Moment-Rotation Relation of Top- and Seat-Angle Connections

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Abstract

In this paper, the moment-rotation relationship of the top- and seat- angle steel beam-to-column building connection is developed. In this development, the initial elastic stiffness and ultimate moment capacity of the connection are determined by a simple analytical procedure. Using the initial stiffness and the ultimate moment capacity so obtained, a three-parameter power model similar to that of Richard and Abbott (1975), was adopted here to represent the moment-rotation relationship of the connection. The analytical model is found in a good agreement with the experimental results.

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1. Introduction

In the analysis of steel frame structure it is customary to assume that the beam-to-column connections are either perfectly pinned or perfectly rigid. However, it is recognized that an actual beam-to-column connection in a building frame always possesses some flexibility in its moment-rotation behavior.

The newly published AISC/LRFD specification (1986) designates two types of construction in its provision; Type FR (Fully Restrained) construction and Type PR (Partially Restrained) construction. If the type PR construction is used, the effects of the connection flexibility on the behavior and strength of these frame structures should be considered in the analysis and design procedures. The semi-rigid joints will have a destabilizing effect on the overall stability of frame structures, since additional drift will occur in the joints as a result of the decrease in the effective stiffness of the members to which the connections are attached. Such effect has been studied by Lui and Chen (1986), and Goto and Chen (1987), among others.

The semi-rigid beam-to-column connections play a very important role in the LRFD procedure. Though several researchers have published papers discussing the connection rigidity for all connection types in steel frames, since C.R. Young performed experiments to estimate the rigidity of steel frame connections in 1917; however, the connection behavior has not been standardized yet. At present, the significance of the data base, that is the collection of experiments for beam-to-column connections conducted worldwide, is much emphasized. Nethercot (1985) conducted a literature survey for the period 1915–1985 and reviewed all steel beam-to-column connection test data and their corresponding curve representations. Goverdhan (1983), Kishi and Chen (1987) collected extensively the available test data on moment-rotation characteristics and compared the experimental results with various prediction equations.

In this paper, an analytical procedure is developed to predict the moment-rotation characteristics of the top- and seat- angle connections by determing first the initial stiffness and these ultimate moment capacity of the connections. The three-parameter elastic-plastic stress-strain model proposed previously by Richard and Abbott (1975) is then used to represent the moment-rotation behavior of the connection. The experimental results reported by Hechtman *et al.* (1947) are used here to verify the procedure.

2. Formulation of the Prediction Equation

2.1 General

A typical top- and seat- angle steel connection is shown in Fig. 1. In the design of such connections, the following assumptions are usually made: 1) the seat angle transfers only vertical

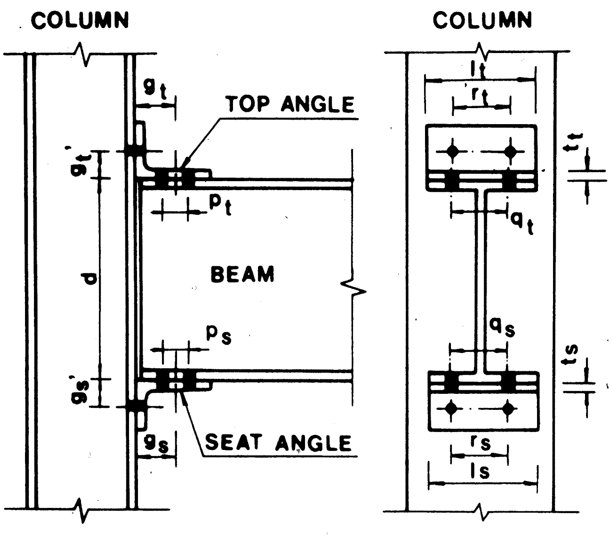

Fig. 1. Typical Top-and Seat-Angle Connection.

