Predictive Calculation for Deflections of Reinforced Concrete Floor Slab Systems —Part 1 Procedure—

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— Part 1 Procedure —

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Abstract

In this report, we incorporate equations for a respective estimation of causally different types of deflection for r. c. beams and one-way slabs into our system for the deflection calculation of two-way floor structures, so that, after the system has been suitably modified, it may be of extended and generalized use for examining such structures.

This permits us to express chronic (or longtime) deflection as the total effect of a variety of agents.

Then, we examine whether and how the actually observed chronic transition of slab behaviors is consistent overall with their follow-up predictions established by our procedure, and we test its utility by resorting to some available previous test examples of slab models under sustained service loading. We note that our system can be maintained in practice.

Predictive Calculation for Deflections of Reinforced Concrete Floor Slab Systems Part 1 : Procedure

1. Introduction

Almost a quarter of a century has passed since the earliest domestic cases of deflection damage and excessive cracking to reinforced concrete floor slabs was first noticed in Hokkaido[5]. Initially the cause of similar types of structural deterioration was attributed to the presumed likelihood of defective material and inappropriate regional construction practice. Later on, however, being mainly made in Britain and West Germany, material scientific researches[6] into related types of floor slab deflection and detailed analyses[7] based on field measurements of the conditions of relevant manifold examples revealed that such deflection combined with cracking is in fact both common and of worldwide incidence and refers to the most frequent structural maintenance problem ; an information which led us to have known many other cases in point all over this country.

There has been a dearth of empirical means to explore the causes of impaired structural serviceability other than finding them out by analyzing observed sets of data of a number of collected, relevantly damaged instances, as was the case with 〔8〕, or by quasi-permanent loading tests of full-scale slab models, while all the time satisfying the designated set of ambient atomospheric conditions ; the latter means being too idealistic in general to be economically maintained.

In recent years, however, significant progress has been made in experimentally clarifying some of the elemental factors that comprise the subject matter of predictive calculation such as effects of the bond-slip of edge reinforcement on predictions of deflection[16,17] and what is called stratified values[25] of intensity of the loads imposed at construction work. Also, rational treatment in static calculations of major detrimental phenomena of cracking, creep and drying shrinkage have become somehow possible by use of findings through the pertinent long postwar research activities fundamentally engaged in Europe. Such achievements have served for solid prospects to be opened for elucidating both material and static phases of the structure.

In other countries, notably Euramerican, major studies on the time dependencies of the deformation of r. c. horizontal members have started largely in 1960s[9] and provided results which are embodied by the ACI, CEB or other typical building codes in their pertinent clauses.

Primarily referring to beams or one-way slabs such building code methods for predictive estimation of slab deflections must practically depend on beam approximations as is the case typically with ACI's equivalent frame method and accordingly remain too coarse approaches whereby to go into two-way anisotropic behaviors of the floor slab structure especially relevant to its introduced damaged cases above where the reinforcement around supports is known to cause a significant amount of bond-slip the code methods are unaccountable for.

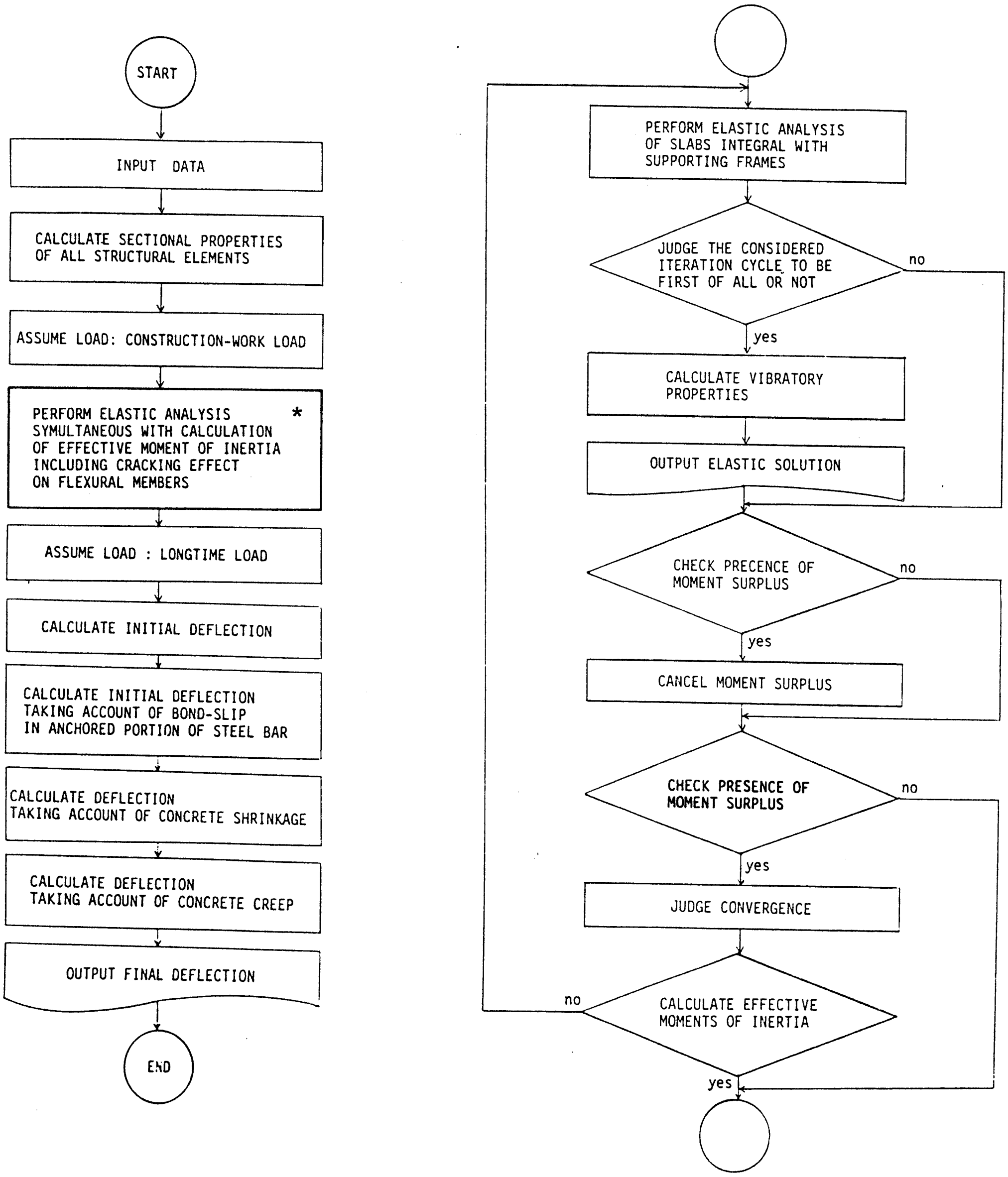
Actually, longtime deflections predicted by use of them is accepted in most cases to be less than half the corresponding direct measurements.

