



クロソイド曲線桁の解析に関する研究

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杉 本 博 之

On the Analysis of Clothoidal Beams

Hiroyuki Sugimoto

Abstract

It is said that a clothoidal curve is one of the important alignments in the geometric design of a express highway. Actually the curve is used in a part of overhead lines.

This paper presents the exact solutions of simply supported statical and non-statical clothoidal beams loaded by concentrative loads.

1. 緒 言

道路線形の設計において、自動車を高速で走らせる場合ハンドルを滑らかに切ることを可能にするために緩和曲線が用いられる。この緩和曲線としては、クロソイド曲線が最も適当であるとされており、高速道路の高架部分にも実際に用いられている。

クロソイド曲線桁の理論を研究したものとして、山崎等がたわみ角式を用いて行なっている¹⁾

本論文はそれとは別に、平面内の静定および一次不静定クロソイド曲線桁の理論式を閉じた形で誘導し、若干の計算例と共に発表するものである。

2. 記号の定義

R : 曲 率 半 径

L : 曲 線 長

A : クロソイドのパラメーター (基本クロソイド曲線では、 $RL=A^2$ である。)

τ_0 : クロソイド曲線桁の始点の基本クロソイド曲線中におけるらせん角

τ_1 : クロソイド曲線桁の終点の始点からのらせん角

ρ : 断面力, 変形を考える点の始点からのらせん角

ω : 荷重載荷点の始点からのらせん角

P, M, T : 外力としての垂直力, 曲げモーメント, ねじりモーメント

$V_{\xi}^0(\omega), V_{\xi}(\omega), T_{\xi}^0(\omega)$: 外力 (ξ) が ω に作用した場合の始点および終点垂直反力およびねじり反力

$Q_{\xi}(\rho, \omega), M_{\xi}(\rho, \omega), T_{\xi}(\rho, \omega)$: 外力 (ξ) が ω に作用した場合の ρ のせん断力, 曲げモーメント, ねじりモーメント

$f_{\xi}(\zeta, \omega), \beta_{\xi}(\zeta, \omega), \theta_{\xi}(\zeta, \omega)$: 外力 (ξ) が ω に作用した場合の ρ の垂直変位, たわみ角, ねじり角

$X_{\xi T}$: 外力 (ξ) が作用した場合の始点における不静定ねじり反力

EI : 曲げ剛性

GI_T : ねじり剛性

k : 剛比 (EI/GI_T)

$c, s, u_1, u_2, \dots, u_9$: クロソイド曲線桁の解析に必要な各係数

応力, 変形等の符号は図-1を正とする。

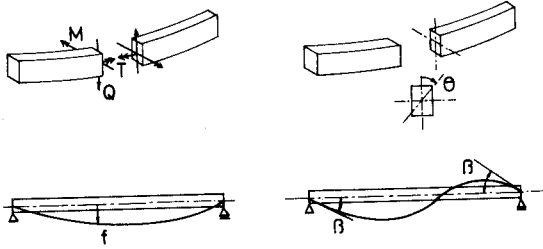


図-1

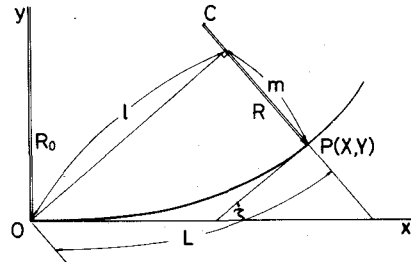


図-2

3. クロソイド曲線

図-2に示す一般クロソイド曲線において次の関係が存在する。

$$\frac{1}{R} = \frac{L}{A^2} + \frac{1}{R_0} \tag{1}$$

また, 点 P 近傍の微少部分において次の三式が成立する。

$$dL = R d\tau \tag{2}$$

$$dX = dL \times \cos \tau \tag{3}$$

$$dY = dL \times \sin \tau \tag{4}$$

式(1), (2)より曲線長 L は次のように導かれる。

$$L = -\frac{A^2}{R_0} + \sqrt{2} A \sqrt{r_0 + \tau} \tag{5}$$

式(2)~(5)より点Pの座標X, Yは次のようになる。

$$\left. \begin{aligned} X &= \frac{A}{\sqrt{2}} \int_0^\tau \frac{\cos \tau}{\sqrt{\tau_0 + \tau}} d\tau \\ Y &= \frac{A}{\sqrt{2}} \int_0^\tau \frac{\sin \tau}{\sqrt{\tau_0 + \tau}} d\tau \end{aligned} \right\} \quad (6)$$

また、図-2におけるl, mは次のようになる。

$$\left. \begin{aligned} l &= X \cos \tau + Y \sin \tau = \\ &= \frac{A}{\sqrt{2}} \int_0^\tau \frac{\cos(\tau - \theta)}{\sqrt{\tau_0 + \theta}} d\theta \\ m &= X \sin \tau - Y \cos \tau = \\ &= \frac{A}{\sqrt{2}} \int_0^\tau \frac{\sin(\tau - \theta)}{\sqrt{\tau_0 + \theta}} d\theta \end{aligned} \right\} \quad (7)$$

4. 各係数の定義および計算

$$(1) \quad c(\alpha, \beta) = \int_\beta^\alpha \frac{\cos(\alpha - \theta)}{\sqrt{\tau_0 + \theta}} d\theta \quad (8)$$

このように定義されたc(α, β)は以下のように展開することができる。

$$\begin{aligned} \int_\beta^\alpha \frac{\cos(\alpha - \theta)}{\sqrt{\tau_0 + \theta}} d\theta &= \int_{\sqrt{\tau_0 + \beta}}^{\sqrt{\tau_0 + \alpha}} \frac{\cos(\tau_0 + \alpha - v^2)}{v} \cdot 2v \cdot dv = \\ &= 2 \left\{ \cos(\tau_0 + \alpha) \int_{\sqrt{\tau_0 + \beta}}^{\sqrt{\tau_0 + \alpha}} \cos v^2 dv + \sin(\tau_0 + \alpha) \int_{\sqrt{\tau_0 + \beta}}^{\sqrt{\tau_0 + \alpha}} \sin v^2 dv \right\} \end{aligned}$$

ここで、

$$f(v) = \int \cos v^2 dv = v - \frac{v^2}{5 \cdot 2!} + \frac{v^4}{9 \cdot 4!} - \frac{v^6}{13 \cdot 6!} + \dots \quad (9)$$

$$g(v) = \int \sin v^2 dv = \frac{v^3}{3} - \frac{v^5}{7 \cdot 3!} + \frac{v^7}{11 \cdot 5!} - \frac{v^9}{15 \cdot 7!} + \dots \quad (10)$$

$$x = \sqrt{\tau_0 + \alpha}, \quad y = \sqrt{\tau_0 + \beta}$$

とおくと、c(α, β)は結局次のようになる。

$$c(\alpha, \beta) = 2 \{ \cos x^2 (f(x) - f(y)) + \sin x^2 (g(x) - g(y)) \} \quad (11)$$

$$(2) \quad s(\alpha, \beta) = \int_\beta^\alpha \frac{\sin(\alpha - \theta)}{\sqrt{\tau_0 + \theta}} d\theta \quad (12)$$

(1)と同様にして、s(α, β)は結局次のようになる。

$$s(\alpha, \beta) = 2 \{ \sin x^2 (f(x) - f(y)) - \cos x^2 (g(x) - g(y)) \} \quad (13)$$

$$(3) \quad u_1(\alpha, \beta, \gamma) = \int_a^{\tau_1} \left\{ \int_\beta^v \frac{\cos(v-\theta)}{\sqrt{\tau_0+\theta}} d\theta \times \int_\gamma^v \frac{\cos(v-\theta)}{\sqrt{\tau_0+\theta}} d\theta \right\} \frac{dv}{\sqrt{\tau_0+v}} \quad (14)$$

このように定義された $u_1(\alpha, \beta, \gamma)$ は, (1) より $x = \sqrt{\tau_0+v}$ とすると次のようになる。

$$u_1(\alpha, \beta, \gamma) = 2 \int_{\sqrt{\tau_0+\alpha}}^{\sqrt{\tau_0+\tau_1}} c(v, \beta) \times c(v, \gamma) dx \quad (15)$$

$$(4) \quad u_2(\alpha, \beta, \gamma) = \int_a^{\tau_1} \left\{ \int_\beta^v \frac{\sin(v-\theta)}{\sqrt{\tau_0+\theta}} d\theta \times \int_\gamma^v \frac{\sin(v-\theta)}{\sqrt{\tau_0+\theta}} d\theta \right\} \frac{dv}{\sqrt{\tau_0+v}} \quad (16)$$

このように定義された $u_2(\alpha, \beta, \gamma)$ は, (2) より $x = \sqrt{\tau_0+v}$ とすると次のようになる。

$$u_2(\alpha, \beta, \gamma) = 2 \int_{\sqrt{\tau_0+\alpha}}^{\sqrt{\tau_0+\tau_1}} s(v, \beta) \times s(v, \gamma) dx \quad (17)$$

$$(5) \quad u_3(\alpha, \beta, \gamma) = \int_a^{\tau_1} \left\{ \cos(v-\beta) \int_\gamma^v \frac{\cos(v-\theta)}{\sqrt{\tau_0+\theta}} d\theta \right\} \frac{dv}{\sqrt{\tau_0+v}} \quad (18)$$

このように定義された $u_3(\alpha, \beta, \gamma)$ は, (1) より $x = \sqrt{\tau_0+v}$ とすると次のようになる。

$$u_3(\alpha, \beta, \gamma) = 2 \int_{\sqrt{\tau_0+\alpha}}^{\sqrt{\tau_0+\tau_1}} \cos(x^2 - \tau_0 - \beta) \times c(v, \gamma) dx \quad (19)$$

$$(6) \quad u_4(\alpha, \beta, \gamma) = \int_a^{\tau_1} \left\{ \cos(v-\beta) \int_\gamma^v \frac{\sin(v-\theta)}{\sqrt{\tau_0+\theta}} d\theta \right\} \frac{dv}{\sqrt{\tau_0+v}} \quad (20)$$

(5) と同様にして $u_4(\alpha, \beta, \gamma)$ は次のようになる。

$$u_4(\alpha, \beta, \gamma) = 2 \int_{\sqrt{\tau_0+\alpha}}^{\sqrt{\tau_0+\tau_1}} \cos(x^2 - \tau_0 - \beta) \times s(v, \gamma) dx \quad (21)$$

$$(7) \quad u_5(\alpha, \beta, \gamma) = \int_a^{\tau_1} \left\{ \sin(v-\beta) \int_\gamma^v \frac{\cos(v-\theta)}{\sqrt{\tau_0+\theta}} d\theta \right\} \frac{dv}{\sqrt{\tau_0+v}} \quad (22)$$

(5) と同様にして $u_5(\alpha, \beta, \gamma)$ は次のようになる。

$$u_5(\alpha, \beta, \gamma) = 2 \int_{\sqrt{\tau_0+\alpha}}^{\sqrt{\tau_0+\tau_1}} \sin(x^2 - \tau_0 - \beta) \times c(v, \gamma) dx \quad (23)$$

$$(8) \quad u_6(\alpha, \beta, \gamma) = \int_a^{\tau_1} \left\{ \sin(v-\beta) \int_\gamma^v \frac{\sin(v-\theta)}{\sqrt{\tau_0+\theta}} d\theta \right\} \frac{dv}{\sqrt{\tau_0+v}} \quad (24)$$

(5) と同様にして $u_6(\alpha, \beta, \gamma)$ は次のようになる。

$$u_6(\alpha, \beta, \gamma) = 2 \int_{\sqrt{\tau_0+\alpha}}^{\sqrt{\tau_0+\tau_1}} \sin(x^2 - \tau_0 - \beta) \times s(v, \gamma) dx \quad (25)$$

$$(9) \quad u_7(\alpha, \beta, \gamma) = \int_a^{\tau_1} \cos(v-\beta) \cos(v-\gamma) \frac{dv}{\sqrt{\tau_0+v}} \quad (26)$$

