A Procedure for the Analysis of Long-Team Deflection of Reinforced Concrete Members and Its Adaptability

A Procedure for the Analysis of Long-Term Deflection of Reinforced Concrete Members and Its Adaptability

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Abstract

For deformation analysis of partially cracked floor members of reinforced concrete, a procedure with some modifications to our earlier system is proposed.

Instead of its having relied on that concept of average for the oft-cited effective member stiffness long used thus far at least in ancillary form by the ACI Code, the present system can afford to account for the detail of steel arrangement along a member by treating all its sections assumed attendant on finite difference subdivision for beam analysis.

In a consistent attempt, the whole lengthwise sectional variation is considered as well in the long-time case of analysis. Then, a long standing notion of increased modular ratio is adopted, alternatively to the ACI's time-dependent multiplier which is given also in average form by way of direct inclusion of the effect of the creep resistant compression steel.

Relative adequacy of the proposed procedure is discussed in comparison of many cases of earlier test results in the literature with deflection estimates by our method and commonly available code methods.

1. Introduction

Our earlier proposed procedure[1)] for predictive deflection analysis of an r/c floor structure as a whole inclusive of its supporting frame, while taking account of the time-dependency of concrete, depends for post-cracking member stiffness on the effective moment of inertia[2)], long used so far in the ACI[3)] or certain major structural design codes, as well as on the auxiliary assumptions to facilitate its practically simplified estimation.

These include assumed average uniform effective stiffness of a linear member over the whole span and considered effects of reinforcement only at mid-span sections for calculating deflections, hence the result being not satisfactorily responsible for the overall crack distribution and the entire reinforcement detail of the member.

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At this time we presents an improved system of procedure being of use for a member with an arbitrary crack distribution or a so-called variable cross-sectioned structure by taking advantage of its finite difference treatment. Initially we explain the fundamentals of the currently adopted procedure and the calculation detail based on iterated elastic analysis to result in immediate deflections due to iteratively modified sectional properties; followed by our formulating a calculation system for time-dependent additional deflection at an arbitrary member age, resorting to modified concrete properties by a traditional method for that long-time case.

Further, we review the degree of adaptability of our procedure by employing available long-time test results on beam or one-way slab models. And we lastly make a systematic calculation of the deflection at infinite years of concrete age (terminative deflection) for a number of calculation model beams with various combinations of sectional size, supporting or other conditions, subsequently to discuss the results respecting the notable inclinations of long-time deflection and the serviceability limits for beam members having customarily practicable demensions.

2. Analytical System

2.1 Formulations for Elastic Deflection

For its succeeding development in difference form, now introduced is a known relation between deflection y of a variably cross-sectioned beam and distance x from its left end to the considered section:[4)]

$$\frac{d^2}{dx^2}\left(I_x\frac{d^2y}{d^2x}\right)=\frac{q_x}{E} \tag{1}$$

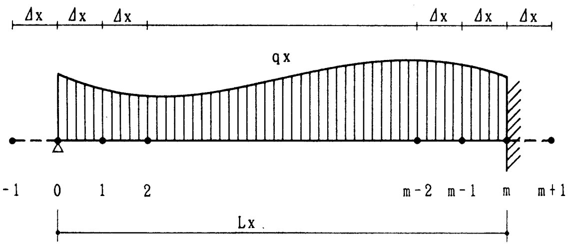

Fig. 1. Difference Subdivision with Numbering System

were, as is well known, $d^2M_x/dx^2 = -q_x$ and $d^2y/dx^2 = -M_x/EI_x$ with M_x = bending moment, q_x = intensity of load of any distribution, E = elastic modulus and I_x = moment of intertia, all respecting this section.

When a beam member is subdivided into m equal parts shown in Figs. 1 and 2 difference expressions for the above bending moment and deflection at section i respectively become:

$$M_i=-EI_i(y_{i-1}-2y_i+y_{i+1})/\Delta x^2 \tag{2}$$

$$k_{i-1}y_{i-2}-2(k_{i-1}+k_i)y_i+(k_{i-1}+4k_i+k_{i+1})y_i-2(k_i+k_{i+1})y_{i+1}+k_{i+1}y_{i+2}=q_i\Delta x^4/EI_0 \tag{3}$$

where : $k_i = I_i/I_0$ = stiffness ratio for section i; I_0 = moment of inertia for reference section and I_i = that for section i.