In this report those standarized or acknowledgedly representative formulations for cracking, creep and other causally different effects on the longtime deflection, derived for the one-way system are incorporated in the authors' calculation system for two-way structures, through its accompanying generalization and modification.

Then, thus far available longtime test results and measured deflection increases with time on model floor slabs are compared with their follow-up solutions afforded by the introduced procedure, in an effort to examine whether the latter results can be reasonably consistent with the former.

2. Method of Analysis

The predictive calculation of the terminative or final deflection of r. c. floor systems may be per-



(a) Main Diagram

(b) Detail of Part in Thick Frame* of Main Diagram

Fig. 1 Diagrammed Flow of Calculation for Predicting Deflections.

formed as for its causally different portions when being pursuant to the flow-diagram in Fig. 1 .

2 . 1 Consideration of Supporting Frames

In the following, the description of the proposed analytical method is, in expectation of its widest possible application, to be so generalized as not just to cover cases of ordinary floor slabs but of constructions with slab or subpanel zones of unform increased thickness.

Hence hereafter to be analyzed is a floor slab with or without beams, which is orthogonally anisotropic due to its differing modes of cracking in the two orthogonal directions.

In effect, a finite difference approach coupled with the slope-deflection fundamentals will now be derived on a whole system of structure with both beam and slab elements. In the present work first subdividing its short and long spans, measured at beam centroidal axes, into equal meshes and then selecting as unknowns deflections at the interior mesh points (inner points), those at the above axes, and the angles of torsion about them, a set of equilibrium equations will be set up in difference form.

1) Orthogonally Anisotropic Slab Equations

Given the slab stiffness in respective short and long directions by D_x and D_y, with $D_x/D_y = k^4$, $D_y/D = \mu$ and Poisson's ratio $\nu = 0$ for simplicity, the governing differential equation for an orthogonally anisotropic rectangular slab is expressed as Eq. (1) and bending moments M_x, M_y, torsional moment M_{xy} as well as reactions V_x, V_y are respectively defined by Eqs. (2) through (6)[10].

$$k^4 \frac{\partial^4 w}{\partial x^4} + 2k^2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - \frac{P}{\mu D} = 0 \tag{1}$$

$$M_x = -\mu k^4 D(\partial^2 w/\partial x^2) \tag{2}$$

$$M_y = -\mu D(\partial^2 w/\partial y^2) \tag{3}$$

$$M_{xy} = -\mu k^2 D(\partial^2 w/\partial x\,\partial y) \tag{4}$$

$$V_x = -\mu k^2 D(k^2 \partial^3 w/\partial x^3 + 2\,\partial^3 w/\partial x\,\partial y^2) \tag{5}$$

$$V_y = -\mu D(\partial^3 w/\partial y^3 + 2k^2 \partial^3 w/\partial x^2 \partial y) \tag{6}$$

where w = deflection, p = intensity of load of uniform distribution, D = stiffness of a standard slab or $Et^3/12(1-\nu^2)$, t = thickness of the standard slab, E = elastic modulus of concrete and ν = Poisson's ratio of concrete.

Further assuming a width of difference subdivision or, briefly, a mesh width for each of the orthogonal directions as Δx and Δy, with ratio $\gamma = \Delta y/\Delta x$, and any mesh point as a reference point of compatibility of surrounding contiguous subpanels A, B, C and D leads to such a resultant reaction S_{xy} at their common corner (point) as is expressed as follows by using reactions in both directions and concentrated reaction, $F_{xy} = 2 \cdot M_{xy}$.

