Super-objectivist Conception and the Rudiments of Mathematics

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Abstract

Beyond the elementary results produced by the finite combination of primitive procedures promised in the original construction of an objectivism, if we try to extend them we must inevitably use the method of abstraction. The concepts which are abstracted but are not yet convincingly accepted, are said to be super-objectivist. Some of these concepts may reasonably be accepted and incorporated with the construction, though sometimes may possibly cause a revision of the system. Some concepts current in the classical lectures may thereafter turn out to be regarded as nonsensical.

O. Introduction

In the late current of developing the mathematical logic, has been raised the metalanguage, which has partly fostered a world of concepts and statements to be left uncertain over the practical realm, apparently concocting a sort of fictive awareness, which might be called *'pseudo-awareness'*. That some authors in this line use the word 'crisis'. seems to suggest their actual feeling of apprehensions for the direct connection with the pseudo-awareness. This paper is intended to clear up these dubious conceptions and to obtain a totally pellucid aspect of awareness.

In an objectivism (or an objectivist theory), if an event a fulfils a qualification (or a specifying property) S, then we write

$$a \subset S,$$

and the locus

$$C(S) = \{s \mid s \subset S\}$$

is admitted as a (determinate) set of events if and only if the following conditions are satidfied :

(1) S is given by a precise description (or a precise formulation) ;

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(2) there previously is given a (determinate) universal set of events (or a universe) U and $C(S)$ is destined to be contained in U;

(3) $\forall a \in U : a \subset S . \vee . a \not\subset S$.

When (3) is satisfied, S is said to be *descriptive*. If all of (1), (2) and (3) are satisfied, U is called the *level* of S. Even when S is not found descriptive, $C(S)$ may yet be called a *class* in the current usage. If the condition (1) is admitted, we shall take $C(S)$ as an assembly, whether S is descriptive or not.

In an objectivism the primitive universe is very essential and 'observation' is taken to be developed over the events concretely defined on the universe. As the observation advances, it will possibly be elevated toward the objects of higher level therefrom defined. The total construction made up by the axioms, definitions and the results therefrom attained is the *proto-construction*[1](of the intended objectivism), which shall possibly be revised in the future if needed. Sets, classes and assemblies suggestsd in the proto-construction will naturally build up a set theory which is called the *annexed set theory*[2] (to the intended objectivism). Propositions composed and announced in the annexed set theory will inversely be interpreted into propositions in the proto-construction. However, these interpreted propositions may not always present significant contents.

If a concept put forward in reference to the proto-construction cannot candidly be considered to promise a content convincingly determinate, then it is called a *super-objectivist concept* or a *super-concept*, and the conception of such one is taken as a *super-objectivist conception*. If a super-concept is proved to give no objection, in application, to the proto-construction, then it may be additionally incorporated into the proto-construction as an *objectivist result*.

1. Source of Recognition

The term *'finitary'* proposed by D. Hilbert has, by most authors, been introduced simply to mean *'intuitively convincing'*. However, for instance, by G. T. Kneebone, what Hilbert explained when he firstly introduced this word is interpreted as follows[3] : We shall always use the word 'finitary' to indicate that the discussion, assertion, or definition in question is kept within the bounds of through-going producibility of objects and through-going practibility of processes, and may accordingly be carried out within the domain of concrete inspection.

The word 'intuition' may possibly cause a world of unavailing difficulties if we work with it in the general sense used in philosophy. This word may not be explained out but dialectically, and its action may be put in rational inspection (under rational restrictions) only in connoction with the intellectual awareness. It may not be denied that even a mere delusion has its composition to be caused through the action of 'intuition', while its falsity is revealed only when it is related to the intellectual awareness. Conversely, the intellectual awareness cannot be separated from 'intuition'. For instance, natural integers, volumes, and sizes are all considered as possible concepts based on the facts intuitively convincing.