With end conditions for a simple beam, $y_0 = 0$ and $y_m = 0$, both difference expressions for $M = dy^2/d^2x = 0$ at its end sections are written as : $(y_{-1} - 2y_0 + y_1)/\Delta x^2 = 0$ as well as $(y_{m-1} - 2y_m + y_{m+4})/\Delta x^2 = 0$, hence $y_{-1} = -y_1$ and $y_{m+1} = -y_{m-1}$.

For a both-end fixed beam the difference expressions for $y = 0$ and $dy/dx = 0$ at both ends are respectively $y_0 = 0$ or $y_m = 0$ and $(y_1 - y_{-1})/2\Delta x = 0$ or $(y_{m-1} - y_{m+1})/2\Delta x = 0$; naturally $y_{-1} = y_1$ and $y_{m+1} = y_{m-1}$.

Assuming the stiffness ratio for any section i as $k_{i(i=1...m)} = 1.0$ and solving the resulting simultaneous equation system for deflections $y_{i(i=1,...m-1)}$, obtained by using Eq. (3) and the boundary conditions above, gives an elastic deflection Δ_e at each section.

2.2 Immediate Deflection

We use the following well-documented equation for member stiffness, proposed by D.E. Branson[2] and adopted in a similar form by the design code initially referred to, in which M_a is currently obtained as M_i of Eq. (2) for each section, i, by substituting in the equation the values of suffixed y's i.e., the deflections obtained by solving Eq. (3).

$$I_e = (M_{cr}/M_a)^4 I_{g0} + [1 - (M_{cr}/M_a)^4] I_{cr} \tag{4}$$

provided $I_e = I_{g0}$ for $|M_a| < M_{cr}$

where : M_a = bending moment, M_{cr} = cracking moments both acting on section i, and of this section, I_e = effective moment of inertia, I_{g0} = moment of inertia of gross concrete section ignoring the steel and I_{cr} = that of cracked transformed section.

Involving partial discretion in certain assumptions if immaterial in most cases, specific calculations we depend on for those properties will be affirmed next: i.e., for a rectangular section:

$$I_{cr} = b(cd)^3/12 + bcd(cd/2)^2 + nA'_s(cd - d')^2 + nA_s(d - cd)^2 \tag{5}$$

where: b = beam width, c = relative depth of neutral axis, d = effective depth of section, d′ = distance from compression face to centroid of compression steel, n = modular ratio, A_s = tension steel area, A'_s = compression steel area and, when expressed in terms of steel ratios ρ ($= A_s/bd$) and ρ' ($= A'_s/bd$):

$$c = \sqrt{2n(\rho + \rho d'/d) + n^2(\rho + \rho')^2} - n(\rho + \rho') \tag{6}$$

The effective width, B, required in case of counting in the effect of slab panels in positive bending regions is provided by the corresponding Japanese r/c code equations. In this case I_{cr} for a tee-section is as follows when the neutral axis, calculated by Eq. (6), on putting $\rho = A_s/Bd$ and $\rho' = A'_s/Bd$, is inside the flange:

$$I_{cr} = B(cd)^3/12 + Bcd(cd/2)^2 + nA'_s(cd-d')^2 + nA_s(d-cd)^2 \quad (7)$$

When the neutral axis is inside the web, with t flange thickness

$$I_{cr} = Bt^3/12 + Bt(cd-t/2)^2 + nA'_s(cd-d')^2 + nA_s(d-cd)^2 \quad (8)$$

$$\text{where } c = [t^2/d^2 + 2n(\rho + \rho' d'/d)]/2[t/d + n(\rho + \rho')] \quad (9)$$

On the other hand, for member sections where $|M_a| < M_{cr}$, I_g ($= I_e$) is as follows, taking account of the reinforcement and referring to Fig. 2 for sectional dimensioning:

$$I_g = bh^3/12 + bh(h/2-cd)^2 + (B-t)t^3/12 + (B-b)t(cd - t/2)^2 + nA_s(d-cd)^2 + nA_s'(cd-d')^2 \quad (10)$$

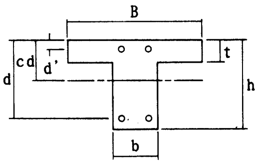

Fig. 2. Sectional Assumptions

$$c = [t^2(1-b/B)/d^2 + bh^2/Bb^2 + 2n(\rho + \rho' d'/d)] / \{2[t + b(h-t)/B]/d + 2n(\rho + \rho')\} \quad (11)$$

Eqs. (10) and (11) for a tee-section are used with t = 0 and B = b when applied to a rectangular section.