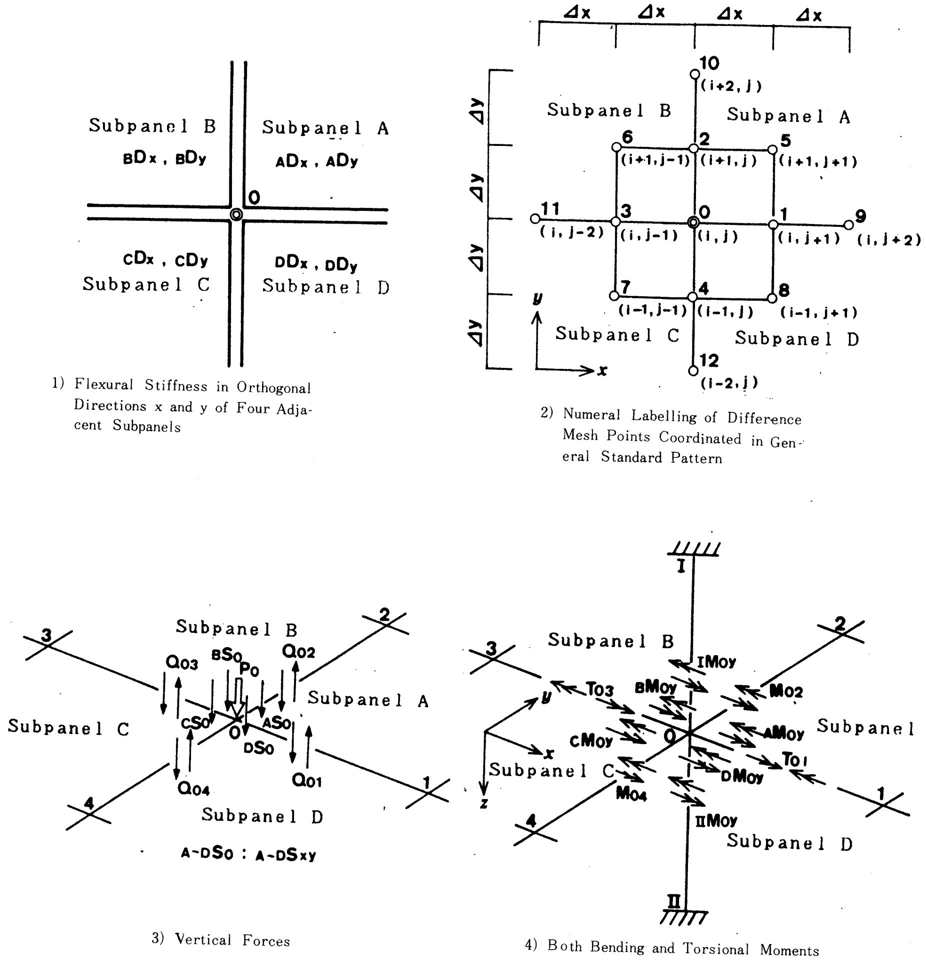


Fig. 2 Vertical Forces and Moments Acting at Point of Intersection of Four Adjacent Subpanels of a Slab; A, B, C and D

$$S_{xy}=(V_x+\gamma V_y)\Delta x/2+2M_{xy} \tag{7}$$

Expansion of Eqs. (1) through (6) in finite difference form with subsequent subtraction of a fourth of the identity Eq. (1) from Eq. (7) gives schematized Eq. (8) of Fig. 3 ; provided that it refers to preceding subpanels A, B, C and D respectively when m = 1 with n = 1 , m = 1 with n = − 1 , m = − 1 with n = − 1 and m = − 1 with n = 1 . Also in the same equation w''s are those deflections at imaginary points which become, in case of the structure having a beam along the above contiguous edge, using angles of torsinal rotation θ_x and θ_y about its sectional axis, such as

$$w'_{i,j-n}=w_{i,j+n}-2n\Delta x\theta_{x_{i,j}} \tag{9}$$

$$w'_{i-m,j}=w_{i+m,j}+2m\Delta y\theta_{y_{i,j}} \tag{10}$$

$$
{}_{A-D}S_{xy}=\frac{\mu D}{\lambda^2}
\begin{array}{|c|c|c|c|}
\cline{2-2}
\multicolumn{1}{c|}{} & -\frac{1}{2r^3}\cdot w_{i+2m,j} & \multicolumn{1}{c}{} & \multicolumn{1}{c}{} \\
\cline{2-3}
\multicolumn{1}{c|}{} & \left(\frac{2k^2}{r}+\frac{3}{2r^3}\right)w_{i+m,j} & -\frac{2k^2}{r}\cdot w_{i+m,j+n} & \multicolumn{1}{c}{} \\
\hline
\frac{k^4 r}{2}\cdot w_{i,j-n} & -\left(\frac{3k^4 r}{2}+\frac{2k^2}{r}+\frac{3}{2r^3}\right)w_{i,j} & \left(\frac{3k^4 r}{2}+\frac{2k^2}{r}\right)w_{i,j+n} & -\frac{k^4 r}{2}\cdot w_{i,j+2n} \\
\hline
\multicolumn{1}{c|}{} & \frac{1}{2r^3}\cdot w_{i-m,j} & \multicolumn{1}{c}{} & \multicolumn{1}{c}{} \\
\cline{2-2}
\end{array}
+\frac{p r \lambda^2}{4} \qquad (8)
$$

Fig. 3 Resultant Reaction at a Corner Point of a Slab Panel

When this edge has no beam but is the boundary of two subpanels with different slab stiffnesses the following equations of continuity for an above-mentioned slab with subpanel thickenings are to be used[11].

$$w'_{i,j-n}=\frac{\xi-1}{\xi+1}w_{i,j+n}-2\frac{\xi-1}{\xi+1}w_{i,j}+\frac{2\xi}{\xi+1}w_{i,j-n} \tag{11}$$

$$w'_{i-m,j}=\frac{\xi'-1}{\xi'+1}w_{i+m,j}-2\frac{\xi'-1}{\xi'+1}w_{i,j}+\frac{2\xi'}{\xi'+1}w_{i-m,j} \tag{12}$$

where ξ and ξ' are ratios of adjacent to considered panel stiffness, e. g. in deflection equations for imaginary points of subpanel A : with

$\xi={}_BD_x/{}_AD_x$ and $\xi'={}_BD_y/{}_AD_y$

2) Equilibrium of Vertical Forces

In cases with a beam along a common edge of preceding subpanels the equation of equibrium at their common corner point of vertical forces becomes

$$(Q_{01}-Q_{03})+(Q_{02}-Q_{04})+({}_AS_{xy}+{}_BS_{xy}+{}_CS_{xy}+{}_DS_{xy})=P_o \tag{13}$$

where $Q_{01}-Q_{04}$= shearing forces at the ends of beam members, ${}_{A-D}S_{xy}$ = reaction resultants at corner points of the subpanels, P_o = concentrated load acting at point O.