Hilbert's 'concrete inspection' may be regarded to be possible only in that the intended formal system has a concrete model. Thus an accumulation of estimated results cannot produce but a pseudo-awareness unless a formal system is proved to have a model containing these results. However, in an objectivism, its substantial model is precedently given, so that the pseudo-awareness may be precluded.

An assembly, as an object merely abstracted from the annexed set theory, may not be but a pseudo-concept (i.e. a super-concept) unless it proves to correspond to a (determinate) set of events in the proto-construction. When a super-concept can be regarded to be an additional concept as an objectivist result, it is that the concept is admitted at least to be manupulatively[4)] convincing in reference to the proto-construction. Such a concept may possibly be said to be intuitively convincing, that is, to gibe a finitary one. On the other side, it is asserted, in the *empricist pragmatism*[5)], that a mere abstract object must be renounced unless any way is found to accept it as an objectivist result. However, if no objection is found against the propriety of the object in question but mere sceptic one which has no rudiment in the proto-construction, then the object may be considered as manipulatively convincing and be accepted.

2. Benefit of the Super-objectivist Conception

For example, the content of the assumption "there can be but a finite number of prime numbers", which Euclid posited as a hypothetical premise, was but super-objectivist, and from this was concluded a constraction, so that it was convinced that there must be infinitely many prime numbers. All members involved in this argument could be found within the proto-construction which Euclid had had, except the above-mentioned hypothetical premise. Thus there was left no way but renounce the premise, to avoid the contradiction. Such a decision is the key of a 'reduction to absurdity', and it is notable that, in this sort of argument, renouncible premise is always supplied through super-objectivist conception.

A substantially objectivist concept should, in actual practice, designate an objectivist set of events. However, the set thereby designated must necessarily be but a finite set. Thus the conception of an infinite set is essentially super-objectivist. But, if an infinite set is, in any way, convinced to be undeniable, then it may be incorporated with the proto-construction as an objectivist result (with epistemologically sufficient explanation).

In acception of the concept of Ω the initial number (or the cardinal) of the 3rd class, following two facts are necessarily accompanied :

(i) the class or the ordinal numbers of the 2nd class makes up a well-ordered set, which must be admitted as determinate if to be accepted ;

(ii) Ω cannot be the limit of an enumerable sequence of sections of it.

Both (i) and (ii) are, at this stage, may not be considered but give super-objectivist conceptions. Because of the historical property of the ordering process, it seems rather impertinent to regard Ω exactly to be a determinate class. The condition (ii) apparently prevents us from attaining Ω by means of an enumerable stepping which is considered, in an objectivism, to be the only way to reach an infinite set as a limiting destination.

As an example of an assembly of the type Ω, we have the family of Borel sets, but there has not yet been discovered any determinate example of the type Ω in the domain of real numbers.*

The concept of ω the initial number of enumerable infinity may not be considered to be so easy a one, either. In effect every remainder of this aggregate has the same size with the original body. However, this aggregate is considered to provide the primitive model of the human process of numeration (i.e., the natural numeration). So, if renounce this, mathematical devices will extremely be limited. Being pushed out to the stage not bounded by any finite integer, we have decided to take it up in the meaning that the $(n+1)$th element is determined when the nth is given. This may be taken as an objectivist result in methodology.

Given a set M, if there is found a sequence of disjoint subsets $M_k (k=1, 2, \cdots)$ such that

$$(\forall x \in M)(\exists k)(x \in M_k)$$

is proved, then the family (M_k) $(k=1, 2, \cdots)$ is a partition of M, so that M is

*) This view is made in that we should not admit any oracle which produces the answers only by 'historical option'.

considered as the union of (M_k) and is written as

$$M = \cup M_k.$$

However, in our objectivism, M is not said to be the sum of (M_k) but for the assurance that the size (adequately defined) of the remainder

$$M - \bigcup_{k=1}^{n} M_k$$

tends to zero as n tends to ∞.