Using the I_e obtained by Eqs. (4) through (11) in the course of iterating the calculation explained in the preceding section results in the immediate deflection Δ_i of a beam member, including the effect of concrete cracking.

2.3 Additional Deflection due to Bond-Slip of the Steel

The additional deflection Δ_s due to the bond-slip of a reinforcement anchorage at a member end section can be of relative significance in case of a slab with a larger span/depth ratio, especially when its end top reinforcement is liable to be lowered during construction work, while in beam the bond-slip effect is usually negligibly small[1]. Accordingly it will not be considered on beams but on slabs alone.

An angle of rotation θ due to the bond-slip by an amount u in the top steel anchorage at encastered ends of a floor slab may be estimated by the following equation on assuming that the axis of that slab rotation is the neutral axis obtained for a cracked section by Eq. (9).

$$\theta = u/(d - cd) \tag{12}$$

The relevant deflection Δ_s is calculated from the result of solving a slab panel restrained with a forced rotation θ along all its edges, i.e., depending on such a simultaneous set of difference equations in terms of Δ_s as can be formulated by putting $q_i = 0$ in Eq. (3) and eliminating the terms concerning external points of subdivision by making use of either of the relations: $(y_1 - y_{-1})/2\Delta x = \theta$ and $(y_{m-1} - y_{m+1})/2\Delta x = \theta$.

2. 4 Long-Time Deflection

Use is made of principal ideas of the increased-n method, referred to as in Ref. 1), whose concepts are given in publications of Large[5)], Branson[2)], Mayer[6)], Beeby[7)] and others; a method based on an introduced nominal elastic modulus called effective modulus thereby to account for the effect of time-dependent creep strain on a long-time deflection Δ_{i+cp}, earlier being a sum of immediate portion and that affected by creep, which may now be provided in one through a procedure similar to the calculation of elastic or immediate deflections.

In this case, supposed beforehand is a creep coefficient of the concrete, ϕ_t, dependent on concrete age at the start of loading, loading period, atomospheric and other conditions; together with an effective elastic modulus E_{ct} of the concrete and modular ratio n_t of the steel; both put in such forms as:

$$E_{ct} = E_c/(1+\phi_t) \tag{13}$$ $$n_t = n(1+\phi_t) \tag{14}$$

Then we calculate again I_{cr} and I_g respectively of Eqs. (5) and (10). And from the I_e value now obtained for each section i on assuming moment M_i is constant the corresponding stiffness ratio is calculated as k_i of Eq. (3) when putting $E = E_c = E_{ct}$ in it, to decide as a result y_i therein for the required solution Δ_{i+cp}.

Next, we calculate $\Delta_{i+cp+sh}$, a long time deflection so far separated into portions, Δ_{cp} and Δ_{sh}, respectively owing to creep and shrinkage, in essentially the same way as the derivation of Δ_{i+cp}, currently in directly combined form. The calculation of effective elastic modulus E'_{ct} and modular ratio n'_t of reinforcing bar is here made by the following equation proposed by Yu and Winter[8)].

$$E'_{ct} = E_c/(1+0.93\alpha E_c Y t^{1/3}/a^{1/2.5}) \tag{15}$$ $$n'_t = E_s/E'_{ct} \tag{16}$$

where: E_c = elastic modulus of concrete, Y = multiplying factor due to years of duration of loading, t = days of duration of loading; provided t is assumed to be 365 in case of t over that number, a = age in days of loaded member at the start of loading and α coefficient by which to include the effect of concrete slump (or α =1.25 for a customary slump of 1.5 or so)[9] and E_s = elastic modulus of reinforcing bars.

The second term of the denominator of Eq. (15) is identical with Shank's experimental equation[5], regarding both creep and shrinkage strain of the concrete, being given by him specific Y-values in year unit, which may alternatively be described by the following polinomial as a result of a pertinent regression analysis thereof[3]; i.e., with N = years of duration of loading:

$$Y = 1.259+0.0592(N-3)-0.0236(N-3)^2+0.058(N-3)^3 \tag{17}$$

3. Procedure

There follows the necessary steps to be taken for the present beam or one-way slab deflection analysis in consideration of the cracking and time-dependency of the concrete and the bond-slip of the anchored steel, i.e. :

1) subdivide member lengthways, followed by its elastic analysis, with the difference method in use, to result in elastic deflection Δ_e.