$x = \sqrt{\tau_0+v}$ とすると,

$$\int_a^{\tau_1} \cos(v-\beta) \cos(v-\gamma) \frac{dv}{\sqrt{\tau_0+v}} =$$

$$\begin{aligned}
 &= \int_{\sqrt{\tau_0+\alpha}}^{\sqrt{\tau_0+\tau_1}} \left\{ \cos(2\tau_0+\beta+\gamma) \cos 2x^2 + \sin(2\tau_0+\beta+\gamma) \sin 2x^2 + \cos(\beta-\gamma) \right\} dx = \\
 &= \cos(2\tau_0+\beta+\gamma) \int_{\sqrt{\tau_0+\alpha}}^{\sqrt{\tau_0+\tau_1}} \cos 2x^2 dx + \sin(2\tau_0+\beta+\gamma) \times \\
 &\quad \times \int_{\sqrt{\tau_0+\alpha}}^{\sqrt{\tau_0+\tau_1}} \sin 2x^2 dx + \cos(\beta-\gamma) \int_{\sqrt{\tau_0+\alpha}}^{\sqrt{\tau_0+\tau_1}} dx
 \end{aligned}$$

式(9), (10)を参考にして, 結局 $u_7(\alpha, \beta, \gamma)$ は次のようになる。

$$\begin{aligned}
 u_7(\alpha, \beta, \gamma) &= \frac{1}{\sqrt{2}} \{ \cos(2\tau_0+\beta+\gamma) \times (f(\sqrt{2(\tau_0+\tau_1)}) - f(\sqrt{2(\tau_0+\alpha)})) + \\
 &\quad + \sin(2\tau_0+\beta+\gamma) (g(\sqrt{2(\tau_0+\tau_1)}) - g(\sqrt{2(\tau_0+\alpha)})) \} + \\
 &\quad + \cos(\beta-\gamma) \times (\sqrt{\tau_0+\tau_1} - \sqrt{\tau_0+\alpha})
 \end{aligned} \tag{27}$$

$$(10) \quad u_8(\alpha, \beta, \gamma) = \int_a^{\tau_1} \cos(v-\beta) \sin(v-\gamma) \frac{dv}{\sqrt{\tau_0+v}} \tag{28}$$

(9)と同様にして,

$$\begin{aligned}
 u_8(\gamma, \beta, \gamma) &= \frac{1}{\sqrt{2}} \{ \cos(2\tau_0+\beta+\gamma) + (g(\sqrt{2(\tau_0+\tau_1)}) - g(\sqrt{2(\tau_0+\alpha)})) - \\
 &\quad - \sin(2\tau_0+\beta+\gamma) \times (f(\sqrt{2(\tau_0+\tau_1)}) - f(\sqrt{2(\tau_0+\alpha)})) \} + \\
 &\quad + \sin(\beta-\gamma) \times (\sqrt{\tau_0+\tau_1} - \sqrt{\tau_0+\alpha})
 \end{aligned} \tag{29}$$

$$(11) \quad u_9(\alpha, \beta, \gamma) = \int_a^{\tau_1} \sin(v-\beta) \sin(v-\gamma) \frac{dv}{\sqrt{\tau_0+v}} \tag{30}$$

(9)と同様にして,

$$\begin{aligned}
 u_9(\alpha, \beta, \gamma) &= -\frac{1}{\sqrt{2}} \{ \cos(2\tau_0+\beta+\gamma) \times (f(\sqrt{2(\tau_0+\tau_1)}) - f(\sqrt{2(\tau_0+\alpha)})) + \\
 &\quad + \sin(2\tau_0+\beta+\gamma) + (g(\sqrt{2(\tau_0+\tau_1)}) - g(\sqrt{2(\tau_0+\alpha)})) \} + \\
 &\quad + \cos(\beta-\gamma) \times (\sqrt{\tau_0+\tau_1} - \sqrt{\tau_0+\alpha})
 \end{aligned} \tag{31}$$

5. クロソイド曲線静定単純桁の理論

5-1 集中荷重 P, M, T による断面力

(1) 垂直力 P が作用する場合の断面力

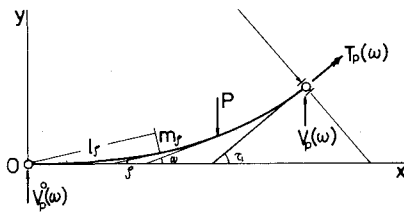


図-3

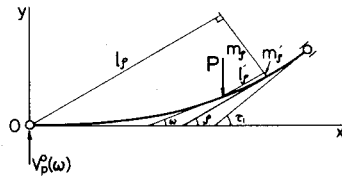


図-4

(a) 反力

図-3より次式が得られる。

$$\left. \begin{aligned} V_P^\circ(\omega) + V_P(\omega) - P &= 0 \\ P X_\omega - V_P(\omega) \cdot X_{\tau_1} + T_P(\omega) \cdot \sin \tau_1 &= 0 \\ P \cdot X_\omega - V_P(\omega) \cdot X_{\tau_1} + T_P(\omega) \cdot \sin \tau_1 &= 0 \end{aligned} \right\}$$

以上の三式を連立に解いて、

$$V_P^\circ(\omega) = P \frac{(X_{\tau_1} - X_\omega) \cos \tau_1 + (Y_{\tau_1} - Y_\omega) \sin \tau_1}{X_{\tau_1} \cos \tau_1 + Y_{\tau_1} \sin \tau_1} = P \frac{c(\tau_1, \omega)}{c(\tau_1, 0)} \quad (32)$$

$$V_P(\omega) = P \frac{X_\omega \cos \tau_1 + Y_\omega \sin \tau_1}{X_{\tau_1} \cos \tau_1 + Y_{\tau_1} \sin \tau_1} = P \frac{(c(\tau_1, 0) - c(\tau_1, \omega))}{c(\tau_1, 0)} \quad (33)$$

$$\begin{aligned} T_P(\omega) &= P \frac{X_{\tau_1} Y_\omega - Y_{\tau_1} X_\omega}{X_{\tau_1} \cos \tau_1 + Y_{\tau_1} \sin \tau_1} = \\ &= \frac{AP}{\sqrt{2}} \frac{(s(\tau_1, \omega) \times c(\tau_1, 0) - c(\tau_1, \omega) \times s(\tau_1, 0))}{c(\tau_1, 0)} \end{aligned} \quad (34)$$

(b) 曲げモーメント

(i) $0 \leq \rho \leq \omega$ の場合 (図-3 参照)

$$\begin{aligned} V_P^\circ(\omega) \times l_\rho + M_P(\rho, \omega) &= 0 \\ M_P(\rho, \omega) &= -V_P^\circ(\omega) \times l_\rho = -\frac{AP}{\sqrt{2}} \frac{AP}{c(\tau_1, 0)} \times c(\tau_1, \omega) \times c(\rho, 0) \end{aligned} \quad (35)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合 (図-4 参照)

$$\begin{aligned} V_P^\circ(\omega) \times l_\rho - P \times l'_\rho + M_P(\rho, \omega) &= 0 \\ M_P(\rho, \omega) &= -V_P^\circ(\omega) \times l_\rho + P \times l'_\rho = \\ &= -\frac{AP}{\sqrt{2}} \frac{AP}{c(\tau_1, 0)} \times (c(\tau_1, \omega) \times c(\rho, 0) - c(\rho, \omega) \times c(\tau_1, 0)) \end{aligned} \quad (36)$$

(c) ねじりモーメント

(i) $0 \leq \rho \leq \omega$ の場合 (図-3 参照)

$$\begin{aligned} V_P^\circ(\omega) \times m_\rho + T_P(\rho, \omega) &= 0 \\ T_P(\rho, \omega) &= -V_P^\circ(\omega) \times m_\rho = -\frac{AP}{\sqrt{2}} \frac{AP}{c(\tau_1, 0)} \times c(\tau_1, \omega) \times s(\rho, 0) \end{aligned} \quad (37)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合 (図-4 参照)

$$\begin{aligned} V_P^\circ(\omega) \times m_\rho - P \times m'_\rho + T_P(\rho, \omega) &= 0 \\ T_P(\rho, \omega) &= -V_P^\circ(\omega) \times m_\rho + P \times m'_\rho = \end{aligned}$$

$$= -\frac{AP}{\sqrt{2}c(\tau_1, 0)} \times (c(\tau_1, \omega) \times s(\rho, 0) - s(\rho, \omega) \times c(\tau_1, 0)) \quad (38)$$

(2) 曲げモーメントMが作用する場合の断面力

(a) 反力

図-5より次式が得られる。

$$\left. \begin{aligned} V_M^\circ(\omega) + V_M(\omega) &= 0 \\ M \cdot \cos\omega + T_M(\omega) \cdot \sin\tau_1 - V_M(\omega) \cdot X_{\tau_1} &= 0 \\ M \cdot \sin\omega - T_M(\omega) \cdot \cos\tau_1 - V_M(\omega) \cdot Y_{\tau_1} &= 0 \end{aligned} \right\}$$

以上の三式を連立に解いて、

$$V_M^\circ(\omega) = -M \frac{\cos\omega \cos\tau_1 + \sin\omega \sin\tau_1}{X_{\tau_1} \cos\tau_1 + Y_{\tau_1} \sin\tau_1} = -\frac{\sqrt{2}M}{A} \frac{\cos(\tau_1 - \omega)}{c(\tau_1, 0)} \quad (39)$$

$$V_M(\omega) = M \frac{\cos\omega \cos\tau_1 + \sin\omega \sin\tau_1}{X_{\tau_1} \cos\tau_1 + Y_{\tau_1} \sin\tau_1} = \frac{\sqrt{2}M}{A} \frac{\cos(\tau_1 - \omega)}{c(\tau_1, 0)} \quad (40)$$

$$\begin{aligned} T_M(\omega) &= M \frac{X_{\tau_1} \sin\omega - Y_{\tau_1} \cos\omega}{X_{\tau_1} \cos\tau_1 + Y_{\tau_1} \sin\tau_1} = \\ &= \frac{M}{c(\tau_1, 0)} (\cos(\tau_1 - \omega) \cdot s(\tau_1, 0) - \sin(\tau_1 - \omega) \cdot c(\tau_1, 0)) \end{aligned} \quad (41)$$

(b) 曲げモーメント

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} V_M^\circ(\omega) \times l_\rho + M_M(\rho, \omega) &= 0 \\ M_M(\rho, \omega) &= -V_M^\circ(\omega) \times l_\rho = \frac{M}{c(\tau_1, 0)} \cos(\tau_1 - \omega) \cdot c(\rho, 0) \end{aligned} \quad (42)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} V_M^\circ(\omega) \times l_\rho + M_M(\rho, \omega) + M \times \cos(\rho - \omega) &= 0 \\ M_M(\rho, \omega) &= -V_M^\circ(\omega) \times l_\rho - M \times \cos(\rho - \omega) = \\ &= \frac{M}{c(\tau_1, 0)} (\cos(\tau_1 - \omega) \times c(\rho, 0) - \cos(\rho - \omega) \times c(\tau_1, 0)) \end{aligned} \quad (43)$$

(c) (ねじりモーメント)

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} V_M^\circ(\omega) \times m_\rho + T_M(\rho, \omega) &= 0 \\ T_M(\rho, \omega) &= -V_M^\circ(\omega) \times m_\rho = \frac{M}{c(\tau_1, 0)} \cos(\tau_1 - \omega) \times s(\rho, 0) \end{aligned} \quad (44)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
 V_M^\circ(\omega) \times m_\rho + T_M(\rho, \omega) + M \times \sin(\rho - \omega) &= 0 \\
 T_M(\rho, \omega) &= -V_M^\circ(\omega) \times m_\rho - M \times \sin(\rho - \omega) = \\
 &= \frac{M}{c(\tau_1, 0)} (\cos(\tau_1 - \omega) + s(\rho, 0) - \sin(\rho - \omega) \times c(\tau_1, 0)) \quad (45)
 \end{aligned}$$

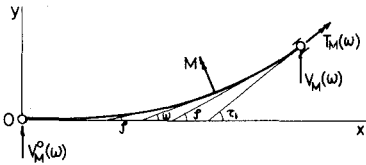


図-5

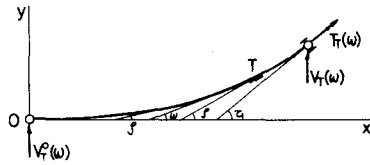


図-6

(3) ねじりモーメント T が作用する場合の断面力

(a) 反力

図-6 より次式が得られる。

$$\left. \begin{aligned}
 V_T^\circ(\omega) + V_T(\tau_1, \omega) &= 0 \\
 V_T(\omega) \times X_{\tau_1} - T \times \sin \omega - T_T(\omega) \times \sin \tau_1 &= 0 \\
 V_T(\omega) \times Y_{\tau_1} + T \times \cos \omega + T_T(\omega) \times \cos \tau_1 &= 0
 \end{aligned} \right\}$$

