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メタデータ	言語: English
	出版者: 室蘭工業大学
	公開日: 2014-03-04
	キーワード (Ja):
	キーワード (En):
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URL	http://hdl.handle.net/10258/771

Admissible Sectional Dimensions of R/C Floor Elements to be Designed without Deflection Check Part 1: Transverse Beams

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Abstract

By use of our proposed modified method, a parametric deflection analysis is attempted for transverse beam models with their dimensions varied within a major practical range in typical cases of their end condition depending on whether or not they have, at one end and/or both, adjoining beams in a slab-beam-girder floor system; the analysis being intended to result in necessary criteria for beam section sizes admissible in floor design without intricacy of deflection check.

Some other design criteria needed for maintained serviceability are derived at the same time.

1. Introduction

Our most recent report indicated that a tenably adequate or practicable estimation system for long-time deflections of r/c flexural members had been provided by use our renovated method.[1]

Its earlier form [2] was developed mainly incorporating D. E. Branson's accepted quasi-empirical formula and using the ACI's time-dependent multiplier; later to be modified reflecting Yu-Winter's well-documented test results combined with sustained elastic modulus used in the original paper for their evaluation.[3] And correspondingly the modified version adopted a more rigorous treatment at difference mesh level of member stiffness in preference to its average whose expediential use is common in most design codes, generally tolerated as practically sufficing.

By use of this improved approach the subject matter has already been examined at the least for discrete beams; the result having been obtained and checked for three elementary cases of end restraint of a member viz. due to to its being supported and/or fixed at one and/or both member ends.[1] [3]

Succeeding to this adequacy check of our method in such simpler cases, herein to be examined

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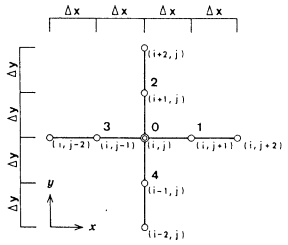
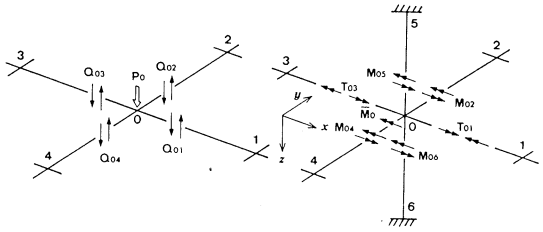


Fig. 1 Used System of Numbered Difference Points.



(a) Vertical Forces.　(b) Bending and Torsional Moments.

Fig. 2 Vertical Forces and Moments Acting on a Joint.

are beam elements of two-way slab-beam-girder systems; specifically regarding major serviceability limiting data including admissible beam section size ranges in case of their being designed in default of deflection check.

For this purpose, long-time deflection analysis at the infinite member age is carried out on a regularly graduated set of member sizes within a commonly well-used range of total system models under working loads. Preliminarily, the present analysis requires some additional finite difference formulation as follows.

2. Equations of Equilibrium at Member Intersections [3]

2. 1 Vertical Forces

For vertical forces acting on four members spanning in orthogonal directions x and y, as shown in Fig. 2(a), the following condition of equilibrium holds at their mutual joint.

$$(Q_{01}-Q_{03})+(Q_{02}-Q_{04})=P_0 \tag{1}$$

where $Q_{01}\sim Q_{04}$=member end shearing forces and P_0=concentrated load acting at the joint.

For any member in the x-direction, the finite difference forms of end shearing force Q_x are to be obtained from the governing equation (2) and its first integral (3) for beam deflection w.

$$\frac{d^2}{dx^2}\left(I_x\frac{d^2w}{dx^2}\right)=\frac{q_x}{E_c} \tag{2}$$

$$\frac{d}{dx}\left(I_x\frac{d^2w}{dx^2}\right)=-\frac{Q_x}{E_c} \tag{3}$$

where I_x=moment of inertia and q_x=intensity of load distribution corresponding to Q_x.