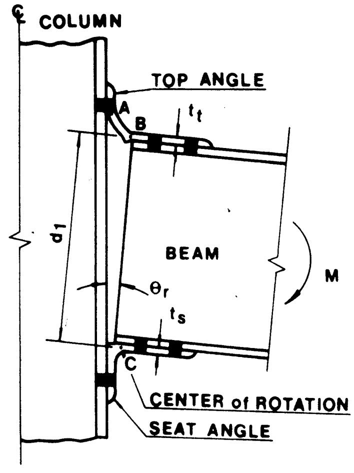

Fig. 2. Deflected Configuration of Top-and Seat-Angles at the Elastic Condition.

reaction and does not provide any restraining to the beam. 2) the top angle is provided merely for lateral stability of the beam and is not considered to carry any gravity loads. However, according to experimental results conducted by Hechtman *et al.* (1947), Altman *et al.* (1982) and Azizinamini *et al.* (1985), it has been clearly shown that this connection rotates at the critical section of the seat angle, and that the top angle provides resistance to the bending forces at the end of the beam as shown in Fig. 2. Thus, the top- and seat- angle connection belongs to the Type PR construction in the AISC/LRFD specification.

2.2 Initial Stiffness

To determine the initial elastic stiffness Rki, we assume that the top- and seat- angle connection behaves in the following manner:

1. Materials of the top and seat angles are linearly elastic and their displacements are small.
2. The center of rotation for the connection is located at the leg adjacent to the compression beam flange at the end of the beam, (Point C in Fig. 2).
3. The top angle acts as a cantilever beam in which the fixed support is assumed to be at the fastener-hole edge near the beam flange in the leg adjacent to the column face as shown in Fig. 3.
4. The resisting moment at the center of rotation is so small that it can be neglected.

Based on these assumptions and considering the shear deformation in leg of the top angle, the hori-

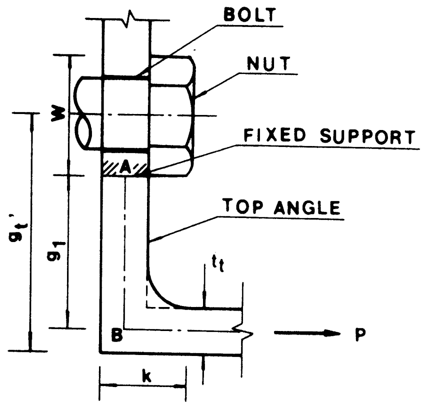

Fig. 3. Cantilever Model of the Top Angle.

zontal displacement Δ of the heel of the top angle corresponding to the beam flange force P (Fig. 3) is

$$\Delta = \frac{P\cdot(g_1)^3}{3\cdot(EI)}\left(1+\frac{0.78\cdot(t_t)^2}{(g_1)^2}\right) \tag{1}$$

in which

EI = bending stiffness of the leg adjacent to the column face,

g_1 = $g_t^3 - D/2 - t_t/2$ (Fig. 3)

D = d_b, the case using rivets as fasteners

= W, the case using bolts as fasteners

d_b = fastener's diameter

W = nut's width across flats

t_t = thickness of the top angle (Fig. 2)

g_t' = gage distance from the top angle's heel to the center of fastener holes in the leg adjacent to the column face (Fig. 1 or 3).

Here, the coefficient of shear deformation is taken as $k=6/5$ (Gere and Timoshenko, 1984).

The relationships between the horizontal displacement Δ and the end rotation θr, and the connection moment M and the beam force acting at the leg of the top angle P, are

$$\Delta = d_1\cdot\theta r \tag{2}$$

$$M = d_1\cdot p \tag{3}$$

in which

d_l = the distance between the centers of the top and bottom angles as shown in Fig. 2.

= $(d+t_t/2+t_s/2)$

where

t_s = thickness of bottom angle, and

d = the total depth of the beam section.