2) from that elastic solution work out maximal bending moment (construction load in the majority of cases), and therefrom effective stiffness E_cI_e, also E_cI_g, for each member section, assuming concrete elastic modulus and modular ratio respectively to be E_c and n;

3) perform deflection analysis for the member with bending stiffness E_cI_e either for cracked regions or E_cI_g including steel effects for those uncracked, to result in M_1 under the maximal load;

4) using the end moment given in above step (3), in case that end sections prove to be cracked, obtain additional deflection Δ_s and M_2, due to slipped anchorage, and add that additional moment to the above M_1 subsequently to modify the member stiffness in the cracked region;

5) iterate previous steps (3) and (4) before convergence of the stiffness value at each section of the member;

6) depending on member stiffness modified by using effective elastic modulus E_{ct} and effective modular ratio n_t, carry out analysis of step (3) to obtain $\Delta_t = \Delta_{i+cp}$ under long-time loads as sustained for t days of concrete age, provided that the n calculating I_e counts on $M_1 + M_2$ above;

7) use E'_{ct} and n'_t in place of E_{ct} and n_t in step (6) so as to provide $\Delta_{i+cp+sh}$ likewise and

8) calculate total long-time deflection $\Delta_s + \Delta_t$.

Table 1. Compared Earlier Deflection Measurements with Corresponding Predictions; on R/C Beams and One-Way Strips from Available Long-Time Loading Test Reports, at Home and Abroad.