A member-end shearing force in the beam in the x-direction is obtained by expanding the governing differential pair of equations for a beam

$$\frac{d^3w}{dx^3}=-\frac{Q_x}{EI_x},\quad \frac{d^4w}{dx^4}=\frac{q_x}{EI_x} \tag{14}$$

in finite difference form, so that

$$Qx=-\frac{EI_x}{2\Delta x^3}(-w_{i,j-2}+2w_{i,j-1}-2w_{i,j+1}+w_{i,j+2}) \tag{15}$$

$$\frac{EI_x}{\Delta x^4}(w_{i,j-2}-4w_{i,j-1}+6w_{i,j}-4w_{i,j+1}+w_{i,j+2})-q_x=0 \tag{16}$$

and by doing such sums as (15)−(16)× Δx/ 2
and (15)+(16)× Δx/ 2 as follows

$$Q_{01},\ Q_{03}=\frac{mEI_x}{\Delta x^3}(w'_{i,j-m}-3w_{i,j}+3w_{i,j+m}-w_{i,j+2m})+mq_x\Delta x/2 \tag{17}$$

where Q_{01} and Q_{03} are respectively used when m = 1 and m = − 1 ;
q_x = self-weight of a beam in the x-direction and I_x = second sectional moment of the beam.

3) Equilibrium Equations for Moments

With the signs of moments in each building element acting at beam-column connection O in the y-direction assumed as shown in Fig. 2 (4), the following equilibrium holds :

$$-({}_AM_y+{}_BM_y-{}_CM_y-{}_DM_y)\Delta x/2+(M_{04}-M_{02})+({}_{\mathrm{I}}M_{0y}+{}_{\mathrm{II}}M_{0y})+(T_{01}+T_{03})=0 \tag{18}$$

where ${}_{A-D}M_y$ = bending moments in each above subpanel in the y-direction, $M_{02,\ 04}$ = bending moments at ends of a beam in the y-direction, ${}_{\mathrm{I.II}}M_{0y}$ = bending moments respectively at upper-column bottom and lower column-top and $T_{01,03}$ = torsional moments in a beam in the x-direction,

$${}_{A\sim D}M_y={}_{A\sim D}D_y(-w_{i+m,j}+2w_{i,j}-w'_{i-m,j})/\Delta y^2 \tag{19}$$

$$M_{02},M_{04}=EI_y(-w_{i+m,j}+2w_{i,j}-w'_{i-m,j})/\Delta y^2 \tag{20}$$

$${}_{\mathrm{I}}M_{0y}=4E_{\mathrm{I}}I_{zy}\theta_{y_{i,j}}/{}_{\mathrm{I}}Lz \tag{21}$$

$${}_{\mathrm{II}}M_{0y}=4E_{\mathrm{II}}I_{zy}\theta_{y_{i,j}}/{}_{\mathrm{II}}Lz \tag{22}$$

$$T_{01},T_{03}=GJ_x(\theta_{y_{i,j}}-\theta_{y_{i,j+m}})/\Delta x \tag{23}$$

where cases of m = 1 and m = − 1 respectively refer to ${}_{A,D}M_y$, M_{02} and T_{01} as well as ${}_{B,C}M_y$, M_{03} and T_{03} ; ${}_{\mathrm{I.II}}I_{zx}$ = second sectional moments repectively of upper and lower columns G = elastic modulus in shear and J_x = coefficient of torsional resistance of a beam[13] in the x-direction.

2 . 2 Consideration of Fiexural Cracking of Concrete

The present deflection analysis of a floor slab together with its supporting frames consists of implementing for a difference system of equations formulated above their symultaneous solution that is to be iterated until its convergence after an initial elastic result, by employing sectional stiffnesses over again whenever they may take further reduced values due to considered effect of cracking, as will be explained in detail ; i.e. leading to the initial deflection, Δ_i, of a slab and a beam, with the considered effect of their cracking.

1) Reduced Slab Stiffness

For a whole span, in either orthogonal direction, of any slab strip with a difference mesh width, henceforth called mesh-width strip, the bending moment distribution along this is checked if both of its positive and negative maximal values, M_a's, exceed a cracking value, M_{cr}, and in an affirmative case using respective moments of inertia for cracked and uncracked sections, I_g and I_{cr}, an effective moment of inertia I_e for a mesh-width strip is calculated by Branson's ensuing equation[14].

$$I_e=\left(\frac{M_{cr}}{M_a}\right)^3 I_g+\left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \tag{24}$$

where for a rectangular section with double reinforcement

$$I_{cr}=b(cd)^3/3+\eta A_s(d-cd)^2+(\eta-1)A'_s(cd-d')^2 \tag{25}$$

with b = difference mesh width, c = ratio of depth of neutal axis of a section to its height, d = distance of centroid of tensile reinforcement from compression face, d′ = distance of centroid of compressive reinforcement from that face, η = modular ratio, A_s = area of tensile reinforcement and A'_s = that of compressive reinforcement ; provided that ratios of tensile and compressive steel area $\rho\,(=A_s/bd)$ and $\rho'\ (=A'_s/bd)$ are respectively used to obtain c, i. e.,

$$c=\sqrt{2\,\eta(\rho+2\rho' d'/d)+\eta^2(\rho+2\rho')^2}-\eta\,(\rho+2\rho') \tag{26}$$

As suggested in 〔14〕 chiefly respecting one-way structure effective moments of inertia I_e's for a positive and two negative moments regions of it are each weighted by multiplying each I_e by a ratio of the moment area of the corresponding region to the sum of the pertinent three areas and are subsequently averaged as usual, resulting for all the mesh-width strip average moments of inertia.

And further, some and the others of these respectively for mesh-width strips in the middle strip and those in the column strip are separately averaged again to provide ${}_{avg}I_e$'s, where both latter wider strips can be those each occupying a half of the whole area of a slab panel as customarily defined, e. g. in 〔12〕, but at this time a similar panel division introduced in Section 3.1 is used deeming it can more reflect actual propensities of cracking.