Le us observe the assembly C of real-valued functions which are defined in the interval [0, 1] and are continuous there. Let an enumerable set $\{x_1, x_2, \cdots)$ ($\subset [0, 1]$) be everywhere dense in [0, 1]. Then, for any two functions f and g from C, if

$$\forall k = 1, 2, \cdots : f(x_k) = g(x_k)$$

we have, as well-known, that

$$\forall x \in [0, 1] : f(x) = g(x).$$

So the elements f of C correspond one-to-one with the sequences (of real numbers)

$$(f(x_1),\ f(x_2),\ \cdots). \tag{2. 1}$$

Thus, if we want to admit C to be a set, the only objection which we may possibly meet will be that the sequence (2. 1) may not be considered as a determinate element (because it might rather be a super-objectivist concept). Neverthless, if we renounce this objection, we may regard C to be a set.

Now we take up the assembly V of propositions whose validities in reference to the proto-construction have been or will possibly in the future be proved. In this case, since the assembly V is the class caused by provability, it appears as if only one super-objectivist conception (i.e., the provability in the future) is involved. But, on thinking over the matters, we find it is not so simple. In effect, the ways of proof are generally not so simple as arithmetic operations, but may possibly need some assumptions which are originally super-objectivist. In addition, if some trial of proving a proposition comes across undecidable elements, the proto-construction itself must possibly be changed out. If then, at least some propositions will have their validities to be promised only by the proofs in terms of the new construction.

If the above assembly $\boldsymbol{V}$ is taken up primarily with the intention of examining the dominating extent of the present proto-construction, then the revised construction may not worth notice. So then, propositions to be proved in the new construction may make only a redundant part for the examination. However, if this part is omitted, the intended objectivism will lose its sense that it must proceed its developing through the revision of the proto-construction if needed.

3. Hypothetical Scheme of Universal Assemblies

Let U_1 be the class of events which can be produced by a finite number of elementary operations (given in the intended objectivism) from the primitive universe U_0, and U_2 be the class of events which can finitely be produced in terms of the language promised on U_1, and so on. Then, through the iteration of the definition, we have a sequence of assemblies

$$U_0,\ U_1, U_2,\ \cdots. \tag{3. 1}$$

If these assemblies are admitted as objectivist results, they are considered to give universes. Thus, in this sense, we have a hierarchy of universes by (3. 1).

If P is a proposition produced by combination of a finite number of operations and terms involved in the language promised on U_0, then there will be assumed an assembly $E(P)$ of events such that

$$E(P) \subseteq U_0 \cup U_1,$$

on the question if

$$(\forall a \in E(P) |\ a \subset P) \wedge (a \notin E(P).\ \Rightarrow.\ a \not\subset P) \tag{3. 2}$$

is verified or not. If $E(P)$ is existent as a (determinate) set fulfiling (3. 2), then P is an *objectivist proposition.* However, what we should at the primary stage inquire is "what event a is to be examined on the relation

$$a \subset P$$

?". The assembly $L(P)$ of such a's is called the *level of* P.

If $L(P)$ is either proved to be a set or admitted to be regarded as on objectivist result, then P is called a *general proposition* (in the intended objectivism). If a general proposition P fulfils the condition

$$\forall a \in L(P) : a \subset P. \quad \vee. \quad a \not\subset P \tag{3. 3}$$

(hence $\sim(a \subset P. \quad \wedge. \quad a \not\subset P)$), then P is an *objectivist* (or *descriptive*) *proposition*.

$L(P)$ may apparently be regarded as a universe, and it is clear that

$$L(P) \subseteq U_1 \cup U_0.$$

Yet we may not always have

$$L(P) = U_0. \quad \vee. \quad L(P) = U_1.$$

We thus see that the construction of U_1 may not be so simple. Incidentally, when (3. 3) does not hold, it must be that there exists at least one event $a \in L(P)$ for which whether $a \subset P$ or not is undecidable. In this case P is an *undecidable proposition*.