Workers with Ref. Nos. &	Desig-nation	Age of Concrt. in Days at :		Properties (kg/cm²)		Span	Sectional Dimensions & Reinforcement					Impsd. Loads	Deflections (cm)				Rel. Values	Slumps of Concrete
													Measured		Predicted			
Types of Model Structures	of Models	Loading	Measurmnt.	F_c	E_c x10^5	L_x m	B cm	b cm	t cm	h cm	A'_s (2)	q_x (3) kg/m	Δ_i	Δ_t	Δ_i	Δ_t	$\Delta_{i\,calc}/\Delta_{i\,test}$	at Placing (cm)
Washa-Fluck	A1, A4	14	900	255	2.08	6.1	20.3	20.3	-	30.5	A_s	563	1.35	2.36	1.50	2.51	1.06	15.2
(5)(1)	A2, A5	14	900	255	2.08	6.1	20.3	20.3	-	30.5	$A_s/2$	563	1.57	3.23	1.57	3.23	1.00	
Simple Beams	A3, A6	14	900	255	2.08	6.1	20.3	20.3	-	20.3	-	563	1.70	4.47	1.65	5.88	1.32	
(Rectangular)	B1, B4	14	900	212	1.91	6.1	15.2	15.2	-	20.3	A_s	159	2.34	5.11	2.57	4.89	0.96	
	B2, B5	14	900	212	1.91	6.1	15.2	15.2	-	20.3	$A_s/2$	159	2.49	6.50	2.63	5.95	0.92	
	B3, B6	14	900	212	1.91	6.1	15.2	15.2	-	12.7	-	159	2.64	8.64	2.70	9.58	1.11	
	C1, C4	14	900	208	1.88	6.3	30.5	30.5	-	12.7	A_s	122	4.01	8.00	4.29	7.48	0.94	
	C2, C5	14	900	208	1.88	6.3	30.5	30.5	-	12.7	$A_s/2$	122	4.34	10.06	4.42	9.27	0.92	
	C3, C6	14	900	208	1.88	6.3	30.5	30.5	-	12.7	-	122	4.78	14.07	4.61	16.00	1.14	
	D1, D4	14	900	205	1.85	3.8	30.5	30.5	-	12.7	A_s	341	1.19	2.77	1.57	2.72	0.93	
	D2, D5	14	900	205	1.85	3.8	30.5	30.5	-	12.7	$A_s/2$	341	1.42	3.30	1.63	3.37	1.02	
	D3, D6	14	900	226	1.92	3.8	30.5	30.5	-	12.7	-	341	1.78	4.45	1.65	5.83	1.31	
	E1, E4	14	900	210	1.88	5.3	30.5	30.5	-	7.6	A_s	57	5.94	12.40	5.23	10.30	0.83	
	E2, E5	14	900	210	1.88	5.3	30.5	30.5	-	7.6	$A_s/2$	57	5.59	12.88	5.39	12.40	0.96	
	E3, E6	14	900	210	1.88	5.3	30.5	30.5	-	7.6	-	57	6.30	18.49	5.53	19.35	1.05	
Yu-Winter	A	30	180	259	1.80	6.1	30.5	15.2	6.4	30.5	-	655	3.40	6.73	3.22	5.66	0.84	3.4- 8.9
(3)	B	29	180	273	1.76	6.1	30.5	15.2	6.4	30.5	$A_s/2$	656	3.14	5.66	3.18	4.89	0.86	
Simple Beams	C	28	180	248	1.76	6.1	30.5	15.2	6.4	30.5	A_s	653	3.02	5.18	3.15	4.44	0.86	
(Tee)	D	31	180	259	1.80	6.1	61.0	15.2	6.4	30.5	-	1196	3.23	6.71	3.51	5.90	0.88	
	E	29	180	299	1.84	4.3	30.5	15.2	6.4	30.5	-	1253	1.30	2.92	1.56	2.83	0.97	
	F	34	180	299	1.84	6.1	30.5	15.2	5.1	20.3	-	387	5.59	10.03	5.91	10.80	1.03	
Washa-Fluck	†X1, X4	14	900	227	1.99	6.1	15.2	15.2	-	20.3	A_s	283	1.42	2.90	1.70	3.20	1.01	10.2-15.2
(6)	†X2, X5	14	900	227	1.99	6.1	15.2	15.2	-	20.3	$A_s/2$	283	1.45	3.23	1.72	3.60	1.11	
Two-Span	†X3, X6	14	900	227	1.99	6.1	15.2	15.2	-	20.3	-	283	1.57	3.78	1.75	4.74	1.25	
Continuous	†Y1, Y4	14	900	236	2.04	6.3	30.5	30.5	-	12.7	A_s	217	2.26	4.00	2.70	4.73	1.03	
(Rectangular)	†Y2, Y5	14	900	236	2.04	6.3	30.5	30.5	-	12.7	$A_s/2$	217	2.36	4.98	2.75	5.35	1.07	
	†Y3, Y6	14	900	236	2.04	6.3	30.5	30.5	-	12.7	-	217	2.54	5.99	2.81	7.28	1.22	
	†Z1, Z4	14	900	232	2.10	5.3	30.5	30.5	-	7.6	A_s	101	2.64	5.89	3.48	6.60	1.12	
	†Z2, Z5	14	900	232	2.10	5.3	30.5	30.5	-	7.6	$A_s/2$	101	2.87	6.78	3.52	7.38	1.09	
	†Z3, Z6	14	900	232	2.10	5.3	30.3	30.3	-	7.6	-	101	3.05	7.93	3.54	9.54	1.20	
Iwahara	SL-1	35	140	278	2.96	3.0	40.0	40.0	-	13.0	-	297	0.37	1.70	0.37	1.62	0.95	12.5(SL)
(7), (8)	SL-2	35	140	278	2.96	3.0	40.0	40.0	-	13.0	-	140	0.02	0.55	0.07	0.45	0.82	19.2(SN)
One-Way	SL-3	35	140	278	2.96	3.0	40.0	40.0	-	13.0	-	297	0.20	0.90	0.32	1.28	1.42	
Slab Strips	SL-4	35	140	204	2.96	3.0	40.0	40.0	-	13.0	-	297	0.68	1.92	0.41	2.06	1.07	
	*SN-1	30	856	204	2.32	4.0	45.0	45.0	-	13.7	-	430	0.14	2.22	0.91	1.94	0.87	
	*SN-2	30	856	204	2.32	4.0	45.0	45.0	-	13.5	-	168	0.37	-	0.53	4.00	-	
	*SN-3	30	856	204	2.32	4.0	45.0	45.0	-	13.5	-	429	0.16	1.57	0.71	1.52	0.97	
	*SN-4	30	856	204	2.32	4.0	45.0	45.0	-	13.3	-	427	0.17	1.78	1.51	2.92	1.64	
Yamamoto	S3	56	350	306	2.42	4.0	40.0	40.0	-	13.0	-	195	-	3.25	0.83	2.78	0.86	18.0
(9)	S4	56	350	306	2.42	4.0	40.0	40.0	-	13.0	-	195	-	3.05	0.83	2.78	0.91	
〃	*S1	56	350	306	2.42	5.3	40.0	40.0	-	13.0	-	231	-	2.30	1.49	2.61	1.13	
	*S2	56	350	306	2.42	5.3	40.0	40.0	-	13.0	-	231	-	2.00	1.49	2.61	1.31	
Matsuzaki	*A1	30	350	163	1.85	4.2	100.	100.	-	12.0	-	576	0.15	1.60	0.76	1.61	1.00	
(10)	*A2	30	350	163	1.85	4.2	100.	100.	-	12.0	-	468	0.10	1.16	0.47	1.23	1.06	
〃	*A3	30	350	163	1.85	4.2	100.	100.	-	12.0	-	288	0.06	0.82	0.10	0.54	0.66	
Komori (11)	*S1-A	56	90	205	1.62	5.3	25.0	25.0	-	10.0	-	60	3.10	6.30	3.38	5.61	0.89	
〃	*S1-B	56	90	205	1.62	5.3	25.0	25.0	-	10.0	-	60	1.30	4.60	3.16	5.46	1.19	