以上の三式を連立に解いて、

$$\begin{aligned}
 V_T^\circ(\omega) &= T \frac{\sin(\tau_1 - \omega)}{X_{\tau_1} \cdot \cos \tau_1 + Y_{\tau_1} \cdot \sin \tau_1} = \frac{\sqrt{2} T \sin(\tau_1 - \omega)}{A c(\tau_1, 0)} \\
 V_T(\omega) &= -T \frac{\sin(\tau_1 - \omega)}{X_{\tau_1} \cdot \cos \tau_1 + Y_{\tau_1} \cdot \sin \tau_1} = -\frac{\sqrt{2} T \sin(\tau_1 - \omega)}{A c(\tau_1, 0)} \\
 T_T(\omega) &= -T \frac{X_{\tau_1} \times \cos \omega + Y_{\tau_1} \times \sin \omega}{X_{\tau_1} \times \cos \tau_1 + Y_{\tau_1} \times \sin \tau_1} = \\
 &= -\frac{T}{c(\tau_1, 0)} (\sin(\tau_1 - \omega) \times s(\tau_1, 0) + \cos(\tau_1 - \omega) c(\tau_1, 0)) \quad (48)
 \end{aligned}$$

(b) 曲げモーメント

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
 V_T^\circ(\omega) \times l_\rho + M_T(\rho, \omega) &= 0 \\
 M_T(\rho, \omega) &= -V_T^\circ(\omega) \times l_\rho = -\frac{T}{c(\tau_1, 0)} \sin(\tau_1 - \omega) \times c(\rho, 0) \quad (49)
 \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
 V_T^2(\omega) \times l_\rho - T \times \sin(\rho - \omega) + M_T(\rho, \omega) &= 0 \\
 M_T(\rho, \omega) &= -V_T^2(\omega) \times l_\rho + T \times \sin(\rho - \omega) = \\
 &= -\frac{T}{c(\tau_1, 0)} (\sin(\tau_1 - \omega) \times c(\rho, 0) - \sin(\rho - \omega) \times c(\tau_1, 0)) \quad (50)
 \end{aligned}$$

(c) ねじりモーメント

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
 V_T^2(\rho, \omega) \times m_\rho + T_T(\rho, \omega) &= 0 \\
 T_T(\rho, \omega) &= -V_T^2(\omega) \times m_\rho = -\frac{T}{c(\tau_1, 0)} \sin(\tau_1 - \omega) \times s(\rho, 0) \quad (51)
 \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
 V_T^2(\omega) \times m_\rho + T \times \cos(\rho - \omega) + T_T(\rho, \omega) &= 0 \\
 T_T(\rho, \omega) &= -V_T^2(\omega) \times m_\rho - T \times \cos(\rho - \omega) = \\
 &= -\frac{T}{c(\tau_1, 0)} (\sin(\tau_1 - \omega) \times s(\rho, 0) + \cos(\rho - \omega) \times c(\tau_1, 0)) \quad (52)
 \end{aligned}$$

5-2 集中荷重 P, M, T による垂直変位

(1) 垂直力 P が作用する場合の垂直変位

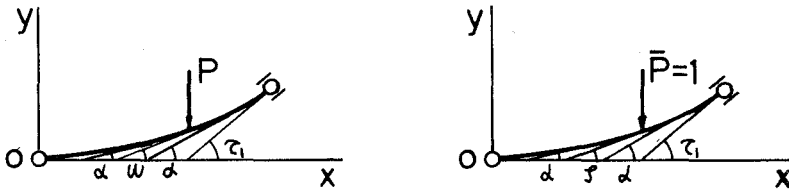


図-7

$$f_P(\rho, \omega) = f_{MP}(\rho, \omega) + f_{TP}(\rho, \omega)$$

(a) $f_{MP}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
 f_{MP}(\rho, \omega) &= \frac{1}{EI} \int_0^{\tau_1} M_P(\alpha, \omega) \times \bar{M}_P(\alpha, \rho) dL = \\
 &= \frac{A^3 P}{2\sqrt{2} EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{c(\tau_1, \omega) \times c(\tau_1, \rho) \times u_1(\alpha, \alpha, \alpha) - \\
 &\quad - c(\tau_1, \omega) \times c(\tau_1, \alpha) \times u_1(\rho, \rho, \alpha) - c(\tau_1, \alpha) \times c(\tau_1, \rho) \times u_1(\omega, \omega, \alpha) + \\
 &\quad + c(\tau_1, \alpha)^2 \times u_1(\omega, \omega, \rho)\} \quad (53)
 \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
 f_{MP}(\rho, \omega) &= \frac{A^3 P}{2\sqrt{2} EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{c(\tau_1, \omega) \times c(\tau_1, \rho) \times u_1(0, 0, 0) - \\
 &\quad - c(\tau_1, \rho) \times c(\tau_1, 0) \times u_1(\omega, \omega, 0) - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_1(\rho, \rho, 0) + \\
 &\quad + c(\tau_1, 0)^2 \times u_1(\rho, \rho, \omega)\} \quad (54)
 \end{aligned}$$

(b) $f_{TP}(\rho, \omega)$ (i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
 f_{TP}(\rho, \omega) &= \frac{1}{GI_T} \int_0^{\tau_1} T_P(\alpha, \omega) \times \bar{T}_P(\alpha, \rho) dL = \\
 &= \frac{A^3 P}{2\sqrt{2} GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{c(\tau_1, \omega) \times c(\tau_1, \rho) \times u_2(0, 0, 0) - \\
 &\quad - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_2(\rho, \rho, 0) - c(\tau_1, 0) \times c(\tau_1, \rho) \times u_2(\omega, \omega, 0) + \\
 &\quad + c(\tau_1, 0)^2 \times u_2(\omega, \omega, \rho)\} \quad (55)
 \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
 f_{TP}(\rho, \omega) &= \frac{A^3 P}{2\sqrt{2} GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{c(\tau_1, \omega) \times c(\tau_1, \rho) \times u_2(0, 0, 0) - \\
 &\quad - c(\tau_1, \rho) \times c(\tau_1, 0) \times u_2(\omega, \omega, 0) - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_2(\rho, \rho, 0) + \\
 &\quad + c(\tau_1, 0)^2 \times u_2(\rho, \rho, \omega)\} \quad (56)
 \end{aligned}$$

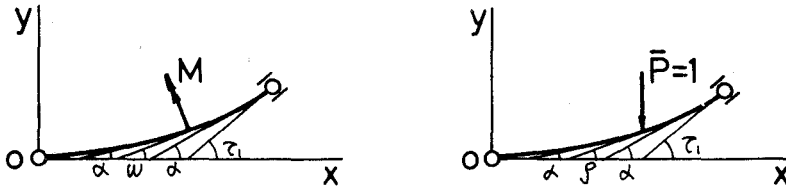
(2) 曲げモーメント M が作用する場合の垂直変位

図-8

$$f_M(\rho, \omega) = f_{MM}(\rho, \omega) + f_{TM}(\rho, \omega)$$

(a) $f_{MM}(\rho, \omega)$ (i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
 f_{MM}(\rho, \omega) &= \frac{1}{EI} \int_0^{\tau_1} M_M(\alpha, \omega) \times \bar{M}_P(\alpha, \rho) dL = \\
 &= -\frac{A^2 M}{2EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{\cos(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_1(0, 0, 0) -
 \end{aligned}$$

(20)

$$\begin{aligned}
 & -\cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_1(\rho, \rho, o) - \\
 & -c(\tau_1, o) \times c(\tau_1, \rho) \times u_3(\omega, \omega, o) + c(\tau_1, o)^2 \times u_3(\omega, \omega, \rho) \} \quad (57)
 \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
 f_{MM}(\rho, \omega) = & -\frac{A^2 M}{2EI} \times \frac{1}{c(\tau_1, o)^2} \times \{ \cos(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_1(o, o, o) - \\
 & -c(\tau_1, \rho) \times c(\tau_1, o) \times u_3(\omega, \omega, o) - \\
 & -\cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_1(\rho, \rho, o) + c(\tau_1, o)^2 \times u_3(\rho, \omega, \rho) \} \quad (58)
 \end{aligned}$$

(b) $f_{TM}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
 f_{TM}(\rho, \omega) = & \frac{1}{GI_T} \int_0^{\tau_1} T_M(\alpha, \omega) \times \bar{T}_P(\alpha, \rho) dL = \\
 = & -\frac{A^3 M}{2GI_T} \times \frac{1}{c(\tau_1, o)^2} \times \{ \cos(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_2(o, o, o) - \\
 & -\cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_2(\rho, \rho, o) - \\
 & -c(\tau_1, o) \times c(\tau_1, \rho) \times u_6(\omega, \omega, o) + c(\tau_1, o)^2 \times u_6(\omega, \omega, \rho) \} \quad (59)
 \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
 f_{TM}(\rho, \omega) = & -\frac{A^2 M}{2GI_T} \times \frac{1}{c(\tau_1, o)^2} \times \{ \cos(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_2(o, o, o) - \\
 & -c(\tau_1, \rho) \times c(\tau_1, o) \times u_6(\omega, \omega, o) - \\
 & -\cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_2(\rho, \rho, o) + c(\tau_1, o)^2 \times u_6(\rho, \omega, \rho) \} \quad (60)
 \end{aligned}$$

(3) ねじりモーメント T が作用する場合の垂直変位

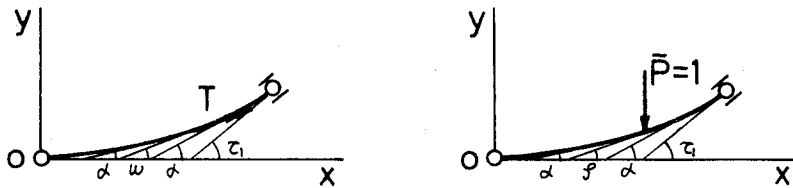


図-9

$$f_T(\rho, \omega) = f_{MT}(\rho, \omega) + f_{TT}(\rho, \omega)$$

(a) $f_{MT}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$f_{MT}(\rho, \omega) = \frac{1}{EI} \int_0^{\tau_1} M_T(\alpha, \omega) \times \bar{M}_P(\alpha, \rho) dL =$$

(21)

$$\begin{aligned}
&= \frac{A^2 T}{2EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_1(0, 0, 0) - \\
&\quad - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_1(\rho, \rho, 0) - \\
&\quad - c(\tau_1, 0) \times c(\tau_1, \rho) \times u_5(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_5(\omega, \omega, \rho) \} \quad (61)
\end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
f_{MT}(\rho, \omega) &= \frac{A^2 T}{2EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_1(0, 0, 0) - \\
&\quad - c(\tau_1, \rho) \times c(\tau_1, 0) \times u_5(\omega, \omega, 0) - \\
&\quad - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_1(\rho, \rho, 0) + c(\tau_1, 0)^2 \times u_5(\rho, \omega, \rho) \} \quad (62)
\end{aligned}$$

(b) $f_{TT}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
f_{TT}(\rho, \omega) &= \frac{1}{GI_T} \int_0^{\tau_1} T_T(\alpha, \omega) \times \bar{T}_P(\alpha, \rho) dL = \\
&= \frac{A^2 T}{2GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_2(0, 0, 0) - \\
&\quad - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_2(\rho, \rho, 0) + \\
&\quad + c(\tau_1, 0) \times c(\tau_1, \rho) \times u_4(\omega, \omega, 0) - c(\tau_1, 0)^2 \times u_4(\omega, \omega, \rho) \} \quad (63)
\end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
f_{TT}(\rho, \omega) &= \frac{A^2 T}{2GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_2(0, 0, 0) + \\
&\quad + c(\tau_1, \rho) \times c(\tau_1, 0) \times u_4(\omega, \omega, 0) - \\
&\quad - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_2(\rho, \rho, 0) - c(\tau_1, 0)^2 \times u_4(\rho, \omega, \rho) \} \quad (64)
\end{aligned}$$

5-3 集中荷重 P, M, T によるたわみ角

(1) 垂直力 P が作用する場合のたわみ角

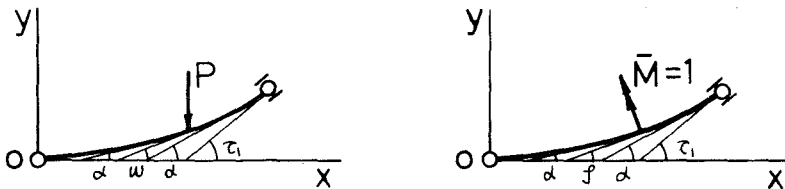


図-10

$$\beta_P(\rho, \omega) = \beta_{MP}(\rho, \omega) + \beta_{TP}(\rho, \omega)$$

(22)