4. Incompleteness

The proposition (in the theory of numbers) "there exist infinitely many pairs of twinprime numbers" must, independently of the human speculation, be either true or otherwise false. A proposition which must, like this example, be absolutely and univalently destined to be true or otherwise false is called a *solid proposition*. If a theory based on certain axiomatics cannot clearify the truth value of at least one solid proposition ocurring in it, it is said to be *incomplete*.

If any of the proposition Q or its negation $\sim Q$ can be added to the axioms without violating the consistency, then Q is an *undecidable proposition* for the original theory. Therefore, if a proposition Q is undecidable, Q can neither be true nor false, so that it may not be solid.

The assembly $\boldsymbol{V}$ of valid* propositions (*: i.e., provable of its truth) referred to the proto-construction may contain not only the actually known valid propositions, but also ones which will possibly in the future turn out to be valid. So, $\boldsymbol{V}$ is essentially a super-concept, whereas, if $\boldsymbol{V}_0$ is the total collection of the actually known valid propositions, $\boldsymbol{V}_0$ is at most a (determinate) finite set.

To tell the truth, the content of the assembly $\boldsymbol{V} - \boldsymbol{V}_0$ is all but nonsensical. If a proposition Q is certainly such that

$$Q \in \boldsymbol{V} - \boldsymbol{V}_0, \tag{4. 1}$$

then Q must turn out actually to be valid through the proof ascertaining (4. 1), so that it must be that $Q \in \boldsymbol{V}_0$. This being so, $\boldsymbol{V}$ may not be regarded as an objectivist result, but rather be regarded to be a mere abstract object as a historical extension of $\boldsymbol{V}_0$. Similarly, the same thing may be concluded about the assembly of invalid propositions.

In an objectivism, valid propositions and invalid ones may both be regarded as solid. Hence, that a proposition Q cannot be solid must mean that Q is undecidable, if not renounced. Since 'validity' is now considered as a historical concept, the essential problem left in here is of undecidability.

Incompleteness and inconsistency (of an objectivism) do not essentially interact each other. The problem of inconsistency cannot be thought so essential. If an objectivism is factually found inconsistent, it must be caused by some human carelessness on selecting the axioms or the definitions.

A solid proposition which is left unsolved may be considered important for the intention to discover an evidence of incompleteness of the proto-construction. However, even though it is certainly unsolvable in the proto-construction, it may possibly turn out to be solvable in a revised construction in the future. Thus, the problem of incompleteness may not be more than a historical pending one. In effect, if we take the example of the twinprime numbers, we may not say "no precise solution can be expected now on through".

In conclusion it shall be noted that an objectivism may be proceeded along a smooth developing course except for the following treatments :

(i) if we come across a contradiction, we eliminate it by adequate revision of the axioms or the definitions ;

(ii) if a proposition P which cannot be laid aside unsettled is found undecidable, then we add to the construction either P or $\sim P$ as an axiom to settle the construction.

These treatments appear to be not only very artificial but rather optional policies. If we yet are to research for any rudiment justifying them, it may not be done in other place than epistemology. On the like stand the treatment of super-objectivist subjects should generally be deeply associated with epistemology.

If a proposition Q is proved to be undecidable, either Q or $\sim Q$ is to be added to the proto-construction as an axiom, so that Q is turned out to be a solid proposition in the new construction. Thus the solidity of a proposition may be considered to be a historical concept relative to the improvement of the construction. However, the discourse may emphatically be thought to produce a branch after the addition of the new axiom, and the part prior to the addition may be taken as the proper part of the intended objectivism. If the construction revised by the addition of Q is found to be inconsistent, then it may be concluded that Q is, in fact, not an undecidable proposition at all, on the proper part.

Mathematical Seminar of the Muroran Inst. Tch., Hokkaido
(Received Apr. 20, 1979)

References

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