Note: (1) Bracketed are Numbers of Reference; * Asterisked being Both-End Fixed Structures, with Spans Measured at Centers of Supports;
† Daggered for One-End-Supported, Other Fixed Cases;
(2), (3) Small Numerals (2), (3) refer respectively to Mid-Span Compr. Steel and Inclusion of Self-Weight.

4. Review of Calculation Results

Adopted to be set against corresponding calculations for immediate and long-time deflections, afforded by our procedure, are eight case of sustained loading test results, i.e., deflection measurements on either r/c beams or one-way slab strips under uniformly distributed loads, conducted by

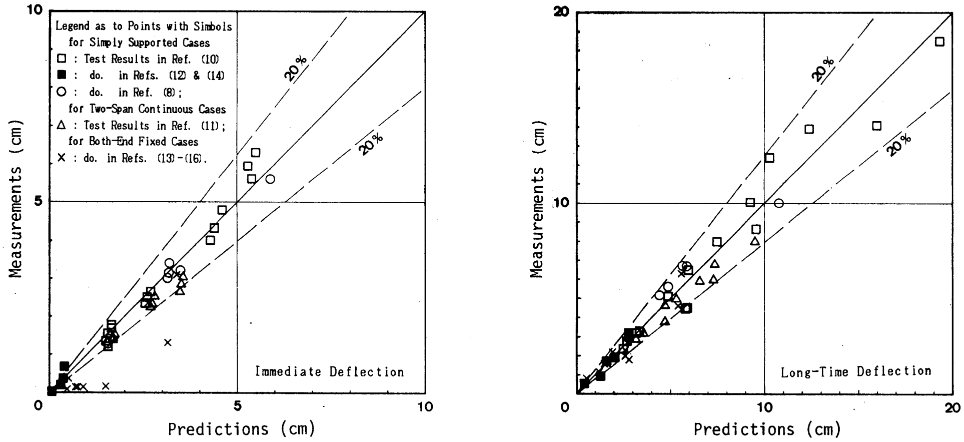

Fig. 3. Examined Degrees of Agreement between Measured and Predicted Deflections

Washa and Fluck,[10),11)] Yu and Winter,[8)] Iwahara,[12),13)] Yamamoto,[14)] Matsuzaki[15)] and Komori.[16)]

Of these considered test models: 62 were simply supported, consisting of six tee-sectioned and all others rectangular; 18 fixed at one end with the other supported; and the rest of six fixed at both member ends.

The check calculation supposed a concrete strength of $1.8\sqrt{F_c}$ (with F_c = compressive concrete

Table 2. Specific Assumptions for the Authors' Calculation Models

	Girders (One-and Two-Beam-Supporting Types)	Lateral Beams	Note
Span L_x(cm)	450,600,750,800	600,750,900	Flange (Slab) Thickness=15 cm
Lateral Span L_y (cm)	200,300,400	450,600,750,900	Beam Sections:
Member Width	25,30,35 (L_x=450) 30,35,40 (L_x=600,750) 35,40,45 (L_x=900)	30,35,40 (L_x=600) 35,40,45 (L_x=750) 40,45,50 (L_x=900)	25×40 (L_x=450), 35×60 (L_x=750) 30×50 (L_x=600), 40×70 (L_x=900)
Material Properties	Concrete : Strength F_c 210kg/cm^2 Concrete : Elast. Modulus E_c 210000kg/cm^2 Concrete : Effective Mod. E_{ct} 26600kg/cm^2 Steel Elast. Mod. E_s 2100000kg/cm^2		Concrete Modulus of Rupture $=1.8\sqrt{Fc}$ kg/cm^2
Loads	Materials for Finish 80kg/m^2 Design Live Load 300kg/m^2 Long-Time Imposed Load 100kg/m^2		Construction Load=2.1 times R/C Self-Weight