Substituting Eq. (1) into Eq. (3) and using Eq. (2), the bending moment M is given by:

$$M = \frac{3\cdot(EI)}{\left(1+\frac{0.78\cdot(t_t)^2}{(g_1)^2}\right)}\cdot\frac{(d_1)^2}{(g_1)^3}\theta r \tag{4}$$

from which the initial connection stiffness Rki is determined as

$$Rki = \frac{3 \cdot (EI)}{\left(1+\frac{0.78 \cdot (t_t)^2}{(g_1)^2}\right)} \cdot \frac{(d_1)^2}{(g_1)^3} \theta r \tag{5}$$

2.3 Ultimate Bending Capacity

Based on the experimental results by Altman *et al.* (1982), we assume the collapse mechanism for the top- and seat- angle connection as shown in Fig. 4. Since the distance between two plastic hinges is rather short compared with the top angle's thickness, we take into account the effect of shear force on the yielding of the material.

The work equation for the mechanism shown in Fig. 4 with the plastic moment Mp, and the shear force in the top angle leg Vp, (force P in Fig. 3) is given by

$$2 \cdot Mp \cdot \theta = Vp \cdot g_2 \cdot \theta \tag{6}$$

Using the Drucker's yield criteria (1956) for the combined bending moment Mp and shear force Vp

$$\left(\frac{Mp}{Mo}\right)+\left(\frac{Vp}{Vo}\right)^4 = 1 \tag{7}$$

in which Mo and Vo are respectively the plastic bending moment capacity and the plastic shear force capacity of the angle leg without coupling. Using the Tresca's yielding criterion, we have

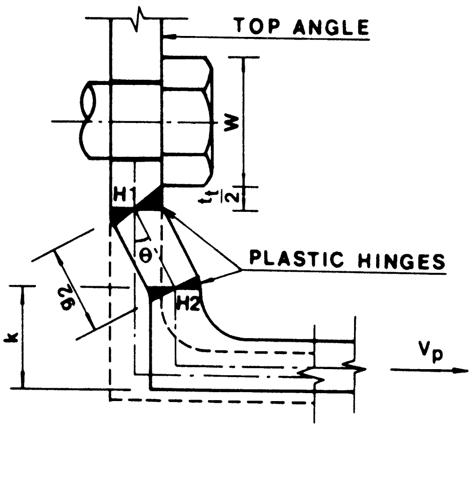

Fig. 4. Mechanism of the Top Angle at the Ultimate Condition.

$$Mo = \sigma y \cdot l_t \cdot (t_t)^2/4 \tag{8}$$

$$Vo = \sigma y \cdot l_t \cdot t_t/2 \tag{9}$$

in which σy is the yield stress of the top angle. Substituting Eqs. (6), (8) and (9) into Eq. (7) and rearranging, (Vp/Vo) is obtained as

$$(\frac{Vp}{Vo})^4 + \frac{g_2}{t_t}(\frac{Vp}{Vo}) - 1 = 0 \tag{10}$$

The ultimate shear strength Vp can be determined by solving Eq. (10).

Taking the moment with respect to the center of rotation in the leg adjacent to the compression beam flange (point C in Fig. 2), the ultimate moment capacity Mu is

$$Mu = Mos + Mp + Vp \cdot d_2 \tag{11}$$

in which

Mos = plastic moment capacity at point C of the seat angle in Fig. 2.

$$= \sigma y \cdot l_s \cdot (t_s)^2/4 \tag{12}$$

Mp = plastic moment capacity at point H_2 of the top angle

$$d_2 = d + t_s/2 + k \tag{13}$$

k = distance from the top angle's heel to the toe of fillet as shown in Fig. 4.

2.4 Modeling the M$-\theta$r Relationship

Using the initial connection-stiffness Rki and the ultimate moment capacity Mu of the connection, the moment rotation (M$-\theta$r) relationship can be represented adequately by the power model

$$M = \frac{R_1 \cdot \theta r}{\{1 + (\theta r/\theta o)^n\}^{1/n}} + Rkp \cdot \theta r \tag{14}$$

in which

Rkp = plastic connection stiffness

R_l = $Rki - Rkp$

θo = a reference plastic rotation

n = shape parameter.