(a) $\beta_{MP}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \beta_{MP}(\rho, \omega) &= \frac{1}{EI} \int_0^{\tau_1} M_P(\alpha, \omega) \times \bar{M}_M(\alpha, \rho) dL = \\ &= -\frac{A^2 P}{2EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_1(0, 0, 0) - \\ &\quad - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_3(\rho, \rho, 0) - \\ &\quad - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_1(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_3(\omega, \rho, \omega) \} \end{aligned} \quad (65)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{MP}(\rho, \omega) &= \frac{A^2 P}{2EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_1(0, 0, 0) - \\ &\quad - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_1(\omega, \omega, 0) - \\ &\quad - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_3(\rho, \rho, 0) + c(\tau_1, 0)^2 \times u_3(\rho, \rho, \omega) \} \end{aligned} \quad (66)$$

(b) $\beta_{TP}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \beta_{TP}(\rho, \omega) &= \frac{1}{GI_T} \int_0^{\tau_1} T_P(\alpha, \omega) \times \bar{T}_M(\alpha, \rho) dL = \\ &= -\frac{A^2 P}{2GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_2(0, 0, 0) - \\ &\quad - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_6(\rho, \rho, 0) - \\ &\quad - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_2(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_6(\omega, \rho, \omega) \} \end{aligned} \quad (67)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{TP}(\rho, \omega) &= -\frac{A^2 P}{2GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_2(0, 0, 0) - \\ &\quad - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_2(\omega, \omega, 0) - \\ &\quad - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_6(\rho, \rho, 0) + c(\tau_1, 0)^2 \times u_6(\rho, \rho, \omega) \} \end{aligned} \quad (68)$$

(2) 曲げモーメントMが作用する場合のたわみ角

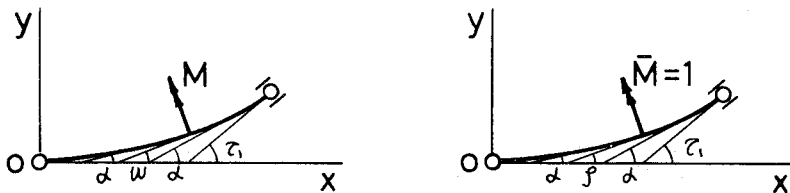


図-11

$$\beta_M(\rho, \omega) = \beta_{MM}(\rho, \omega) + \beta_{TM}(\rho, \omega)$$

(a) $\beta_{MM}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \beta_{MM}(\rho, \omega) &= \frac{1}{EI} \int_0^{\tau_1} M_M(\alpha, \omega) \times \bar{M}_M(\alpha, \rho) dL = \\ &= \frac{AM}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_1(\omega, \omega, 0) - \\ &\quad - \cos(\tau_1 - \omega) \times c(\tau_1, 0) \times u_3(\rho, \rho, 0) - \\ &\quad - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_3(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_7(\omega, \omega, \rho) \} \end{aligned} \quad (69)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{MM}(\rho, \omega) &= \frac{AM}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_1(\omega, \omega, 0) - \\ &\quad - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_3(\omega, \omega, 0) - \\ &\quad - \cos(\tau_1 - \omega) \times c(\tau_1, 0) \times u_3(\rho, \rho, 0) + c(\tau_1, 0)^2 \times u_7(\rho, \omega, \rho) \} \end{aligned} \quad (70)$$

(b) $\beta_{TM}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \beta_{TM}(\rho, \omega) &= \frac{1}{GI_T} \int_0^{\tau_1} T_M(\alpha, \omega) \times \bar{T}_M(\alpha, \rho) dL = \\ &= \frac{AM}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_2(\omega, \omega, 0) - \\ &\quad - \cos(\tau_1 - \omega) \times c(\tau_1, 0) \times u_6(\rho, \rho, 0) - \\ &\quad - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_6(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_9(\omega, \omega, \rho) \} \end{aligned} \quad (71)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{TM}(\rho, \omega) &= \frac{AM}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_2(\omega, \omega, 0) - \\ &\quad - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_6(\omega, \omega, 0) - \\ &\quad - \cos(\tau_1 - \omega) \times c(\tau_1, 0) \times u_6(\rho, \rho, 0) + c(\tau_1, 0)^2 \times u_9(\rho, \omega, \rho) \} \end{aligned} \quad (72)$$

(3) ねじりモーメント T が作用する場合のたわみ角

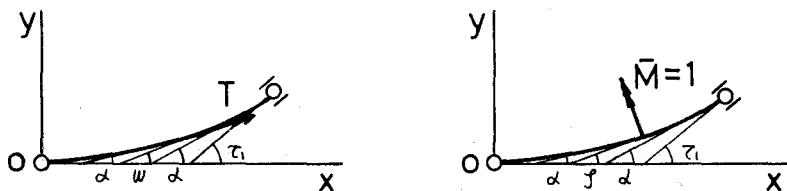


図-12

$$\beta_T(\rho, \omega) = \beta_{MT}(\rho, \omega) + \beta_{TT}(\rho, \omega)$$

(a) $\beta_{MT}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \beta_{MT}(\rho, \omega) &= \frac{1}{EI} \int_0^{\tau_1} M_T(\alpha, \omega) \times \bar{M}_M(\alpha, \rho) dL = \\ &= -\frac{AT}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_1(\omega, \omega, 0) - \\ &\quad - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_3(\rho, \rho, 0) - \\ &\quad - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_5(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_8(\omega, \rho, \omega) \} \quad (73) \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{MT}(\rho, \omega) &= -\frac{AT}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_1(\omega, \omega, 0) - \\ &\quad - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_5(\omega, \omega, 0) - \\ &\quad - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_3(\rho, \rho, 0) + c(\tau_1, 0)^2 \times u_8(\rho, \rho, \omega) \} \quad (74) \end{aligned}$$

(b) $\beta_{TT}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \beta_{TT}(\rho, \omega) &= \frac{1}{GIT} \int_0^{\tau_1} T_T(\alpha, \omega) \times \bar{T}_M(\alpha, \rho) dL = \\ &= -\frac{AT}{\sqrt{2}GIT} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_2(\omega, \omega, 0) - \\ &\quad - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_6(\rho, \rho, 0) + \\ &\quad + \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_4(\omega, \omega, 0) - c(\tau_1, 0)^2 \times u_8(\omega, \omega, \rho) \} \quad (75) \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{TT}(\rho, \omega) &= -\frac{AT}{\sqrt{2}GIT} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_2(\omega, \omega, 0) + \\ &\quad + \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_4(\omega, \omega, 0) - \\ &\quad - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_6(\rho, \rho, 0) - c(\tau_1, 0)^2 \times u_8(\rho, \omega, \rho) \} \quad (76) \end{aligned}$$

5-4 集中荷重 P, M, T によるねじり角

(1) 垂直力 P が作用する場合のねじり角

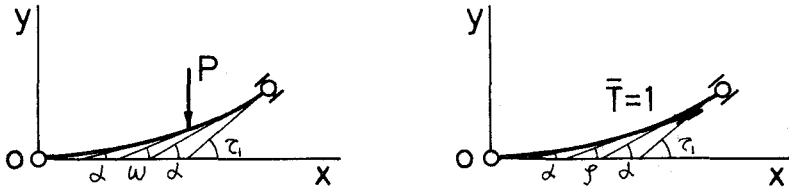


図-13

$$\theta_P(\rho, \omega) = \theta_{MP}(\rho, \omega) + \theta_{TP}(\rho, \omega)$$

(a) $\theta_{MP}(\rho, \omega)$ (i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \theta_{MP}(\rho, \omega) &= \frac{1}{EI} \int_0^{\tau_1} M_P(\alpha, \omega) \times \bar{M}_T(\alpha, \rho) dL = \\ &= \frac{A^2 P}{2EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_1(0, 0, 0) - \\ &\quad - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_5(\rho, \rho, 0) - \\ &\quad - \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_1(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_5(\omega, \rho, \omega) \} \end{aligned} \quad (77)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \theta_{MP}(\rho, \omega) &= \frac{A^2 P}{2EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_1(0, 0, 0) - \\ &\quad - \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_1(\omega, \omega, 0) - \\ &\quad - c(\tau_1, \omega) c(\tau_1, 0) \times u_5(\rho, \rho, 0) + c(\tau_1, 0)^2 \times u_5(\rho, \rho, \omega) \} \end{aligned} \quad (78)$$

(b) $\theta_{TP}(\rho, \omega)$ (i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \theta_{TP}(\rho, \omega) &= \frac{1}{GI_T} \int_0^{\tau_1} T_P(\alpha, \omega) \times \bar{T}_T(\alpha, \rho) dL = \\ &= \frac{A^2 P}{2GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_2(0, 0, 0) + \\ &\quad + c(\tau_1, \omega) \times c(\tau_1, 0) \times u_4(\rho, \rho, 0) - \\ &\quad - \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_2(\omega, \omega, 0) - c(\tau_1, 0)^2 \times u_4(\omega, \rho, \omega) \} \end{aligned} \quad (79)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \theta_{TP}(\rho, \omega) &= \frac{AP}{2GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_2(0, 0, 0) - \\ &\quad - \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_2(\omega, \omega, 0) + \\ &\quad + c(\tau_1, \omega) \times c(\tau_1, 0) \times u_4(\rho, \rho, 0) - c(\tau_1, 0)^2 \times u_4(\rho, \rho, \omega) \} \end{aligned} \quad (80)$$

(2) 曲げモーメントMが作用する場合のねじり角

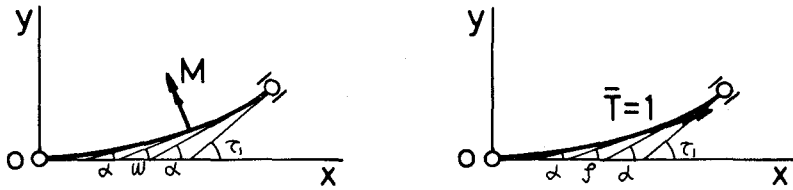


図-14

$$\theta_M(\rho, \omega) = \theta_{MM}(\rho, \omega) + \theta_{TM}(\rho, \omega)$$

(a) $\theta_{MM}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \theta_{MM}(\rho, \omega) &= \frac{1}{EI} \int_0^{\tau_1} M_M(\alpha, \omega) \times \bar{M}_T(\alpha, \rho) dL = \\ &= -\frac{1}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_1(0, 0, 0) - \\ &\quad - \cos(\tau_1 - \omega) \times c(\tau_1, 0) \times u_5(\rho, \rho, 0) - \\ &\quad - \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_3(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_8(\omega, \omega, \rho) \} \end{aligned} \quad (81)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \theta_{MM}(\rho, \omega) &= -\frac{AM}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_1(0, 0, 0) - \\ &\quad - \cos(\tau_1 - \omega) \times c(\tau_1, 0) \times u_5(\rho, \rho, 0) - \\ &\quad - \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_3(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_8(\rho, \omega, \rho) \} \end{aligned} \quad (82)$$

(b) $\theta_{TM}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \theta_{TM}(\rho, \omega) &= \frac{1}{GI_T} \int_0^{\tau_1} T_M(\alpha, \omega) \times \bar{T}_T(\alpha, \rho) dL = \\ &= -\frac{AM}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_2(0, 0, 0) + \\ &\quad + \cos(\tau_1 - \omega) \times c(\tau_1, 0) \times u_4(\rho, \rho, 0) - \\ &\quad - \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_6(\omega, \omega, 0) - c(\tau_1, 0)^2 \times u_8(\omega, \rho, \omega) \} \end{aligned} \quad (83)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\theta_{TM}(\rho, \omega) = -\frac{AM}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \cos(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_2(0, 0, 0) -$$

$$\begin{aligned}
 & -\sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_6(\omega, \omega, 0) + \\
 & -\cos(\tau_1 - \omega) \times c(\tau_1, 0) \times u_4(\rho, \rho, 0) - c(\tau_1, 0)^2 \times u_8(\rho, \rho, \omega) \} \quad (84)
 \end{aligned}$$