strength) as a rule, provided an alternative of 1.2 is used to the customary 1.8 for domestic cases with relatively small spans and limited degrees of cracking; with a difference subdivision of a span numbering ten. The reported deflection measurements of each test model, along with its overall and sectional dimensioning, are summarized in Table 1 in comparison with our corresponding predictive calculations. Connectedly plotted in Fig. 3 are the above results, i.e., immediate as well as long-time measurements against associated calculations, respectively as abscissas and ordinates. As for each such coordinate pair, while certain immediate deflections in the both-end fixed cases show a considerable difference of a predicted value from its measured correspondent, in most of the other cases sufficiently good agreements are generally seen as to immediate as well as long-time values.

Also, we attempted comparisons in the same context by separately using the methods of Koyanagi et al.[15], about to appear in Appendix 13.2 of the latest revision of the r/c design code by the Architectural Institute of Japan, and the two code methods each from the relevant publications by the European Concrete Committee[20], and the British Standard Institution[21]. The result shows a tolerably better adaptation of the current procedure than the others in the majority of the cases of tested structure models reproduced herein.[18]

5. Final Deflections

5.1 Outline of Calculation

The trial calculation models here introduced include beams of three types, i.e., cases simply supported; supported at one end with the other fixed; and fixed at both ends. The others comprise both-end-fixed girders, with one or two lateral beams across each of them, having span L_x, effective width L_y, web width b, flange (or slab) depth t and associated material properties as well as load conditions as assumed in Table 2. There, specifically, the whole depth of the girder is stepped down from the largest $0.1L_x + 20$ cm at 5cm intervals, within a feasible range from single to double arrangement of reinforcement by using the main and the lateral deformed steel of respective D19 and D10 of nominal diameter.

5.2 Deflection Limits

Each deflection thus obtained and divided by the corresponding span length, now simply called final deflection ratio, is rearranged with the associated beam depth/span ratio used as index; which effort leads to Fig. 4 for both discussed beams and girders.

For the reason that our throughout assuming respective end and mid-span sections to be rectangular and tee, though additionally affirmed here, the practicable lower limits of beam depth are considerably low, necessarily causing sharp increases in final deflection factors for depth/span ratios less than 0.0075. Relatedly, assuming say 0.002 of a serviceablity limit deflection factor results in permissible depth/span ratios ensured if they exceed 0.08 even for a simple beams. In the other case of end conditions, for beams and girders, capable of reinforcement in the present concerns, final deflection factors seldom exceed 0,0015.