The connection stiffness Rk in Eq. (14) is

$$Rk = \frac{dM}{d\theta r} = \frac{R_l}{\{1+(\theta r/\theta o)^n\}^{(n+1)/n}} + Rkp \tag{15}$$

For an elastic-perfectly plastic moment-rotation curve, $Rkp=0$, Equations (14) and (15) reduce to

$$M = \frac{Rki \cdot \theta r}{\{1+(\theta r/\theta o)^n\}^{1/n}} \tag{16}$$

$$Rk = \frac{dM}{d\theta r} = \frac{Rki}{\{1+(\theta r/\theta o)^n\}^{(n+1)/n}} \tag{17}$$

in which $\theta o = Mu/Rki$. Equations (16) and (17) represent the M-θr relationship and the stiffness of the top- and seat- angle connections, respectively. The power model was originaly proposed by Richard (1961) and later applied by Goldberg and Richard (1963).

This power model is an effective tool for designers to execute the second-order nonlinear structural analysis quickly and accurately. This is because the connection stiffness can be determined directly from Eq. (16) without iteration. For example, the equation for θr in Eq. (16) can be represented as

$$\theta r = \frac{M}{Rki \cdot \{1-(M/Mu)^n\}^{1/n}} \tag{18}$$

3. Experimental Verifications

To verify the power model proposed here, for representing the M-θr curve of the top- and seat- angle connections, the tests by Hechtman *et al.* (1947) are used. Rivets are used for fasteners in these tests. The comparison on each level of ultimate moment capacity is done. The results are shown in Fig. 5. to 8. In these figures, the experimental results are compared with the analytical power model, the polynominal model proposed by Frye-Morris (1978) and the modified exponential model as the curve-fitting method introduced by Kishi-Chen (1987). Selecting a suitable value for the shape parameter n, the results obtained by the power model agree rather well with the experimental results similar to that of the polynominal and modified exponential models. It can therefore be concluded here that the proposed power model represents adequately the moment-rotation behavior of the top- and seat- angle connections.

4. Conclusions

In this paper, the moment-rotation relationships of the top- and seat- angle connections are developed. The initial stiffness and the ultimate moment capacity of the connections are determined

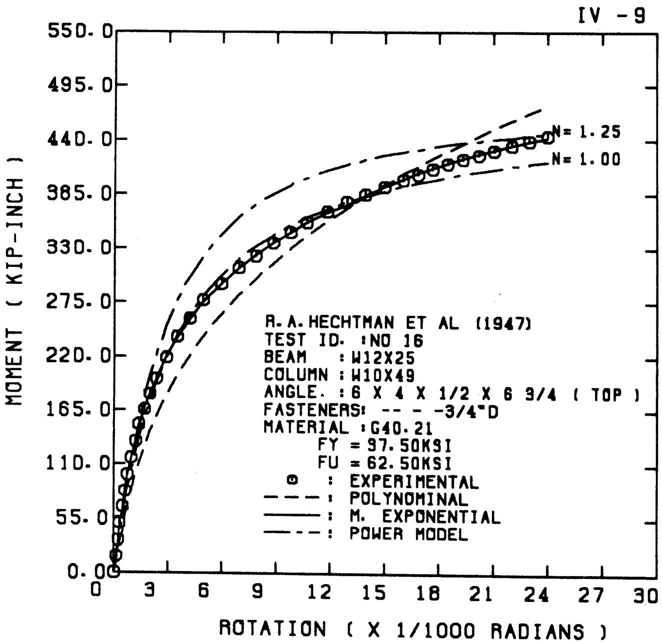

Fig. 5. Comparison Between Proposed Power Model's and Experimental Results (No. 1)