(3) ねじりモーメント T が作用する場合のねじり角

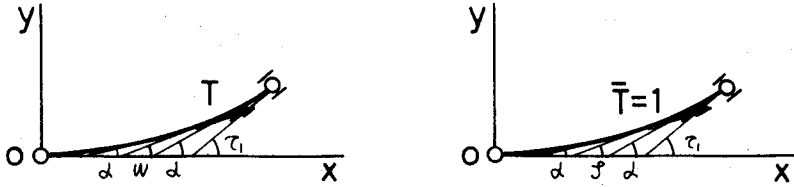


図-15

$$\theta_T(\rho, \omega) = \theta_{MT}(\rho, \omega) + \theta_{TT}(\rho, \omega)$$

(a) $\theta_{MT}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
 \theta_{MT}(\rho, \omega) &= \frac{1}{EI} \int_0^{\tau_1} M_T(\alpha, \omega) \times \bar{M}_T(\alpha, \rho) dL = \\
 &= \frac{AM}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_1(\alpha, \alpha, 0) - \\
 &\quad - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_5(\rho, \rho, 0) - \\
 &\quad - \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_5(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_9(\omega, \omega, \rho) \} \quad (85)
 \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
 \theta_{MT}(\rho, \omega) &= \frac{AT}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_1(\alpha, \alpha, 0) - \\
 &\quad - \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_5(\omega, \omega, 0) - \\
 &\quad - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_5(\rho, \rho, 0) + c(\tau_1, 0)^2 \times u_9(\rho, \omega, \rho) \} \quad (86)
 \end{aligned}$$

(b) $\theta_{TT}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
 \theta_{TT}(\rho, \omega) &= \frac{1}{GI_T} \int_0^{\tau_1} T_T(\alpha, \omega) \times \bar{T}(\alpha, \rho) dL = \\
 &= \frac{AT}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_2(\alpha, \alpha, 0) + \\
 &\quad + \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_4(\rho, \rho, 0) + \\
 &\quad + \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_4(\omega, \omega, 0) + c(\tau_1, 0)^2 \times u_7(\omega, \omega, \rho) \} \quad (87)
 \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \theta_{TT}(\rho, \omega) = & \frac{1}{\sqrt{2} GI_T} \times \frac{1}{c(\tau_1, 0)^2} \times \{ \sin(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_2(a, a, 0) + \\ & + \sin(\tau_1 - \rho) \times c(\tau_1, 0) \times u_4(\omega, \omega, 0) + \\ & + \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_4(\rho, \rho, 0) + c(\tau_1, 0)^2 \times u_7(\rho, \omega, \rho) \} \end{aligned} \quad (88)$$

6. クロソイド曲線一次不静定単純桁の理論

6-1 不静定力の影響線

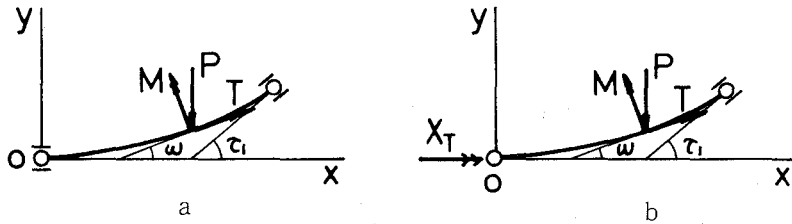


図-16

いま、図-16, a に示すクロソイド曲線一次不静定桁の解析を行なう。まず、図16, b のように始点におけるねじり抵抗を解放したもの、つまり、5の静定桁を基本系にえらび、解放点に不静定力としてねじりモーメント $X_T = 1$ を挿入する。次に、外力として $P = 1, M = 1, T = 1$ が同時に作用しながら動く場合を考えると、次の仕事方程式をうる。

$$\delta_{11} X_T = -(P \times f_{10} + M \times \beta_{10} + T \times \theta_{10}) \quad (89)$$

ここで、 δ_{11} は基本系に $X_T = 1$ が作用している状態において $X_T = 1$ がなす仕事である。また、 $f_{10}, \beta_{10}, \theta_{10}$ は状態 $X_T = 1$ における基本系の垂直変位図、たわみ角図、ねじり角図である。

5-4 を参考にして δ_{11} は以下のようになる。

$$\begin{aligned} \delta_{11} = & (+1) \times (\theta_{MT}(a, 0) + \theta_{TT}(a, 0)) = \\ = & \frac{EI}{\sqrt{2}} \times \frac{1}{c(\tau_1, 0)^2} \times Z \end{aligned} \quad (90)$$

ここで、

$$\left(\begin{aligned} Z = & c(\tau_1, 0)^2 \times u_9(a, a, 0) - 2 \sin \tau_1 \times c(\tau_1, 0) \times u_5(a, a, 0) + \sin^2 \tau_1 \times u_1(a, a, 0) + \\ & + k \times (c(\tau_1, 0)^2 \times u_7(a, a, 0) + 2 \sin \tau_1 \times c(\tau_1, 0) \times u_4(a, a, 0) + \sin^2 \tau_1 \times u_2(a, a, 0)) \\ k = & EI/GI_T \end{aligned} \right)$$

(1) 垂直力 $P=1$ が作用する場合の不静定力影響線

いま、外力として $P=1$ のみが作用する場合の不静定力影響線は、式 (89) の右辺において $P=1$, $M=T=0$ とし、式 (62), (64) を代入して次式のように誘導される。

$$\begin{aligned}
 X_{PT}(\omega) &= \frac{-f_{10}}{\delta_{11}} = \frac{-(f_{MT}(\omega, o) + f_{TT}(\omega, o))}{\delta_{11}} = \\
 &= -\frac{A}{\sqrt{2}Z} \times [c(\tau_1, \omega) \times (\sin \tau_1 \times u_1(o, o, o) - c(\tau_1, o) \times u_5(o, o, \omega)) - \\
 &\quad - c(\tau_1, o) \times (\sin \tau_1 \times u_1(\omega, \omega, o) - c(\tau_1, o) \times u_5(\omega, o, \omega)) + \\
 &\quad + k \times \{c(\tau_1, \omega) \times (\sin \tau_1 \times u_2(o, o, o) + c(\tau_1, o) \times u_4(o, o, o)) - \\
 &\quad - c(\tau_1, o) \times (\sin \tau_1 \times u_2(\omega, \omega, o) + c(\tau_1, o) \times u_4(\omega, o, \omega))\}] = \\
 &= -\frac{A}{\sqrt{2}Z} \times Z_P(\omega) \tag{91}
 \end{aligned}$$

(2) 曲げモーメント $M=1$ が作用する場合の不静定力影響線

(1) と同様に、式 (72), (76) を参考にして次式が誘導される。

$$\begin{aligned}
 X_{MT}(\omega) &= \frac{-\beta_{10}}{\delta_{11}} = \frac{-(\beta_{MT}(\omega, o) + \beta_{TT}(\omega, o))}{\delta_{11}} = \\
 &= \frac{1}{Z} \times [\cos(\tau_1 - \omega) \times (\sin \tau_1 \times u_1(o, o, o) - c(\tau_1, o) \times u_5(o, o, o)) - \\
 &\quad - c(\tau_1, o) \times (\sin \tau_1 \times u_3(\omega, \omega, o) - c(\tau_1, o) \times u_8(\omega, \omega, o)) + \\
 &\quad + k \times \{\cos(\tau_1 - \omega) \times (\sin \tau_1 \times u_2(o, o, o) + c(\tau_1, o) \times u_4(o, o, o)) - \\
 &\quad - c(\tau_1, o) \times (\sin \tau_1 \times u_6(\omega, \omega, o) + c(\tau_1, o) \times u_8(\omega, o, \omega))\}] = \\
 &= \frac{1}{Z} \times Z_M(\omega) \tag{92}
 \end{aligned}$$

(3) ねじりモーメント $T=1$ が作用する場合の不静定力影響線

(1) と同様に、式 (86), (88) を参考にして次式が誘導される。

$$\begin{aligned}
 X_{TT}(\omega) &= \frac{-\theta_{10}}{\delta_{11}} = \frac{-(\theta_{MT}(\omega, o) + \theta_{TT}(\omega, o))}{\delta_{11}} = \\
 &= -\frac{1}{Z} \times [\sin(\tau_1 - \omega) \times (\sin \tau_1 \times u_1(o, o, o) - c(\tau_1, o) \times u_5(o, o, o)) - \\
 &\quad - c(\tau_1, o) \times (\sin \tau_1 \times u_5(\omega, \omega, o) - c(\tau_1, o) \times u_9(\omega, \omega, o)) + \\
 &\quad + k \times \{\sin(\tau_1 - \omega) \times (\sin \tau_1 \times u_2(o, o, o) + c(\tau_1, o) \times u_4(o, o, o)) + \\
 &\quad + c(\tau_1, o) \times (\sin \tau_1 \times u_4(\omega, \omega, o) + c(\tau_1, o) \times u_7(\omega, \omega, o))\}] = \\
 &= -\frac{1}{Z} \times Z_T(\omega) \tag{93}
 \end{aligned}$$

6-2 外力 (ξ) が作用した場合の断面力, 変形の計算

一次不静定桁の場合, 外力 (ξ) が作用した場合の断面力あるいは変形は, 一般的に次式で計算できる。

$$S_{\xi}(\rho, \omega) = S_{\xi}^{\circ}(\rho, \omega) + S_T(\rho) \times X_{\xi T}(\omega) \tag{94}$$

ここで,

$S_{\xi}^{\circ}(\rho, \omega)$: 基本系において, $\xi = 1$ が作用する場合の点 ρ の断面力あるいは変形。

$S_T(\rho)$: 基本系において, 状態 $X_T = 1$ における点 ρ の断面力あるいは変形。

式 (94) はすべてに共通しているので, 以下には結果のみを示す。

6-3 集中荷重 P, M, T による断面力

(1) 垂直力 P が作用する場合の断面力

(a) 反力

$$V_P^{\circ}(\omega) = \frac{P}{c(\tau_1, 0) \times Z} \times (c(\tau_1, \omega) \times Z - \sin \tau_1 \times Z_P(\omega)) \tag{95}$$

$$V_P(\omega) = \frac{P}{c(\tau_1, 0) \times Z} \times \{(c(\tau_1, 0) - c(\tau_1, \omega)) \times Z + \sin \tau_1 \times Z_P(\omega)\} \tag{96}$$

$$T_P^{\circ}(\omega) = -\frac{A \times P \times Z_P(\omega)}{\sqrt{2} Z} \tag{97}$$

$$T_P(\omega) = \frac{AP}{\sqrt{2} c(\tau_1, 0) \times Z} \times \{(s(\tau_1, \omega) \times c(\tau_1, 0) - c(\tau_1, \omega) \times s(\tau_1, 0)) \times Z + (\sin \tau_1 \times s(\tau_1, 0) + \cos \tau_1 \times c(\tau_1, 0)) \times Z_P(\omega)\} \tag{98}$$

(b) 曲げモーメント

(i) $0 \leq \rho \leq \omega$ の場合

$$M_P(\rho, \omega) = -\frac{A}{\sqrt{2}} \times \frac{P}{c(\tau_1, 0) \times Z} \times \{c(\tau_1, \omega) \times c(\rho, 0) \times Z - (\sin \tau_1 \times c(\rho, 0) - \sin \rho \times c(\tau_1, 0)) \times Z_P(\omega)\} \tag{99}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$M_P(\rho, \omega) = -\frac{A}{\sqrt{2}} \times \frac{P}{c(\tau_1, 0) \times Z} \times \{(c(\tau_1, \omega) \times c(\rho, 0) - c(\rho, \omega) \times c(\tau_1, 0)) \times Z - (\sin \tau_1 \times c(\rho, 0) - \sin \rho \times c(\tau_1, 0)) \times Z_P(\omega)\} \tag{100}$$

(c) ねじりモーメント

(i) $0 \leq \rho \leq \omega$ の場合

$$T_P(\rho, \omega) = -\frac{A}{\sqrt{2}} \times \frac{P}{c(\tau_1, o) \times Z} \times \{c(\tau_1, \omega) \times s(\rho, o) \times Z - (\sin \tau_1 \times s(\rho, o) + \cos \rho \times c(\tau_1, o)) \times Z_P(\omega)\} \quad (101)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$T_P(\rho, \omega) = -\frac{A}{\sqrt{2}} \times \frac{P}{c(\tau_1, o) \times Z} \times \{(c(\tau_1, \omega) \times s(\rho, o) - S(\rho, \omega) \times c(\tau_1, o)) \times Z - (\sin \tau_1 \times s(\rho, o) + \cos \rho \times c(\tau_1, o)) \times Z_P(\omega)\} \quad (102)$$