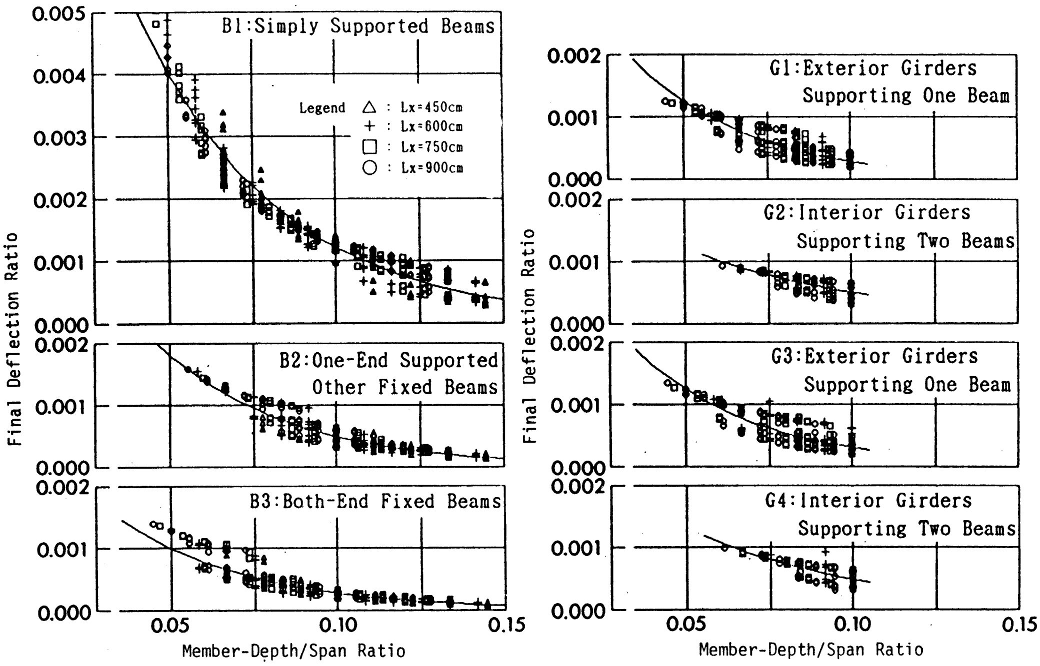

Fig. 4. Member-Depth/Span Ratios vs. Final Deflection Ratios

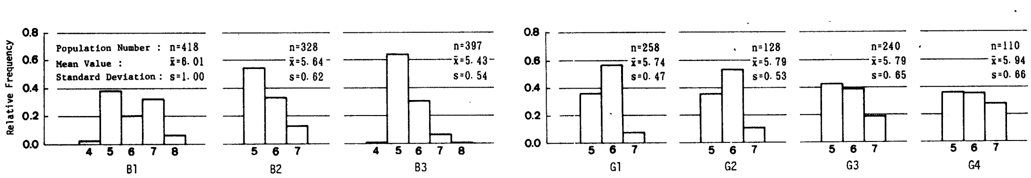

Fig. 5. Relative Frequency Distributions of Final Deflection Factors in terms of Ratios of Final to Elastic Deflections for Cases in Fig. 4

5.3 Estimation of Final Deflections

Fig. 5 shows the distribution of ratio, Δ_t / Δ_e, i.e., the ratio of the final to elastic deflection for all the beams analyzed. Using curves plotted there Δ_t may roughtly be estimated by a multiplied Δ_e value by the upper limit of that ratio which generally varies depending on end restraint or other external conditions.

Also found to be possible is a sufficiently accurate, quartic approximation of final deflection transition of a beam with any definite boundary conditions as its whole depth h only varies with L_x, L_y and b kept constant.

A practical version of the above is illustrated in Fig. 6 where the relations obtained by that quartic between depth/span ratios and corresponding deflection ratios, immediate and final, is shown to be representable by a curve; specifically, as a result of using, by way of example, L_x = 6.0, 7.5 and 9.0 m; with b being any of the three central values in Table 2. And using these sets of data the preceding relation of Δ_t/Δ_e may be expressed as well in equation form, which we have found by an appropriate statistical analysis of the foregoing data.

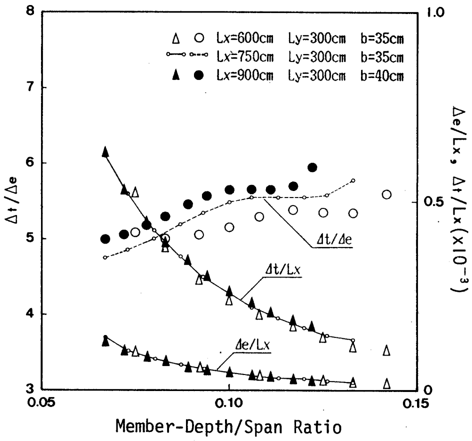

Fig. 6. h/L_x plotted against Δ_t/Δ_e, Δ_e/L_x and Δ_t/L_x

6. Conclusions

Specific items currently clarified by the foregoing facts and figures are: 1) the ratio of elastic Δ_e to final deflection Δ_t ranges from ca. four to eight, averaging six; 2) as far as reinforcing them all is feasible, beams or girders fixed at both ends and those continuous are free from deflection damage; 3) one-span structures have such a possibility even in case their reinforcement is executable. The foregoing findings for one-span members suggest the need for any comprehensive long-time deflection analysis thereof, which is currently under way including the torsion of leteral girders.

A conclusion to serve for a proposal deduced from the calculation results on simply supported members may be that depth/span ratio should be at least 0.08 for one-span beams or girders.

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