Finally, reduced slab stiffnesses in both orthogonal directions, for either such a middle or column strip are obtained as :

$$D_e=D_g({}_{avg}I_e/I_g) \tag{27}$$

naturally differing in value after the first sequence of iteration and so requiring a solution as an orthogonally anisotropic strucure.

2) Reduced Beam Stiffness in Bending and Torsion

The average effective moment of inertia for a beam moment can be obtained in the same manner as in the case of a slab strip of the mesh width, using the average of moment areas at beam ends and center, after being weighted proportional to each area.

Only in its positive bending region the structure needs to be considered to be integral with that part of the slab panel called cooperative width which is in the current case taken from the corresponding equation in 〔15〕 for a T-beam. Then for its cracked section

$$I_{cr}=B(cd)^3/3+\eta A_s(d-cd)^2 \tag{28}$$

where B = flange width of a T-beam with the ratio of depth of neutral axis c being obtained by Eq. (26) putting $S = A_s/(Bd)$ and $S' = A'_s/(Bd)$.

On the other hand, far less work having been available on the torsional stiffness of a T-beam affected by cracking of the concerned type its value is expediently assumed to decrease proportional to the corresponding reduction in flexural stiffness.

2.3 Consideration of Bond-Slip of Reinforcement Anchorage

Both groups of Higashi-Komori and Takahashi-Koyanagi have made short and long-term loading tests on one-way slab strips and cantilever structures in order to account for sustained deformational action of r. c. floor slabs and as a result pointed out that in addition to its being comparable in magnitude to the effects of cracking, creep and drying shrinkage on such deflectional behaviors that of the bond-slip of the portion of anchorage of the reinforcing steel was found to be far larger than an amount which had generally been regarded as being of an ignorable order[16,17].

In this work deflections so caused are to be analyzed in the following process. Denoting the stress in the reinforcement at the support of a slab by σ_s, the length of its portion of anchorage L_d is given by

$$L_d=A_s\sigma_s/\tau_b\psi \tag{29}$$

where τ_b = average bond stress in the above part of reinforcement and ψ = perimetric length of reinforcement.

Assuming the distribution of the bond stress is triangular with σ_s and 0 as its values respectively at the root and tip of anchorage, the elongation of the steel, i. e., the amount of its bond-slip for a length of anchorage L_d becomes

$$u=L_d\,\sigma_s/2E_s \tag{30}$$

What is caused thereby, the angle of of rotation θ of the middle plane of a slab about the axis of its support may be calculated by the following equation, assuming the neutral axis for cracked section decided by Eq. (26) as the above axis

$$\theta=\frac{u}{(1-c)d}=\frac{A_s\sigma_s^2}{2(1-c)dE_s\,\tau_b\,\psi} \tag{31}$$

The additional deflection due to the bond-slip at the support, Δ_s, may be calculated as a solution for the structure with rotations θ's that are thus worked out at each mesh point along its edges forced back again at the same position.

2.4 Consideration of Creep and Shrinkage of Concrete

For r. c. floor slabs their longtime deflections caused by creep and shrinkage of concrete may be calculated as follows by making a generalized application of Branson's method[14].

In his original equation the deflection due to creep is expressed in terms of the initial deflection Δ_i alone that includes the effect of flexural cracking. But the effect of bond-slip, being one of the signficant factors controlling depth and width of cracks, concerned mainly with early stages of structural deflection is currently added to the preceding Δ_i. Thus the deflection due to creep may be expressed as

$$\Delta_{cp}=K_r\phi_t(\Delta_i+\Delta_s) \tag{32}$$

with $$K_r=0.85/(1+50\rho') \tag{33}$$

where ϕ_t = creep coefficient of concrete at age t, ρ' = ratio of compressive steel at midspan of a flexural member or $A'_s/(bd)$ for a slab and average of $A'_s/(Bd)$ and $A_s'/(bd)$ for a T-beam ; and b = member width or, unit width for a slab and web width for a beam.

Lastly, the shrinkage deflection is estimated by the following[14].

$$\Delta_{sh}=\alpha\beta A_{sh}\varepsilon_{st}Lx^2/h \tag{34}$$

together with

$$A_{sh}=3.25(\rho-\rho')^{1/3}(1-\rho'/\rho)^{1/2} \tag{35}$$

where : ε_{st} = shrinkage strain of concrete at age t, α = coefficient of shrinkage deflection[18] dependent on conditions of edge restraint ; e. g. .09, .065, .063, .125 and .5 respectively for exterior and interior span of a continuous structure, both-end built-in beam, simple beam and cantilever, β =multiplying factor due to aspect ratios as later explained, L_x = short span length for slabs measured center-to-center of supports, h = overall thickness of members, ρ =tension steel ratio for the central section, or $A_s/(bd)$ for slabs and average[18] of $A_s/(bd)$ and $A_s/(Bd)$ for beams.

In the above, multiplying factor β may be approximated to be

$$\beta=1+({}_y\Delta_{sh}/{}_x\Delta_{sh}-1)/(1+\lambda^4) \tag{36}$$

by using shrinkage deflections ${}_x\Delta_{sh}$ and ${}_y\Delta_{sh}$ respectively for one-way structures spanning in the respective x- and y-directions and being otherwise the same as the considered encastered slab.

For floor slabs of practicably normal size β is at most 1.2 or so even if such is the case at an aspect ratio as large as 1.5 or so.

In connection with the above calculations the treatment of time-dependent action of concrete is resorted to Rüsch-Jungwirth's method,[19] the CEB-value[20] is adopted as basic shrinkage strain and the effect of concrete slump on the creep coefficient is considered by use of the corresponding ACI's modifying equation[21] that is accountable for plastic or high-slump concretes.