(2) 曲げモーメント M が作用する場合の断面力

(a) 反力

$$V_M^o(\omega) = -\frac{\sqrt{2}}{A} \times \frac{M}{c(\tau_1, o) \times Z} \times \cos(\tau_1 - \omega) \times Z - \sin \tau_1 \times Z_M(\omega) \quad (103)$$

$$V_M(\omega) = \frac{\sqrt{2}}{A} \times \frac{M}{c(\tau_1, o) \times Z} \times (\cos(\tau_1 - \omega) \times Z - \sin \tau_1 \times Z_M(\omega)) \quad (104)$$

$$T_M^o(\omega) = \frac{M}{Z} \times Z_M(\omega) \quad (105)$$

$$T_M(\omega) = \frac{M}{c(\tau_1, o) \times Z} \times \{(\cos(\tau_1 - \omega) \times s(\tau_1, o) - \sin(\tau_1 - \omega) \times c(\tau_1, o)) \times Z - (\sin \tau_1 \times s(\tau_1, o) + \cos \tau_1 \times c(\tau_1, o)) \times Z_M(\omega)\} \quad (106)$$

(b) 曲げモーメント

(i) $0 \leq \rho \leq \omega$ の場合

$$M_M(\rho, \omega) = \frac{M}{c(\tau_1, o) \times Z} \times \{\cos(\tau_1 - \omega) \times c(\rho, o) \times Z - (\sin \tau_1 \times c(\rho, o) - \sin \rho \times c(\tau_1, o)) \times Z_M(\omega)\} \quad (107)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$M_M(\rho, \omega) = \frac{M}{c(\tau_1, o) \times Z} \times \{(\cos(\tau_1 - \omega) \times c(\rho, o) - \cos(\rho - \omega) \times c(\tau_1, o)) \times Z - (\sin \tau_1 \times c(\rho, o) - \sin \rho \times c(\tau_1, o)) \times Z_M(\omega)\} \quad (108)$$

(c) ねじりモーメント

(i) $0 \leq \rho \leq \omega$ の場合

$$T_M(\rho, \omega) = \frac{M}{c(\tau_1, o) \times Z} \times \{ \cos(\tau_1 - \omega) \times s(\rho, o) \times Z - (\sin \tau_1 \times s(\rho, o) + \cos \rho \times c(\tau_1, o)) \times Z_M(\omega) \} \quad (109)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$T_M(\rho, \omega) = \frac{M}{c(\tau_1, o) \times Z} \times \{ (\cos(\tau_1 - \omega) \times s(\rho, o) - \sin(\rho - \omega) \times c(\tau_1, o)) \times Z - (\sin \tau_1 \times s(\rho, o) + \cos \rho \times c(\tau_1, o)) \times Z_M(\omega) \} \quad (110)$$

(3) ねじりモーメント T が作用する場合の断面力

(a) 反力

$$V_T^o(\omega) = \frac{\sqrt{2}}{A} \times \frac{T}{c(\tau_1, o) \times Z} \times (\sin(\tau_1 - \omega) \times Z - \sin \tau_1 \times Z_T(\omega)) \quad (111)$$

$$V_T(\omega) = -\frac{\sqrt{2}}{A} \times \frac{T}{c(\tau_1, o) \times Z} \times (\sin(\tau_1 - \omega) \times Z - \sin \tau_1 \times Z_T(\omega)) \quad (112)$$

$$T_T^o(\omega) = -\frac{T}{Z} \times Z_T(\omega) \quad (113)$$

$$T_T(\omega) = -\frac{T}{c(\tau_1, o) \times Z} \times \{ (\sin(\tau_1 - \omega) \times s(\tau_1, o) + \cos(\tau_1 - \omega) \times c(\tau_1, o)) \times Z - (\sin \tau_1 \times s(\tau_1, o) + \cos \tau_1 \times c(\tau_1, o)) \times Z_T(\omega) \} \quad (114)$$

(b) 曲げモーメント

(i) $0 \leq \rho \leq \omega$ の場合

$$M_T(\rho, \omega) = -\frac{T}{c(\tau_1, o) \times Z} \times \{ \sin(\tau_1 - \omega) \times c(\rho, o) \times Z - (\sin \tau_1 \times c(\rho, o) - \sin \rho \times c(\tau_1, o)) \times Z_T(\omega) \} \quad (115)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$M_T(\rho, \omega) = -\frac{T}{c(\tau_1, o) \times Z} \times \{ (\sin(\tau_1 - \omega) \times c(\rho, o) - \sin(\rho - \omega) \times c(\tau_1, o)) \times Z - (\sin \tau_1 \times c(\rho, o) - \sin \rho \times c(\tau_1, o)) \times Z_T(\omega) \} \quad (116)$$

(c) ねじりモーメント

(i) $0 \leq \rho \leq \omega$ の場合

$$T_T(\rho, \omega) = -\frac{T}{c(\tau_1, o) \times Z} \times \{ \sin(\tau_1 - \omega) \times s(\rho, o) \times Z - (\sin \tau_1 \times s(\rho, o) + \cos \rho \times c(\tau_1, o)) \times Z_T(\omega) \} \quad (117)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$T_T(\rho, \omega) = -\frac{T}{c(\tau_1, o) \times Z} \times \{ (\sin(\tau_1 - \omega) \times s(\rho, o) + \cos(\rho - \omega) \times c(\tau_1, o)) \times Z - (\sin \tau_1 \times s(\rho, o) + \cos \rho \times c(\tau_1, o)) \times Z_T(\omega) \} \quad (118)$$

6-4 集中荷重 P , M , T による垂直変位

(1) 垂直力 P が作用する場合の垂直変位

(a) $f_{MP}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$f_{MP}(\rho, \omega) = \frac{A^3 P}{2\sqrt{2} EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (c(\tau_1, \omega) \times c(\tau_1, \rho) \times u_1(o, o, o) - c(\tau_1, \omega) \times c(\tau_1, o) \times u_1(\rho, \rho, o) - c(\tau_1, o) \times c(\tau_1, \rho) \times u_1(\omega, \omega, o) + c(\tau_1, o)^2 \times u_1(\omega, \omega, \rho)) \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_1(o, o, o) - c(\tau_1, \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_1(\rho, \rho, o) + c(\tau_1, o)^2 \times u_5(\rho, o, \rho)) \times Z_P(\omega) \} \quad (119)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$f_{MP}(\rho, \omega) = \frac{A^3 P}{2\sqrt{2} EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (c(\tau_1, \omega) \times c(\tau_1, \rho) \times u_1(o, o, o) - c(\tau_1, \rho) \times c(\tau_1, o) \times u_1(\omega, \omega, o) - c(\tau_1, \omega) \times c(\tau_1, o) \times u_1(\rho, \rho, o) + c(\tau_1, o)^2 \times u_1(\rho, \rho, \omega)) \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_1(o, o, o) - c(\tau_1, \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_1(\rho, \rho, o) + c(\tau_1, o)^2 \times u_5(\rho, o, \rho)) \times Z_P(\omega) \} \quad (120)$$

(b) $f_{TP}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$f_{TP}(\rho, \omega) = \frac{A^3 P}{2\sqrt{2} GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (c(\tau_1, \omega) \times c(\tau_1, \rho) \times u_2(o, o, o) - c(\tau_1, \omega) \times c(\tau_1, o) \times u_2(\rho, \rho, o) - c(\tau_1, o) \times c(\tau_1, \rho) \times u_2(\omega, \omega, o) + c(\tau_1, o)^2 \times u_2(\omega, \omega, \rho)) \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_2(o, o, o) + c(\tau_1, \rho) \times c(\tau_1, o) \times u_4(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_2(\rho, \rho, o) -$$

$$-c(\tau_1, o)^2 \times u_4(\rho, o, \rho) \times Z_P(\omega) \} \quad (121)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} f_{TP}(\rho, \omega) = & \frac{A^3 P}{2\sqrt{2} GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (c(\tau_1, \omega) \times c(\tau_1, \rho) \times u_2(o, o, o) - \\ & - c(\tau_1, \rho) \times c(\tau_1, o) \times u_2(\omega, \omega, o) - c(\tau_1, \omega) \times c(\tau_1, o) \times u_2(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_2(\rho, \rho, \omega)) \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_2(o, o, o) + \\ & + c(\tau_1, \rho) \times c(\tau_1, o) \times u_4(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_2(\rho, \rho, o) - \\ & - c(\tau_1, o)^2 \times u_4(\rho, o, \rho)) \times Z_P(\omega) \} \quad (122) \end{aligned}$$

(2) 曲げモーメント M が作用する場合の垂直変位

(a) $f_{MM}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} f_{MM}(\rho, \omega) = & -\frac{A^2 M}{2EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\cos(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_1(o, o, o) - \\ & - \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_1(\rho, o, \rho) - c(\tau_1, o) \times c(\tau_1, \rho) \times u_3(\omega, \omega, o) + \\ & + c(\tau_1, o)^2 \times u_3(\omega, \omega, \rho)) \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_1(o, o, o) - \\ & - c(\tau_1, \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_1(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_5(\rho, o, \rho)) \times Z_M(\omega) \} \quad (123) \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} f_{MM}(\rho, \omega) = & -\frac{A^2 M}{2EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\cos(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_1(o, o, o) - \\ & - c(\tau_1, \rho) \times c(\tau_1, o) \times u_3(\omega, \omega, o) - \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_1(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_3(\rho, \omega, \rho)) \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_1(o, o, o) - \\ & - c(\tau_1, \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_1(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_5(\rho, o, \rho)) \times Z_M(\omega) \} \quad (124) \end{aligned}$$

(b) $f_{TM}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} f_{TM}(\rho, \omega) = & -\frac{A^2 M}{2GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\cos(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_2(o, o, o) - \\ & - \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_2(\rho, \rho, o) - c(\tau_1, o) \times c(\tau_1, \rho) \times u_6(\omega, \omega, o) + \\ & + c(\tau_1, o)^2 \times u_6(\omega, \omega, \rho)) \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_2(o, o, o) + \\ & + c(\tau_1, \rho) \times c(\tau_1, o) \times u_4(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_2(\rho, \rho, o) - \end{aligned}$$

$$-c(\tau_1, o)^2 \times u_4(\rho, o, \rho) \times Z_M(\omega) \} \quad (125)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} f_{TM}(\rho, \omega) = & -\frac{A^2 M}{2GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\cos(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_2(o, o, o) - \\ & - c(\tau_1, \rho) \times c(\tau_1, o) \times u_6(\omega, \omega, o) - \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_2(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_6(\rho, \omega, \rho) \} \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_2(o, o, o) + \\ & + c(\tau_1, \rho) \times c(\tau_1, o) \times u_4(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_2(\rho, \rho, o) - \\ & - c(\tau_1, o)^2 \times u_4(\rho, o, \rho) \} \times Z_M(\omega) \} \quad (126) \end{aligned}$$

(3) ねじりモーメント T が作用する場合の垂直変位

(a) $f_{MT}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} f_{MT}(\rho, \omega) = & \frac{A^2 T}{2EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\sin(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_1(o, o, o) - \\ & - \sin(\tau_1 - \omega) \times c(\tau_1, o) \times u_1(\rho, \rho, o) - c(\tau_1, o) \times c(\tau_1, \rho) \times u_5(\omega, \omega, o) + \\ & + c(\tau_1, o)^2 \times u_5(\omega, \omega, \rho) \} \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_1(o, o, o) - \\ & - c(\tau_1, \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_1(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_5(\rho, o, \rho) \} \times Z_T(\omega) \} \quad (127) \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} f_{MT}(\rho, \omega) = & \frac{A^2 T}{2EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\sin(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_1(o, o, o) - \\ & - c(\tau_1, \rho) \times c(\tau_1, o) \times u_5(\omega, \omega, o) - \sin(\tau_1 - \omega) \times c(\tau_1, o) \times u_1(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_5(\rho, \omega, \rho) \} \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_1(o, o, o) - \\ & - c(\tau_1, \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_1(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_5(\rho, o, \rho) \} \times Z_T(\omega) \} \quad (128) \end{aligned}$$