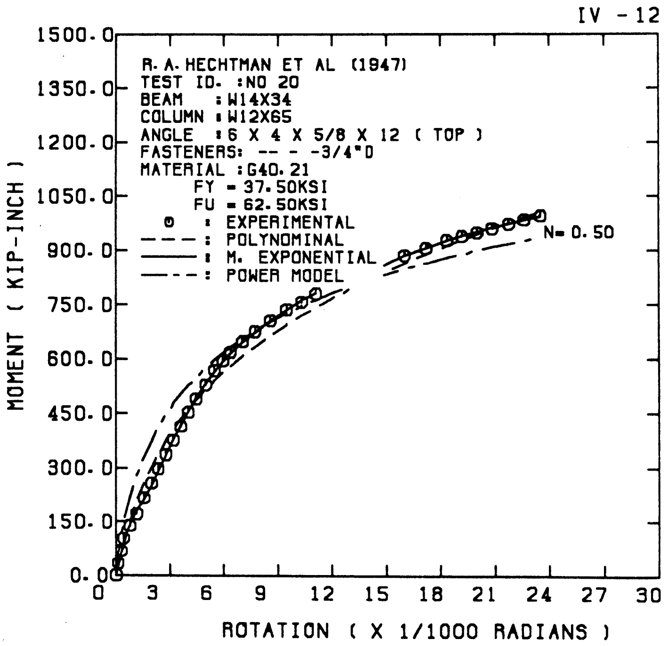

Fig. 6. Comparison Between Proposed Power Model's and Experimental Results (No. 2)

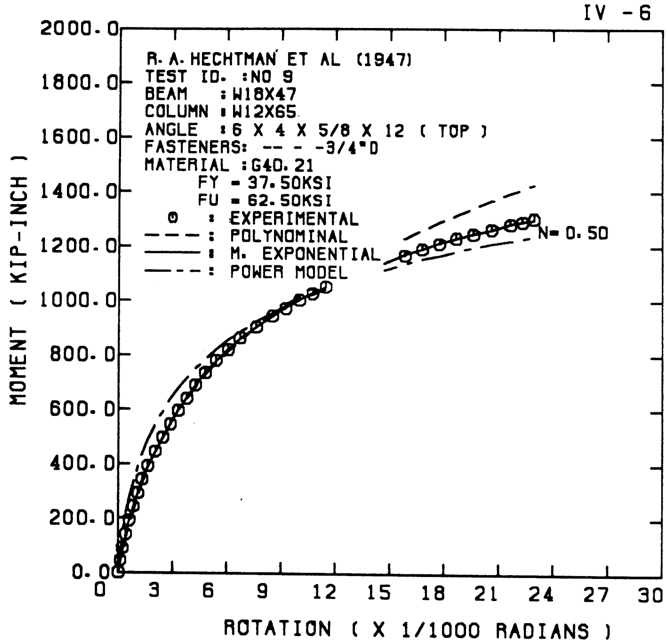

Fig. 7. Comparison Between Proposed Power Model's and Experimental Results (No. 3)

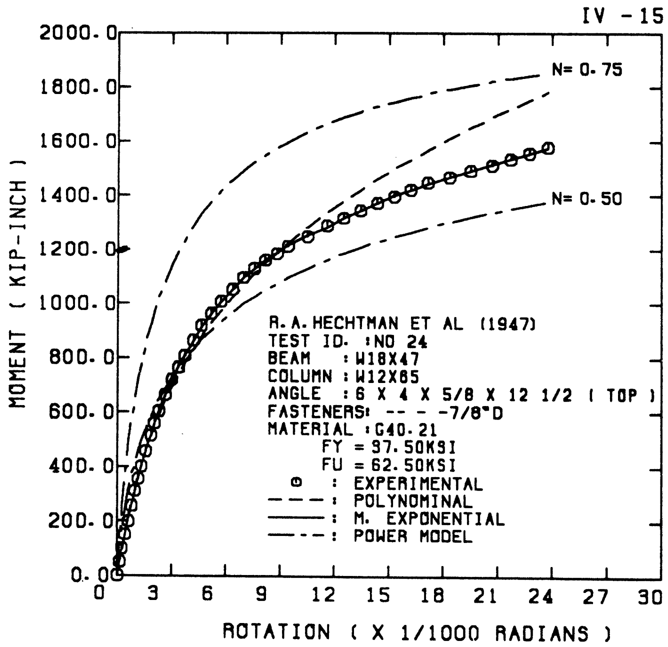

Fig. 8. Comparison Between Proposed Power Model's and Experimentel Results (No. 4)