Consequently the above portions of a total longtime deflection are added to give that as

$$\Delta_t=\Delta_i+\Delta_s+\Delta_{cp}+\Delta_{sh} \tag{37}$$

for loading period $t=\infty$

3. Inferred Tenableness of the Procedure from Test Results

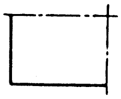
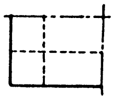
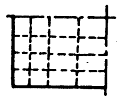
The reasoning whether the proposed system of procedure is soundly accountable for the longtime structural performance now considered will be made through a few or more trials of comparison between accessible test results and their present analytical equivalents.

Beforehand, necessary for it to be implemented some partly discretional assumption and incidental technique are to be introduced.

3.1 Effects of Width Difference between Middle and Column Strips

What partly features the present approach is that a floor slab may be analyzed as a structure having subpanel strips with different effective stiffness, in practice as one with drop panels and/or slab bands so that the effect of its overall crack distribution may be taken into account as much in detail. In the same respect a comparison is attempted in Table 1 among analytical values of midpanel intial deflection for an all-edge-encastered apartment floor slab, 3.6 by 7.05 m of panel size, in case of different panel divisions. An ordinary division for Case 2 is throughout adopted while deflections tend to increase with multiplying subpanels.

Table 1 Effect of Different Allocation of Widths of Beam- and Column-Strips on Short-Term Deflection of Slabs, With Their Cracking Considered

sectional assumptions		deflection mm			relative figure	
slab thickness mm	top steel covering mm	CASE 1 	CASE 2 	CASE 3 	CASE.2 / CASE.1	CASE.3 / CASE.1
110	20	1.11	1.18	1.22	1.06	1.10
110	54	1.16	1.24	1.28	1.07	1.10
95	39	2.14	2.39	2.52	1.12	1.18
80	24	4.40	4.87	5.08	1.11	1.15

3.2 Converging Process of Effective Slab Stiffness

The proposed analytical means when initial slab deflections affected by cracking is thereby to be calculated may not always provide any final convergent results. Caused by large differences possible between average effective stiffness values in the two orthogonal directions, mainly at the earliest iteration stages, oscilations between successive intermediate reduced stiffness values can be precluded by using an average of the above two sets of values for any further sequence of iteration, assuring after only several of its cycles sufficient convergence.

3.3 Examples of Pursuing Time-Dependent Deflection Change on Test Floor Slabs

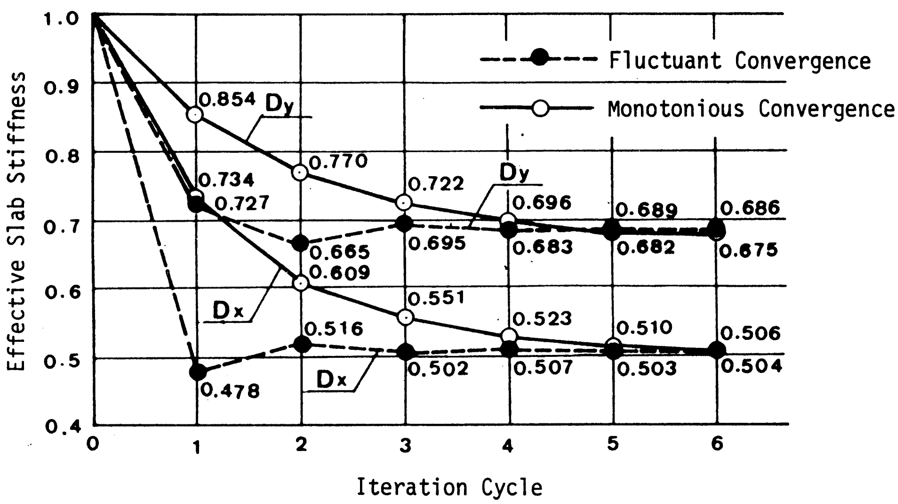


Fig. 4　Typical Converging Process for Effective Moments of Inertia of Slabs in Deflection Analysis [from Calculation Example in Case 1 of Preceding Section].

Examples to be discussed comprise a one-way slab model[22] under two concentrated loads, two square slabs[23] different in edge restraint, and a rectangular model[24] respectively tested under long-term loading by Takahashi-Koyanagi, Yamamoto et al. and Building Constructors' Society (B. C. S.), in order to investigate longtime structural movements, the preceding last case being related to prediction of the time for formwork removal and the rest conducted only for the proper purpose.

In Table 2 are shown geometric shape and dimensions, material properties and loading conditions for each model.

1) One-Way Floor Slab

Fig. 5 shows the relevant record of readings of laboratory temperature and relative humidity ; both taken about 450 days after the start of loading. The analysis resulted in a midpanel deflection for the model by using an average temperature of 7 ℃ on a concrete of eight weeks of effective age, a creep coefficient and a shrinkage strain both in their extreme cases of 40 and 80 percent of average relative humidity.

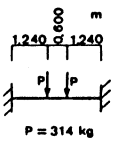
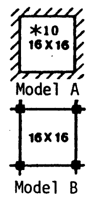
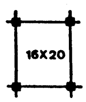
In Fig. 6 measurements of deflection are compared with the present calculations as their correspondents, where longtime test results are located nearly midway between the above referential extremities of deflections.

Analyses in the ensuing examples will use constant values for humidity as averages for the whole test period.

