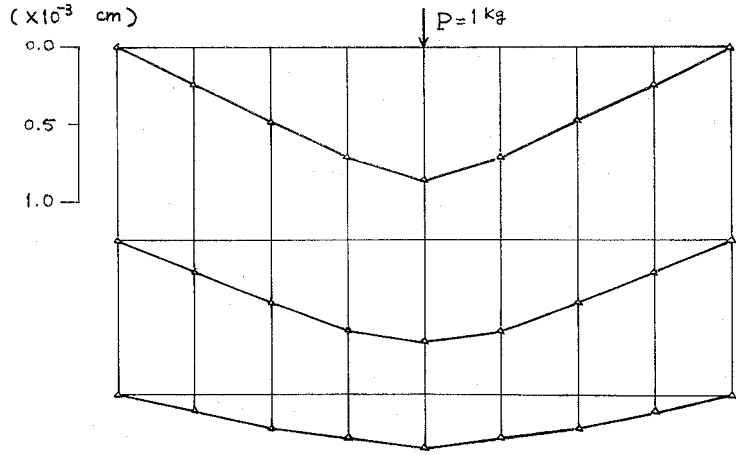

Fig. 6. ω Diagraw

The computation was carried on by FACOM 230–60 in HOKKAIDO university which is an electric digital computor with 80 K core memories. CPU time occupied in a cycle of stress calculation and its output was only 20 seconds for each cases.

a) **Simply supported two-way ribbed plate subjected to a lateral concentrated load at the center of plate**

$E=34800$ kg/cm², $\nu=0.0$, $t=0.3$ cm, $t_{0x}=t_{0y}=0.5$ cm, $\lambda_1=\lambda_2=10$ cm, $h_{0x}=h_{0y}=6$ cm, $n=8$, $k=6$, $P=1$ kg.

b) **Simply supported two-way ribbed plate subjected to surface tractions parallel to xy plane**

$E=34800$ kg/cm², $\nu=0.0$, $t=0.3$ cm, $t_{0x}=t_{0y}=0.5$ cm, $\lambda_1=\lambda_2=10$ cm, $h_{0x}=h_{0y}=6$ cm, $n=8$, $k=6$, $P_x=1$ kg/cm.