(b) $f_{TT}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} f_{TT}(\rho, \omega) = & \frac{A^2 T}{2GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\sin(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_2(o, o, o) - \\ & - \sin(\tau_1 - \omega) \times c(\tau_1, o) \times u_2(\rho, \rho, o) + c(\tau_1, o) \times c(\tau_1, \rho) \times u_4(\omega, \omega, o) - \\ & - c(\tau_1, o)^2 \times u_4(\omega, \omega, \rho) \} \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_2(o, o, o) + \\ & + c(\tau_1, \rho) \times c(\tau_1, o) \times u_4(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_2(\rho, \rho, o) - \\ & - c(\tau_1, o)^2 \times u_4(\rho, o, \rho) \} \times Z_T(\omega) \} \quad (129) \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
 f_{TT}(\rho, \omega) = & \frac{A^2 T}{2GI_T} \times \frac{1}{c(\tau_1, 0)^2 \times Z} \times \{(\sin(\tau_1 - \omega) \times c(\tau_1, \rho) \times u_2(0, 0, 0) + \\
 & + c(\tau_1, \rho) \times c(\tau_1, 0) \times u_4(\omega, \omega, 0) - \sin(\tau_1 - \omega) \times c(\tau_1, 0) \times u_2(\rho, \rho, 0) - \\
 & - c(\tau_1, 0)^2 \times u_4(\rho, \omega, \rho)) \times Z - (\sin \tau_1 \times c(\tau_1, \rho) \times u_2(0, 0, 0) + \\
 & + c(\tau_1, \rho) \times c(\tau_1, 0) \times u_4(0, 0, 0) - \sin \tau_1 \times c(\tau_1, 0) \times u_2(\rho, \rho, 0) - \\
 & - c(\tau_1, 0)^2 \times u_4(\rho, 0, \rho)) \times Z_T(\omega)\} \tag{130}
 \end{aligned}$$

6-5 集中荷重 P, M, T によるたわみ角

(1) 垂直力 P が作用する場合のたわみ角

(a) $\beta_{MP}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
 \beta_{MP}(\rho, \omega) = & -\frac{A^2 P}{2EI} \times \frac{1}{c(\tau_1, 0)^2 \times Z} \times \{(\cos(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_1(0, 0, 0) - \\
 & - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_3(\rho, \rho, 0) - \cos(\tau_1 - \xi) \times c(\tau_1, 0) \times u_1(\omega, \omega, 0) + \\
 & + c(\tau_1, 0)^2 \times u_3(\omega, \rho, \omega)) \times Z - (\sin \tau_1 \times \cos(\tau_1 - 0) \times u_1(0, 0, 0) - \\
 & - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_5(0, 0, 0) - \sin \tau_1 \times c(\tau_1, 0) \times u_3(\rho, \rho, 0) + \\
 & + c(\tau_1, 0)^2 \times u_8(\rho, \rho, 0)) \times Z_P(\omega)\} \tag{131}
 \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned}
 \beta_{MP}(\rho, \omega) = & -\frac{A^2 P}{2EI} \times \frac{1}{c(\tau_1, 0)^2 \times Z} \times \{(\cos(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_1(0, 0, 0) - \\
 & - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_1(\omega, \omega, 0) - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_3(\rho, \rho, 0) + \\
 & + c(\tau_1, 0)^2 \times u_3(\rho, \rho, \omega)) \times Z - (\sin \tau_1 \times \cos(\tau_1 - \rho) \times u_1(0, 0, 0) - \\
 & - \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_5(0, 0, 0) - \sin \tau_1 \times c(\tau_1, 0) \times u_3(\rho, \rho, 0) + \\
 & + c(\tau_1, 0)^2 \times u_8(\rho, \rho, 0)) \times Z_P(\omega)\} \tag{132}
 \end{aligned}$$

(b) $\beta_{TP}(\rho, \omega)$

(i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned}
 \beta_{TP}(\rho, \omega) = & -\frac{A^2 P}{2GI_T} \times \frac{1}{c(\tau_1, 0)^2 \times Z} \times \{(\cos(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_2(0, 0, 0) - \\
 & - c(\tau_1, \omega) \times c(\tau_1, 0) \times u_6(\rho, \rho, 0) - \cos(\tau_1 - \xi) \times c(\tau_1, 0) \times u_2(\omega, \omega, 0) + \\
 & + c(\tau_1, 0)^2 \times u_6(\omega, \rho, \omega)) \times Z - (\sin \tau_1 \times \cos(\tau_1 - \rho) \times u_2(0, 0, 0) + \\
 & + \cos(\tau_1 - \rho) \times c(\tau_1, 0) \times u_4(0, 0, 0) - \sin \tau_1 \times c(\tau_1, 0) \times u_6(\rho, \rho, 0) - \\
 & - c(\tau_1, 0)^2 \times u_8(\rho, 0, \rho)) \times Z_P(\omega)\} \tag{133}
 \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{TP}(\rho, \omega) = & -\frac{A^2 P}{2GI_T} \times \frac{1}{c(\tau_1, \rho)^2 \times Z} \times \{(\cos(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_2(o, o, o) - \\ & - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_2(\omega, \omega, o) - c(\tau_1, \omega) \times c(\tau_1, o) \times u_6(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_6(\rho, \rho, \omega)) \times Z - (\sin \tau_1 \times \cos(\tau_1 - \rho) \times u_2(o, o, o) + \\ & + \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_6(\rho, \rho, o) - \\ & - c(\tau_1, o)^2 \times u_8(\rho, o, \rho)) \times Z_P(\omega)\} \end{aligned} \quad (134)$$

(2) 曲げモーメント M が作用する場合のたわみ角(a) $\beta_{MM}(\rho, \omega)$ (i) $o \leq \rho \leq \omega$ の場合

$$\begin{aligned} \beta_{MM}(\rho, \omega) = & \frac{AM}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\cos(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_3(\rho, \rho, o) - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_3(\omega, \omega, o) + \\ & + c(\tau_1, o)^2 \times u_7(\omega, \omega, \rho)) \times Z - \sin \tau_1 \times \cos(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_3(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_8(\rho, \rho, o)) \times Z_M(\omega)\} \end{aligned} \quad (135)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{MM}(\rho, \omega) = & \frac{AM}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\cos(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_3(\omega, \omega, o) - \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_3(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_7(\rho, \omega, \rho)) \times Z - (\sin \tau_1 \times \cos(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_3(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_8(\rho, \rho, o)) \times Z_M(\omega)\} \end{aligned} \quad (136)$$

(b) $\beta_{TM}(\rho, \omega)$ (i) $o \leq \rho \leq \omega$ の場合

$$\begin{aligned} \beta_{TM}(\rho, \omega) = & \frac{AM}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\cos(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_2(o, o, o) - \\ & - \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_6(\rho, \rho, o) - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_6(\omega, \omega, o) + \\ & + c(\tau_1, o)^2 \times u_9(\omega, \omega, \rho)) \times Z - (\sin \tau_1 \times \cos(\tau_1 - \rho) \times u_2(o, o, o) + \\ & + \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_6(\rho, \rho, o) - \\ & - c(\tau_1, o)^2 \times u_8(\rho, o, \rho)) \times Z_M(\omega)\} \end{aligned} \quad (137)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{TM}(\rho, \omega) = & \frac{AM}{\sqrt{2} GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\cos(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_2(o, o, o) - \\ & - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_6(\omega, \omega, o) - \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_6(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_9(\rho, \omega, \rho)) \times Z - (\sin \tau_1 \times \cos(\tau_1 - \zeta) \times u_2(o, o, o) + \\ & + \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_6(\rho, \rho, o) - \\ & - c(\tau_1, o)^2 \times u_8(\rho, o, \rho)) \times Z_M(\omega) \} \end{aligned} \quad (138)$$

(3) ねじりモーメント T が作用する場合のたわみ角

(a) $\beta_{MT}(\rho, \omega)$

(i) $o \leq \rho \leq \omega$ の場合

$$\begin{aligned} \beta_{MT}(\rho, \omega) = & -\frac{AT}{\sqrt{2} EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\sin(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \sin(\tau_1 - \omega) \times c(\tau_1, o) \times u_3(\rho, \rho, o) - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(\omega, \omega, o) + \\ & + c(\tau_1, o)^2 \times u_8(\omega, \rho, \omega)) \times Z - (\sin \tau_1 \times \cos(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \sin \tau_1 \times c(\tau_1, o) \times u_3(\rho, \rho, o) + \\ & + c(\tau_1, o)^2 \times u_8(\rho, \rho, o)) \times Z_T(\omega) \} \end{aligned} \quad (139)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{MT}(\rho, \omega) = & -\frac{AT}{\sqrt{2} EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\sin(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(\omega, \omega, o) - \\ & - \sin(\tau_1 - \omega) \times c(\tau_1, o) \times u_3(\rho, \rho, o) + c(\tau_1, o)^2 \times u_8(\rho, \rho, \omega)) \times Z - \\ & - (\sin \tau_1 \times \cos(\tau_1 - \rho) \times u_1(o, o, o) - \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \\ & - \sin \tau_1 \times c(\tau_1, o) \times u_3(\rho, \rho, o) + c(\tau_1, o)^2 \times u_8(\rho, \rho, o)) \times Z_T(\omega) \} \end{aligned} \quad (140)$$

(b) $\beta_{TT}(\rho, \omega)$

(i) $o \leq \rho \leq \omega$ の場合

$$\begin{aligned} \beta_{TT}(\rho, \omega) = & -\frac{AT}{\sqrt{2} GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\sin(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_2(o, o, o) - \\ & - \sin(\tau_1 - \omega) \times c(\tau_1, o) \times u_6(\rho, \rho, o) + \\ & + \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(\omega, \omega, o) - c(\tau_1, o)^2 \times u_8(\omega, \omega, \rho)) \times Z - \\ & - (\sin \tau_1 \times \cos(\tau_1 - \rho) \times u_2(o, o, o) + \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) - \\ & - \sin \tau_1 \times c(\tau_1, o) \times u_6(\rho, \rho, o) - c(\tau_1, o)^2 \times u_8(\zeta, o, \rho)) \times Z_T(\omega) \} \end{aligned} \quad (141)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \beta_{TT}(\rho, \omega) = & -\frac{AT}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\sin(\tau_1 - \omega) \times \cos(\tau_1 - \rho) \times u_2(o, o, o) + \\ & + \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(\omega, \omega, o) - \\ & - \sin(\tau_1 - \omega) \times c(\tau_1, o) \times u_6(\rho, \rho, o) - c(\tau_1, o)^2 \times u_8(\rho, \omega, \rho)\} \times Z - \\ & - (\sin \tau_1 \times \cos(\tau_1 - \rho) \times u_2(o, o, o) + \cos(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) - \\ & - \sin \tau_1 \times c(\tau_1, o) \times u_6(\rho, \rho, o) - c(\tau_1, o)^2 \times u_8(\rho, o, \rho)\} \times Z_T(\omega) \} \quad (142) \end{aligned}$$

6-6 集中荷重 P , M , T によるねじり角(1) 垂直力 P が作用する場合のねじり角(a) $\theta_{MP}(\rho, \omega)$ (i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \theta_{MP}(\rho, \omega) = & \frac{A^2P}{2EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\sin(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_1(o, o, o) - \\ & - c(\tau_1, \omega) \times c(\tau_1, o) \times u_5(\rho, \rho, o) - \\ & - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_1(\omega, \omega, o) + c(\tau_1, o)^2 \times u_5(\omega, \rho, \omega)\} \times Z - \\ & - (\sin \tau_1 \times \sin(\tau_1 - \rho) \times u_1(o, o, o) - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \\ & - \sin \tau_1 \times c(\tau_1, o) \times u_5(\rho, \rho, o) + c(\tau_1, o)^2 \times u_5(\rho, o, \rho)\} \times Z_P(\omega) \} \quad (143) \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \theta_{MP}(\rho, \omega) = & \frac{A^2P}{2EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\sin(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_1(o, o, o) - \\ & - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_1(\omega, \omega, o) - \\ & - c(\tau_1, \omega) \times c(\tau_1, o) \times u_5(\rho, \rho, o) + c(\tau_1, o)^2 \times u_5(\rho, \rho, \omega)\} \times Z - \\ & - (\sin \tau_1 \times \sin(\tau_1 - \rho) \times u_1(o, o, o) - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \\ & - \sin \tau_1 \times c(\tau_1, o) \times u_5(\rho, \rho, o) + c(\tau_1, o)^2 \times u_5(\xi, o, \xi)\} \times Z_P(\omega) \} \quad (144) \end{aligned}$$