2) Square Slabs

The introduced slab models with different boundary conditions consist of an all-edge-encastered

Table 2 A Brief Summary of Previous Long-Term Loading Test Results for One-and Two-Way Floor Slab Models

ITEMS		ONE-WAY SLAB	TWO-WAY SLABS	
		REF. (22)	(23)	(24)
DATE OF CONC. PLACING		JAN. 21, 1973	—	JUL. 9, 1982
SLAB DIMENSIONS	BEAM-CENT. TO-CENT. SPANS	—	4.800 × 4.800	4.600 × 5.800
	EFFECTIVE SPANS	3.080(0.500)^{*1}	4.500 × 4.500	4.300 × 5.500
	THICKNESS	120	120	130
	EFFECTIVE DEPTH OF TOP STEEL	90	95	100
SLAB STEEL AREA	BEAM STRIP EDGE TOP	3 — D10	D10 ●200	D10 ●150
	BEAM STRIP EDGE BOTTOM	1 — D10	D10 ●400	D10 ●300
	BEAM STRIP CENT. TOP	—	—	—
	BEAM STRIP CENT. BOTTOM	3 — D10	D10 ●200	D10●150(300)^{*8}
	COL. STRIP EDGE TOP	—	D10 ●400	D10 ●300
	COL. STRIP EDGE BOTTOM	—	D10 ●400	D10 ●300
BEAM & COL. SECTIONS	COL. GROSS SECT.		400 × 400	400 × 400
	BEAM GROSS SECT.		300 × 450	300 × 600
	BEAM STEEL AREA		4 — D19	4 — D19
CONCRETE	COMPR. STRENGTH	141	69.4^{*6}, 254^{*7}	223^{*11}, 251^{*7}
	TENSILE STRENGTH	14.1	8.7 , 25.4	22.3
	AVERG. BOND STRESS	14.1	6.8 , 13.8	
	ELASTIC MODULUS	140000	196000	222000 , 225000
	POISSON'S RATIO	0.2	0.2	0.2
	MODULAR RATIO	10	10	10
	SLUMP	10.7	18.0	18.0
	BASIC CREEP COEF.	3.0^{*2} , 1.7^{*3}	2.6	2.8
	BASIC SHRINKAGE STRAIN	52.0 , 26.0	40.0	43.0
AGE AT START OF LOADING	FOR CONSTRUCTION-WORK LOAD	—	2	14
	FOR LONG-TERM SUSTAINED LOAD	56	14	28
IMPOSED LOADS	AS CONSTRUCTION-WORK LOAD	—	288	343
	AS LONG-TERM SUSTAINED LOAD	628 kg^{*4}	144 kg/m²	112 kg/m²
ENVIROMENTAL CONDITIONS	AVERG. TEMPERATURE	7^{*5} , 20	20	20
	AVERG. RELATIVE HUMIDITY	40 , 80	70	65^{*9}
NOTES	* 1: SLAB WIDTH IN PARENTHESES * 2: FOR 40 % RELATIVE HUMIDITY * 3: FOR 80 % RELATIVE HUMIDITY * 4: TOTAL IMPOSED LOAD * 5: 8 WEEKS AFTER CONC. PLACING * 6: VALUE AT AGE OF 2 DAYS * 7: VALUE AT AGE OF 28 DAYS * 8: AREA OF DISTRIBUTION STEEL PARENTHESIZED * 9: AVERG. FROM METEOROLOG. ANNUAL * 10: NUMBER OF DIFFERENCE SUBDIV. * 11: VALUE AT AGE OF 14 DAYS	 m 1.240 0.600 1.240 P P P = 314 kg	 *10 16X16 Model A 16X16 Model B	 16X20

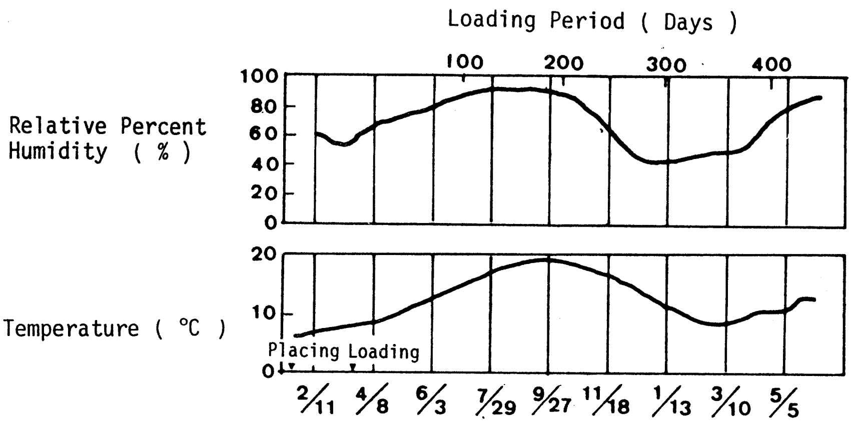


Fig. 5 Laboratory Atomosphere during Long-Term Test of One-Way Floor Slab from Ref. [22]

one (model A) and the other of interior bay-type (model B) that had been supposed to cause deflections without torsional rotation of its edge beams.

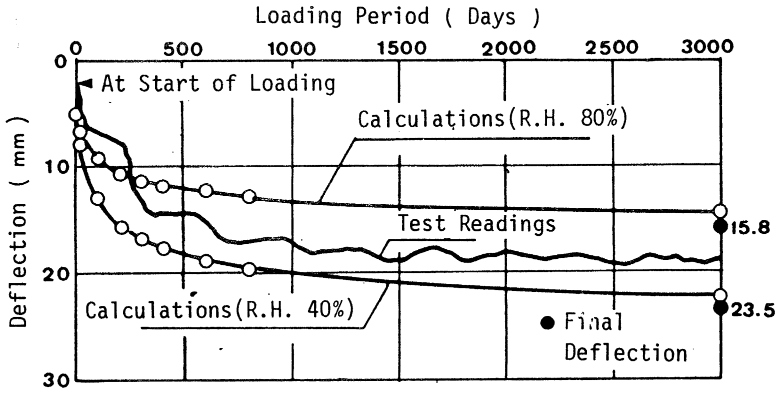


Fig. 6 Progress of Midpanel Deflections for One-Way Slab from Ref. [22]

The adopted way of loading amounts to initially imposing a uniformly distributed construction-work load, two days after concrete placing, and its subsequent shifting to a long-term sustained loading at the age of a fortnight.