Initially the above two are given their corresponding difference expressions (4) and (5).

$$\begin{aligned}&\frac{E_cI_0}{\Delta x^4}\{k_{i\cdot j-1}w_{i\cdot j-2}-2(k_{i\cdot j-1}+k_{i\cdot j})w_{i\cdot j-1}+(k_{i\cdot j-1}+4k_{i\cdot j}+k_{i\cdot j+1})w_{i\cdot j}\\&\quad-2(k_{i\cdot j}+k_{i\cdot j+1})w_{i\cdot j+1}+k_{i\cdot j+1}w_{i\cdot j+2}\}-q_{xi\cdot j}=0\\&Q_x=-\frac{E_cI_0}{2\Delta x^3}\{-k_{i\cdot j-1}w_{i\cdot j-2}+2k_{i\cdot j-1}w_{i\cdot j-1}-(k_{i\cdot j-1}-k_{i\cdot j+1})w_{ij}\end{aligned} \tag{4}$$

$$-2k_{i\cdot j+1}w_{i\cdot j+1}+k_{i\cdot j+1}w_{i\cdot j+2}\} \tag{5}$$

Then performing such operations as (5)−(4)× $\Delta x/2$ and (5)+(4)× $\Delta x/2$ results in the end shearing forces required above in difference form, Q_{01} or Q_{03}.

$$Q_{01},\ Q_{03}=\frac{mE_cI_0}{\Delta x^3}\ \{k_{i\cdot j-1}w'_{i\cdot j-m}-(2k_{i\cdot j}+k_{i\cdot j+m})w_{i\cdot j}$$
$$+(k_{i\cdot j}+2k_{i\cdot j+m})w_{i\cdot j+m}-k_{i\cdot j+m}w_{i\cdot j+2m}\}\ +q_{xi\cdot j}\Delta x/2 \tag{6}$$

provided: Q_{01}=end shearing force for m=1; Q_{03}=that for m=−1; E_c=concrete elastic modulus; $q_{xi\cdot j}$=self weight, acting at joint O, of member in the direction x; $k_{i\cdot j}=l_{xi\cdot j}/l_0$=flexural stiffness ratio of member at any of its points of subdivision; l_0=reference moment of inertia of member; Δx =width of subdivision of member; and w′=imaginary deflection at an exterior point of subdivision.

2. 2 Equations of Moments

On our assuming the signs of moments acting in the y-direction on joint O of members the following requirement for equilibrium is to hold.

$$(M_{04}-M_{02})+(M_{05}-M_{06})+(T_{01}-T_{03})=\overline{M}_0 \tag{7}$$

where: M_{04}, M_{02}=end moments for member in the y-direction; M_{y05}, M_{y06}=those column end moments in the y-direction acting respectively at the column bottom and top; T_{01}, T_{03}=torsional moments for member spanning in the x-direction; and $\overline{M}_0$=external moment acting at the mutual joint of members; with all these given as follows.

$$M_{02},\ M_{04}=E_cI_0k_{i\cdot j}(-w_{i+m\cdot j}+2w_{i\cdot j}-w'_{i-m\cdot j})/\Delta y \tag{8}$$

$$M_{y05}=4E_cI_{y05}\theta_{yi\cdot j}/L_{05}$$

$$M_{y06}=4E_cI_{y06}\theta_{yi\cdot j}/L_{06} \tag{9}$$

$$T_{01},\ T_{03}=G\ J_0\ r_{i\cdot j}(\theta_{yi\cdot j}-\theta_{yi\cdot j+m})/\Delta x \tag{10}$$

with M_{02}, T_{01}=moments for m=1; M_{04}, T_{03}=those for m=−1; Δy=width of subdivision of member in the y-direction; I_{y05}, I_{y06}=moments of inertia in the y-direction of respective upper- and lower-stair columns connected to the mutual joint; L_{05}, L_{06}=repective heights of upper and lower storeys; G=shear elastic modulus; J_0=reference torsional resistance; $r_{i\cdot j}=J_{xi\cdot j}/J_0$=torsional stiffness ratio at any point of subdivision of member; and θ_y=torsional angle of rotation about member axis in the x-direction.

The terms for imaginary exterior points are to be eliminated by use of end condition dw/dy $=\theta_y$, or its difference form in practice, Eq. (11), set up at exterior member ends.

$$w'_{i-m\cdot j}=w_{i+m\cdot j}+2m\Delta y\theta_{yi\cdot j} \tag{11}$$

3. Previous Result

For a subsequent review of our present calculation result as compared with those earlier, its substance is necessarily reproduced beforehand; viz. it is concerned with the above three elementary cases with their span, load allocation width, section sizes shown in Table 1 and material properties and loading conditions in Table 2. Long-time deflections were calculated for the members designed with their tensile reinforcement allowed up to two layers of the steel.

Table 1 Dimensions of Earlier Calculation Models.

Model	Span Lx	Half Bay Width Ly	Beam Web Width b	Additional Annotation
Beam	450 600 750 900	200 300 400	25, 30, 35 (for Lx = 450) 30, 35, 40 (for Lx = 600, 750) 35, 40, 45 (for Lx = 900)	Slab Thickness = 15 with AIJ Code Effective Width Assumed; All Entries in cm.