analytically and used as two of the three parameters in the proposed power model. The proposed power model is found in a good agreement with available results. The power model can be easily implemented in a second-order nonlinear structural analysis.

References

AISC (1986), Load and Resistance Factor Design Manual of Steel Construction, 1st. Ed., AISC, Chicago.

Altman, W. G., Jr., Azizinamini, A., Bradburn, J. H. and Radziminski, J. B. (1982), Moment-Rotation Characteristics of Semi-Rigid Steel Beam-Column connections, Structural Research Studies, Department of Civil Engineering, University of South Carolina, Columbia, South Carolina.

Azizinamini, A., Bradburn, J. B., and Radziminski, J. B. (1985), Static and Cyclic Behavior of Semi-Rigid Steel Beam-Column Connections, Dept. of C. E., U. of South Carolina.

Drucker, D. C. (1956), The Effect of Shear on the Plastic Bending of Beams, Journal of Applied Mechanics, December, 509–514.

Gere, J. M. and Timoshenko, S. P. (1984), Mechanics of Materials, 2nd Ed., Wadsworth, Belmont, CA., 660–666.

Frye, M. J. and Morris, G. A., (1975), Analysis of Flexibily Connected Steel Frames, Canadian Journal of Civil Engineering, Vol. 2, 280-291.

Goldberg, J. E. and Richard, R. M. (1963), Analysis of Nonlinear Structures, Journal of the Structural Division, ASCE, Vol. 89, No. ST4, 333–351.

Goto, Y. and Chen, W. F. (1987), On Second Order Elastic Analysis for Design, Journal of Structural Engineering, ASCE, Vol. 113, New York, 1987.

Goverdhan, A. V. (1983), A Collection of Experimental Moment-Rotation Curves and Evaluation of Prediction Equations for Semi-Rigid Connections, Thesis Presented to Vanderbilt University, Nashville, Tennessee, in Partial Fulfillment of the Requirements for the Degree of Master of Science.

Hechtman, R. A. and Johnston, B. G. (1947), Riveted Semi-Rigid Beam-to-Column Building Connections, Progress Report Number 1, AISC Research at Lehigh University.

Kishi, N. and Chen, W. F. (1986), Data Base of Steel Beam-to-Column Connections, CE–STR–86–26, School of Civil Engineering, Purdue University, West Lafayette, IN. 47907, two Volumes, 653 pages.

Kishi, N. and Chen, W. F. (1987), Data Base of Steel Beam-to-Column Connections, AISC Engineering Journal, Vol. 24, Chicago.

Lui, E. M. and Chen, W. F. (1986), Analysis and Behavior of Flexibly-Jointed Frames, Engineering Structures, Butterworth, U. K., Vol. 8, No. 2, 107–115.

Nethercot, D. A. (1985), Steel Beam-to-Column Connections — a Review of Test Data, CIRIA, London.

Richard, R. M. (1961), A Study of Structural Systems Having Conservative Non-linearity, Thesis Presented to Purdue University, West Lafayette, IN., in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy.

Richard, R. M. and Abbott, B. J. (1975), Versatile Elastic-Plastic Stress-Strain Formula, Journal of the Engineering Mechanics Division, ASCE, Vol. 101, No. EM4, 511–515.

Young, C. R. (1917), Bulletin No. 104, Engineering Experiment Station, University of Illinois, Urbana, IL.