(b) $\theta_{TP}(\rho, \omega)$ (i) $0 \leq \rho \leq \omega$ の場合

$$\begin{aligned} \theta_{TP}(\rho, \omega) = & \frac{A^2P}{2GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\sin(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_2(o, o, o) + \\ & + c(\tau_1, \omega) \times c(\tau_1, o) \times u_4(\rho, \rho, o) - \\ & - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_2(\omega, \omega, o) - c(\tau_1, o)^2 \times u_4(\omega, \rho, \omega)\} \times Z - \\ & - (\sin \tau_1 \times \sin(\tau_1 - \rho) \times u_2(o, o, o) + \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) + \end{aligned}$$

$$+ \sin \tau_1 \times c(\tau_1, o) \times u_4(\rho, \rho, o) + c(\tau_1, o)^2 \times u_7(\rho, o, \rho) \times Z_P(\omega) \} \quad (145)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \theta_{TP}(\rho, \omega) = & \frac{A^2 P}{2GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\sin(\tau_1 - \rho) \times c(\tau_1, \omega) \times u_2(o, o, o) - \\ & - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_2(\omega, \omega, o) + \\ & + c(\tau_1, \omega) \times c(\tau_1, o) \times u_4(\rho, \rho, o) - c(\tau_1, o)^2 \times u_4(\rho, \rho, \omega) \} \times Z - \\ & - (\sin \tau_1 \times \sin(\tau_1 - \rho) \times u_2(o, o, o) + \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) + \\ & + \sin \tau_1 \times c(\tau_1, o) \times u_4(\rho, \rho, o) + c(\tau_1, o)^2 \times u_7(\rho, o, \rho) \} \times Z_P(\omega) \} \quad (146) \end{aligned}$$

(2) 曲げモーメント M が作用する場合のねじり角

(a) $\theta_{MM}(\rho, \omega)$

(i) $o \leq \rho \leq \omega$ の場合

$$\begin{aligned} \theta_{MM}(\rho, \omega) = & -\frac{AM}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\cos(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_5(\rho, \rho, o) - \\ & - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_3(\omega, \omega, o) + c(\tau_1, o)^2 \times u_8(\omega, \omega, \rho) \} \times Z - \\ & - (\sin \tau_1 \times \sin(\tau_1 - \rho) \times u_1(o, o, o) - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \\ & - \sin \tau_1 \times c(\tau_1, o) \times u_5(\rho, \rho, o) + c(\tau_1, o)^2 \times u_9(\rho, o, \rho) \} \times Z_M(\omega) \} \quad (147) \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \theta_{MM}(\rho, \omega) = & -\frac{AM}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\cos(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_3(\omega, \omega, o) - \\ & - \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_5(\rho, \rho, o) + c(\tau_1, o)^2 \times u_8(\rho, \omega, \rho) \} \times Z - \\ & - (\sin \tau_1 \times \sin(\tau_1 - \rho) \times u_1(o, o, o) - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \\ & - \sin \tau_1 \times c(\tau_1, o) \times u_5(\rho, \rho, o) + c(\tau_1, o)^2 \times u_9(\rho, o, \rho) \} \times Z_M(\omega) \} \quad (148) \end{aligned}$$

(b) $\theta_{TM}(\rho, \omega)$

(i) $o \leq \rho \leq \omega$ の場合

$$\begin{aligned} \theta_{TM}(\rho, \omega) = & -\frac{AM}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{ (\cos(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_2(o, o, o) + \\ & + \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_4(\rho, \rho, o) - \\ & - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_6(\omega, \omega, o) - c(\tau_1, o)^2 \times u_8(\omega, \rho, \omega) \} \times Z - \\ & - (\sin \tau_1 \times \sin(\tau_1 - \rho) \times u_2(o, o, o) + \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) + \end{aligned}$$

$$+\sin \tau_1 \times c(\tau_1, o) \times u_4(\rho, \rho, o) + c(\tau_1, o)^2 \times u_7(\rho, o, \rho) \times Z_M(\omega) \} \quad (149)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \theta_{TM}(\rho, \omega) = & -\frac{AM}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\cos(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_2(o, o, o) - \\ & - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_6(\omega, \omega, o) + \\ & + \cos(\tau_1 - \omega) \times c(\tau_1, o) \times u_4(\rho, \rho, o) - c(\tau_1, o)^2 \times u_8(\rho, \rho, \omega)) \times Z - \\ & - (\sin \tau_1 \times \sin(\tau_1 - \rho) \times u_2(o, o, o) + \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) + \\ & + \sin \tau_1 \times c(\tau_1, o) \times u_4(\rho, \rho, o) + c(\tau_1, o)^2 \times u_7(\rho, o, \rho)) \times Z_M(\omega) \} \quad (150) \end{aligned}$$

(3) ねじりモーメント T が作用する場合のねじり角

(a) $\theta_{TM}(\rho, \omega)$

(i) $o \leq \rho \leq \omega$ の場合

$$\begin{aligned} \theta_{MT}(\rho, \omega) = & \frac{AT}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\sin(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \sin(\tau_1 - \omega) \times c(\tau_1, o) \times u_5(\rho, \rho, o) - \\ & - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(\omega, \omega, o) + c(\tau_1, o)^2 \times u_9(\omega, \omega, \rho)) \times Z - \\ & - (\sin \tau_1 \times \sin(\tau_1 - \rho) \times u_1(o, o, o) - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \\ & - \sin \tau_1 \times c(\tau_1, o) \times u_5(\rho, \rho, o) + c(\tau_1, o)^2 \times u_9(\rho, o, \rho)) \times Z_T(\omega) \} \quad (151) \end{aligned}$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \theta_{MT}(\rho, \omega) = & \frac{AT}{\sqrt{2}EI} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\sin(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_1(o, o, o) - \\ & - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(\omega, \omega, o) - \\ & - \sin(\tau_1 - \omega) \times c(\tau_1, o) \times u_5(\rho, \rho, \rho) + c(\tau_1, o)^2 \times u_9(\rho, \omega, \rho)) \times Z - \\ & - (\sin \tau_1 \times \sin(\tau_1 - \rho) \times u_1(o, o, o) - \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_5(o, o, o) - \\ & - \sin \tau_1 \times c(\tau_1, o) \times u_5(\rho, \rho, o) + c(\tau_1, o)^2 \times u_9(\rho, o, \rho)) \times Z_T(\omega) \} \quad (152) \end{aligned}$$

(b) $\theta_{TT}(\rho, \omega)$

(i) $o \leq \rho \leq \omega$ の場合

$$\begin{aligned} \theta_{TT}(\rho, \omega) = & \frac{AT}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\sin(\tau_1 - \omega) \times \sin(\tau_1 - \rho) \times u_2(o, o, o) + \\ & + \sin(\tau_1 - \omega) \times c(\tau_1, o) \times u_4(\rho, \rho, o) + \\ & + \sin(\tau_1 - \rho) \times c(\tau_1, o) \times u_4(\omega, \omega, o) + c(\tau_1, o)^2 \times u_7(\omega, \omega, \rho)) \times Z - \end{aligned}$$

$$-(\sin \tau_1 \times \sin (\tau_1 - \rho) \times u_2(o, o, o) + \sin (\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) + \sin \tau_1 \times c(\tau_1, o) \times u_4(\rho, \rho, o) + c(\tau_1, o)^2 \times u_7(\rho, o, \rho)) \times Z_T(\omega) \quad (153)$$

(ii) $\omega \leq \rho \leq \tau_1$ の場合

$$\begin{aligned} \theta_{TT}(\rho, \omega) = & \frac{AT}{\sqrt{2}GI_T} \times \frac{1}{c(\tau_1, o)^2 \times Z} \times \{(\sin (\tau_1 - \omega) \times \sin (\tau_1 - \rho) \times u_2(o, o, o) + \\ & + \sin (\tau_1 - \rho) \times c(\tau_1, o) \times u_4(\omega, \omega, o) + \\ & + \sin (\tau_1 - \omega) \times c(\tau_1, o) \times u_4(\rho, \rho, o) + c(\tau_1, o)^2 \times u_7(\rho, \omega, \rho)) \times Z - \\ & - (\sin \tau_1 \times \sin (\tau_1 - \rho) \times u_2(o, o, o) + \sin (\tau_1 - \rho) \times c(\tau_1, o) \times u_4(o, o, o) + \\ & + \sin \tau_1 \times c(\tau_1, o) \times u_4(\rho, \rho, o) + c(\tau_1, o)^2 \times u_7(\rho, o, \rho)) \times Z_T(\omega) \} \quad (154) \end{aligned}$$

7. 計 算 例

以上の理論を用いた計算例を円曲線の結果と比較して以下に示す。なお、円曲線の結果は文献2)を参考にしている。

(1) $\tau_0 = 0.0$, $\tau_1 = 0.1$, $A = 100\text{m}$ (つまり, $L = 44.75\text{m}$, $R_0 = \infty$, $R = 224\text{m}^3$) の一次不静定のクロソイド曲線桁 ($k = 1$) の $w = 0.03$ に垂直荷重 $P = 1^t$ が載荷した場合。

曲げモーメント図, ねじりモーメント図を図-17, 18に示す。図中, circle 1 は中心角 $11^\circ 28'$, 半径 224m , circle 2 は $25^\circ 51'$, 半径 100m の円曲線桁の結果である。両円曲線桁ともクロソイド曲線桁と始点, 端点の座標を一致させている。

図より曲げモーメントは良く一致しているが, ねじりモーメントは差が大きい。

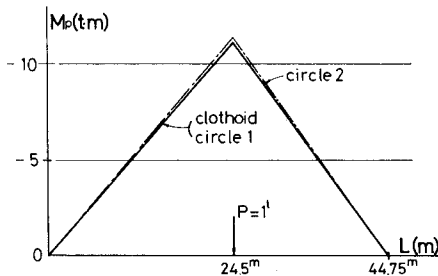


図-17

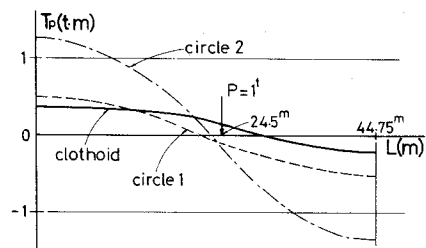


図-18

(2) $\tau_0 = 0.2$, $\tau_1 = 0.4$, $A = 100\text{m}$ (つまり, $L = 46.29\text{m}$, $R_0 = 158\text{m}$, $R = 91\text{m}$) の一次不静定クロソイド曲線桁 ($k = 1$) の $\omega = 0.16$ に垂直荷重 $P = 1^t$ が載荷した場合。

曲げモーメント図, ねじりモーメント図を図-19, 20に示す。図中, circle 1 は, 中心角 $26^\circ 32'$, 半径 100m (つまり $R_{\text{circle}} = A$ としている。), circle 2 は中心角 $21^\circ 10'$, 半径 125m の円曲線桁の結果である。始点, 端点の座標は (1) と同様に一致している。

図より、曲げモーメント、ねじりモーメントとも良く一致していると思われる。

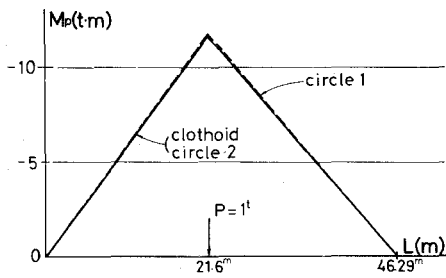


図-19

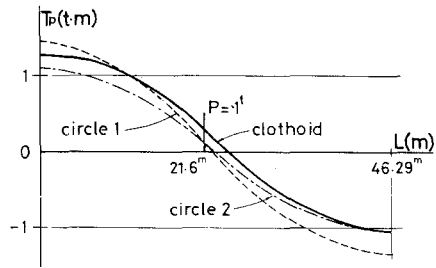


図-20

8. 結 言

静定および一次不静定クロソイド曲線桁の理論式を示し若干の計算例を説明した。紙面の都合で計算例を詳細に説明できなかったが、すでに $\tau_0=0.0\sim 0.7$, $\tau_1=0.1\sim 0.8$ のクロソイド曲線桁の計算は終っており、その一部は文献4), 5) に発表してある。その中で、 $\tau_0=0.0$ 以外のクロソイド曲線桁を円曲線で近似する場合は、

$$R_{\text{circle}} = A$$

なる円曲線で近似すれば良い等の提案を行なっている。

最後に、本論文の研究にあたり終始御指導を賜った北海道大学教授 渡辺 昇博士に深甚の謝意を表する次第であります。

(昭和50年5月8日受理)

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