Though this report uses as an average bond stress the result of substituting the concrete srength at the start of loading into the corresponding Japanese R. C. Code equation for "steel bars for longtime loads or their equivalents"[15], two alternative values of bond strength, one the same Japanese Code value and the other twice that are tried in the analysis, thus implying the possibility of correspondingly large variance of bond property at very early ages at the start of loading.

The results are set against their observed counterparts in Fig. 7 .

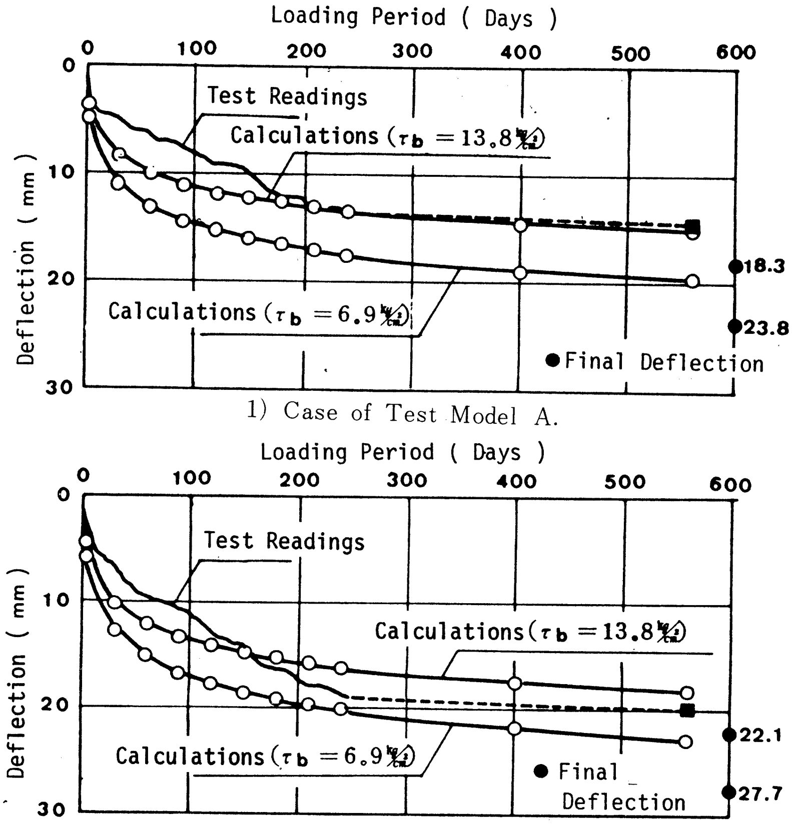
3) Rectangular Floor Slab

The test had been arranged and conducted as follows. Immediately after two weeks of concrete age or the removal then of forms and shuttering the model had been imposed on by a line load about 1.1 times its self-weight until a fortnight later, when the load had further been adjusted to a longtime sustained load, comprising a third of the design live load for office rooms plus weight of finishing materials other than the self-weight, amounting to 117 kg per sq. m., to have been kept applied until 35 weeks of concrete age. The assumed initial part of the observed deflection not being originally included in it, is deduced here as 0.5 mm from a pertinent load-deflection curve for lower load levels, then being added to the measurements.

The result is compared with the present calculation in Fig. 8 .

3 . 4 Comparison between Analytical and Test Results

In the above cases of comparison analytical results in general show some amounts of differences from the comparable test measurements at earlier ages of concrete but fairly good agreements after



1) Case of Test Model A.

2) Case of Test Model B.

Fig. 7 Progress of Midpanel Deflections for Two-Way Slab from Ref. [23]

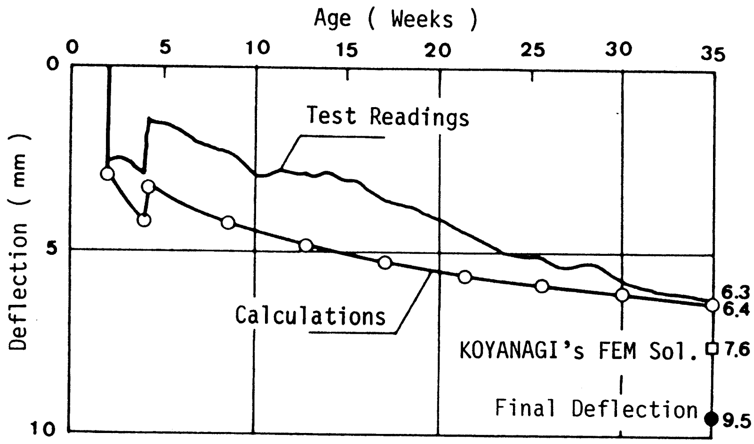


Fig. 8 Progress of Midpanel Deflections for Two-Way Slab from Ref. [24]

200 days of concrete age between both results.

As a matter of computation the partial contradiction is referable to analytical priority given to the prediction of final deflections over their correspondents at an earlier stage, for which purpose relatively low values were assumed for properties of tensile and bond strengths of concrete, in anticipation of its deterioration with time, though factual strength data concerned are not available in the cited test reports.

4. Conclusion

The procedure used here has been shown to have sufficient utility in general, as a result of its specific substantiation using some examples, giving a practically consistent approximation of long-term deflection progress. A related advantage of the method lies in its enabling representation of the causes of chronic slab deflection as the total effect of various agents.

By the nature of things the present analysis must inevitably allow for ill known parametric variables including material properties.

The possibilities of practical application of the present modified approach will be discussed and explored, including the potential extent of utility compared with that of such building code methods as we initially referred to, in a subsequent part of this report.

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