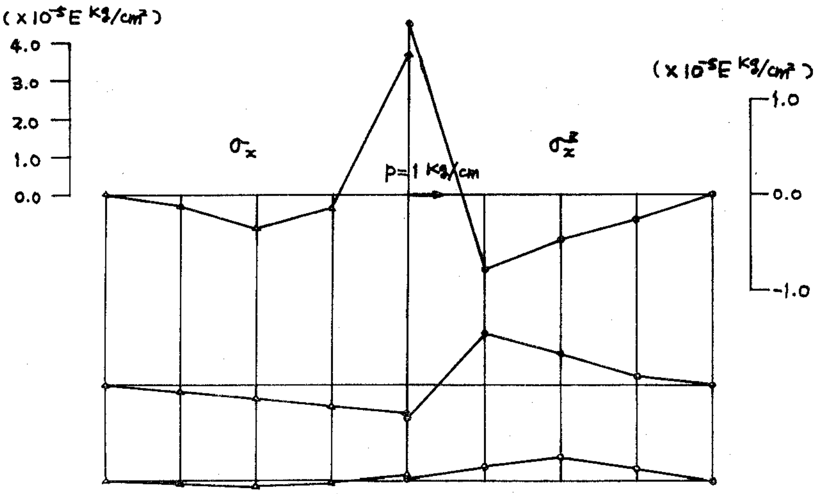

Fig. 7. σ_x^0 and σ_x^{δ} Diagraw

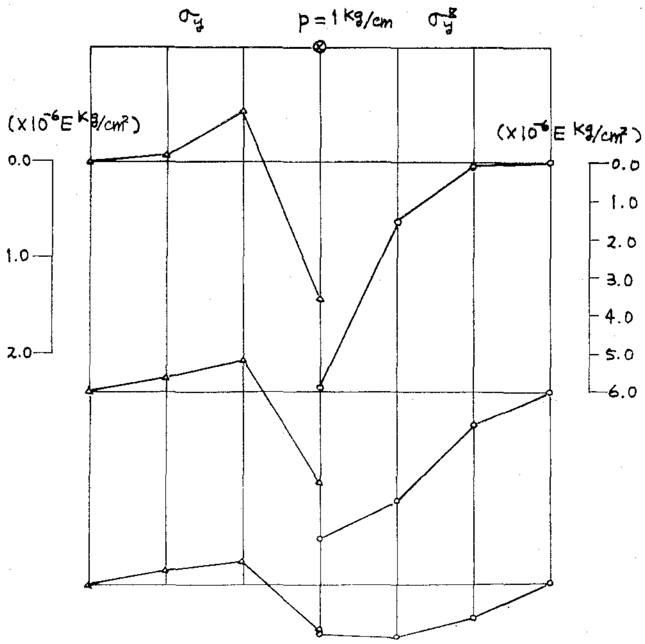

Fig. 8. σ_y^0 and σ_y^s Diagraw

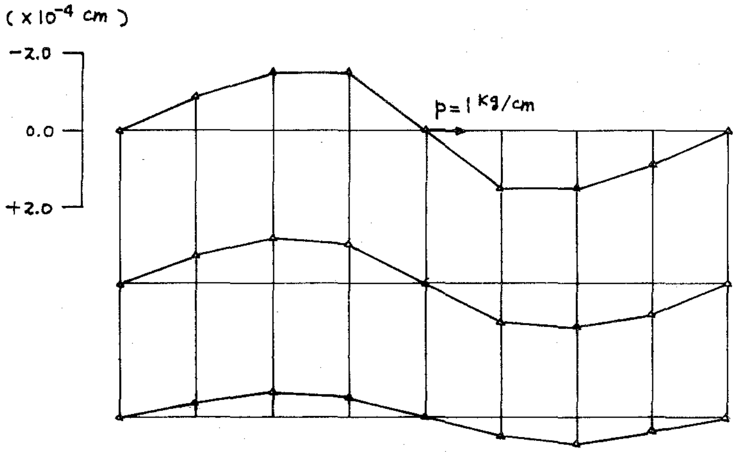

Fig. 9. ω Diagraw

5. Remarks

The solutions of the ribbed plate we have discussed here is analytical method of solving the finite difference equations. The actual system can be reduced to the discrete model starting with the Displacement-shear equation of a folded plate element. Thus obtained equation is easily solved by means of "Finite Integration Transform".

The numerical results will be checked by the experimental one. And the method used in this discussion will be extended to analysis of sandwitch ribbed plate, trussed plate and as such.

The stiffness matrix for the prescribed numerical computation may be said up to $5\times n\times k$ one when we follow the way of usual folded plated theory, whereas the method mentioned above needs only 5×5 matrix.

References

1) Aschenberg, H. u. Freudenberg, G.: "Die Brücke über die Norderelbe im Zuge der Bundesautobahn Sudliche Umgehung Hamcurg" Teil III: Statische Berechnung des Brückenüberbaus Der Stahlbau 1963-9.
2) Daniel, H. u. Schumann, H.: "Die Bundesautobahnbrücke über den Rein bei Leverkusen" Stahlerner Überbau der Strombrücke Der Stahlbau 1967-8.
3) Homberg, H. u. Trenks, K.: Drehsteife Kreuzwerke, Berlin/Göttingen/Heiderberg: Spridger 1962.
4) Leonhardt, F. u. Andrä, W.: Die vereinfachte Trägerrostberechnung, Stuttgart: Julius Hoffmann 1950.
5) Timoschenko, S., Krieger, W.: Theory of plates and schells.
6) Ebner. H.: Die Beanspruchungdünnwandiger kastenträger auf Drillung bei behinderter Quershhnittswölbung Z. F. M. 24, 1933.
Ebner, H., Koller, H.: Über den kraftverlauf inlangs-und querversteifen Scheiben. Luftf-Forsch 15, 1938.
7) Nomachi, S. G.: On torsion bending of thin walled rectangular beams with equidistant rigid diaghrams. Trans. JSCE, 146, 1967.
8) Nomachi, S. G.: A Note on Finite Fourier Transforms concerning Finite Integration. The Memories of the Muroran Institute of Technology. Vol. 5, No. 2, August, 1966.

Appendix

To find the discrete relation of the differential equation, a method of the successive integration by S. G. Nomachi will be introduced.

Suppose the interval of two adjacent point be small enough to assume that a part of linear variation takes a most important one in that reign and the higher order term is neglected in comparison with it.

To make further discussion simple, let us begin with the equation;

$$K\ddot{u}_Y + G_1 u_Y + G_2 \dot{w}_Y = P(x) \tag{39}$$

We take as approximate values of external surface tractions distribute along the line of intersection in the x direction as

$$P(x) = P_X\left(1-\frac{x}{\lambda_1}\right) + P_{X+1}\left(\frac{x}{\lambda_1}\right) \tag{40}$$

The Eq. (39) is rewritten in

$$K\ddot{u}_Y+G_1u_{XY}\left(1-\frac{x}{\lambda_1}\right)+G_1u_{X+.1Y}\left(\frac{x}{\lambda_1}\right)$$
$$+G_2\dot{w}_Y-P_{XY}\left(1-\frac{x}{\lambda_1}\right)-P_{X+.1Y}\left(\frac{x}{\lambda_1}\right)=0 \qquad (41)$$

Integrating it with respect to x, and regulating the integral constant as to satisfy the condition for $x=0$, and taking that the value on the left side of a certain point should be equal to the one on the right side of it, into account, we have

$$K\dot{u}_Y+G_1u_{XY}\left(x-\frac{x^2}{2\lambda_1}\right)+G_1u_{X+.1Y}\left(\frac{x^2}{2\lambda_1}\right)+G_2w_Y$$
$$-P_{XY}\left(x-\frac{x^2}{2\lambda_1}\right)-P_{X+.1Y}\left(\frac{x^2}{2\lambda_1}\right)=K\dot{u}_{XY}+G_2w_{XY} \qquad (42)$$

And substituting

$$w_Y(x)=w_{X.Y}\left(1-\frac{x}{\lambda_1}\right)+w_{X+1.Y}\left(\frac{x}{\lambda_1}\right) \qquad (43)$$

into Eq. (42), and integrating again from $x=0$ to $x=\lambda_1$, we find that

$$K(u_{X+1.Y}-u_{X.Y})+G_1\frac{\lambda_1^2}{3}u_{XY}+G_1\frac{\lambda_1^2}{6}u_{X+1.Y}+G_2\frac{\lambda_1}{2}(w_{XY}+w_{X+1.Y})$$
$$-\left(P_{XY}\frac{\lambda_1^2}{3}+P_{X+1.Y}\frac{\lambda_1^2}{6}\right)=(K\ddot{u}_{XY}+G_2w_{XY})\lambda_1 \qquad (44)$$

Putting λ_1 for x in (42), and multiplying it by λ_1, we have

$$(Ku_{X+1.Y}+G_2w_{X+1.Y})\lambda_1+G_2\frac{\lambda_1^2}{2}(u_{XY}+u_{X+1.Y})-\frac{\lambda_1^2}{2}(P_{X.Y}+P_{X+1.Y})$$
$$=(K\dot{u}_{XY}+G_2w_{XY})\lambda_1 \qquad (45)$$

And substitution of the left side of Eq. (45) into the right side of Eq. (44) becomes

$$K(u_{X+1.Y}-u_{X.Y})-\frac{\lambda_1^2}{6}G_1u_{X.Y}-\frac{\lambda_1^2}{3}G_1u_{X+1.Y}+G_2\frac{\lambda_1}{2}(w_{X.Y}+w_{X+1.Y})$$
$$+\frac{\lambda_1^2}{6}P_{X.Y}+\frac{\lambda_1^2}{3}P_{X+1.Y}=(K\dot{u}_{X+1.Y}+G_2w_{X+1.Y})\lambda_1 \qquad (46)$$

$$K(u_{X.Y}-u_{X-1Y})-\frac{\lambda_1^2}{6}G_1u_{X-1.Y}-\frac{\lambda_1^2}{3}G_1u_{X.Y}+G_2\frac{\lambda_1}{2}(w_{X-1.Y}+w_{X.Y})$$
$$+\frac{\lambda_1^2}{6}P_{X-1.Y}+\frac{\lambda_1^2}{3}P_{XY}=(K\dot{u}_{X.Y}+G_2w_{X.Y})\lambda_1 \qquad (47)$$

Then subtracting Eq. (47) from Eq. (44), the pequired difference equation is obtained as follows.

$$K\Delta_X^2u_{X.Y}+G_1\frac{\lambda_1^2}{6}\Delta_X^2u_{XY}+G_1\lambda_1^2u_{XY}+\frac{\lambda_1}{2}G_2\Delta_Xw_{XY}=\frac{\lambda_1^2}{6}\Delta_X^2P_{XY}+\lambda_1^2P_{XY} \qquad (48)$$

(Received May 21, 1973)