All calculation results may be plotted in terms of final deflection ratios, viz. final or terminative deflections divided by span lengths. Among these f.d. ratios, those for simply supported cases refer to the critical or most adverse conditions for deflection control; thus being shown in Fig. 3 relatively to depth/span ratios as abscissas.

Therein noticed are extremely larger rates of increase in final deflection ratios as beam depth/span ratios become smaller than 0.075. This is ascribable to a significantly lowered minimum depth of members capable of two-layered reinforcement which is pursuant to the design practice of assumed medial sections to be tee-shaped.

Table 2 Material Properties and Loads for the Calculation Models, Earlier and Present.

Assumed Items	Adopted Values with Supplementary Note
For Concrete, in kg/cm^2:	
Compressive Strength	210 = F_c: AIJ Code Value/ A. C. V. for short
Modulus of Rupture	26.1 = 1.8 $\sqrt{F_c}$: Suggested in Code
Bond Strength	F_c/15 for End-Top Deformed Steel: A. C. V.
Elastic Modulus	210,000 : A. C. V.
Sustained Modulus	26,600 at Infinite Age (t = ∞)
Poisson's Ratio	0.2 : A. C. V. ;
For Steel, in kg/cm^2:	
Allowable Tensile Stress	2,000 for Code No. SD300 Steel: A. C. V.
Elastic Modulus	2.1x 10^6 with Modular Ratio n = 10: A. C. V.
Loads, in kg/m^2:	
Construction Load	Conventional 2.1 times Wt. of an R/C Floor
Full Design Live Load -	300 & 100 : A. C. V. for respective Office and Living Room
Long-Time Sustained Portion of Live Load -	= 1/3 of the above Values
Wt. of Ceiling & Finish	80 : A. C. V. (in Ordinary Use)

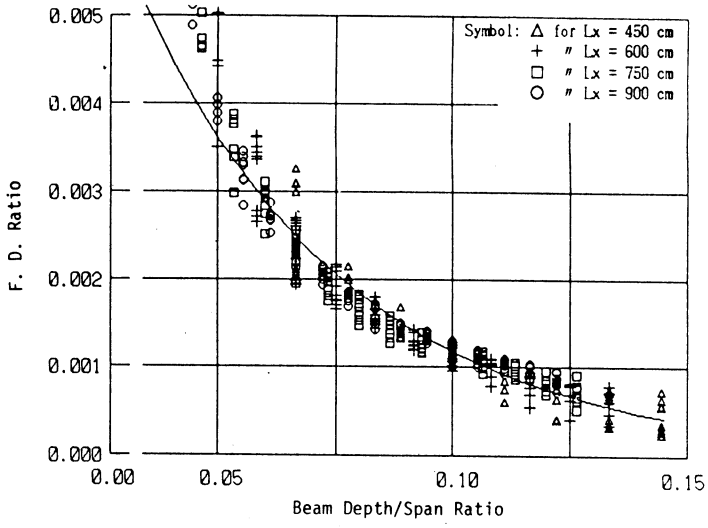


Fig. 3 Final Deflection Ratios plotted vs. Beam Depth/Span Ratios for Simple Beams.

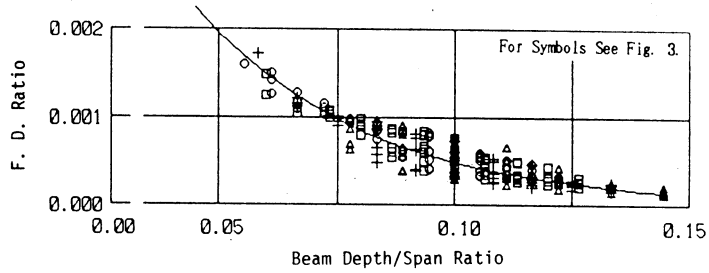


Fig. 4 Final Deflection Ratios plotted vs. Beam Span Ratios for One-End-Fixed Other-Supported Beams.

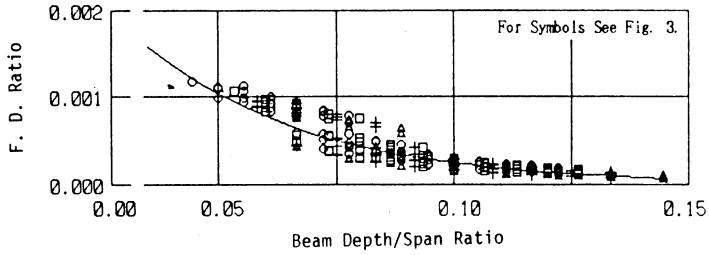


Fig. 5 Final Deflection Ratios plotted vs. Beam Depth/Span Ratios for Both-End-Fixed Beams.

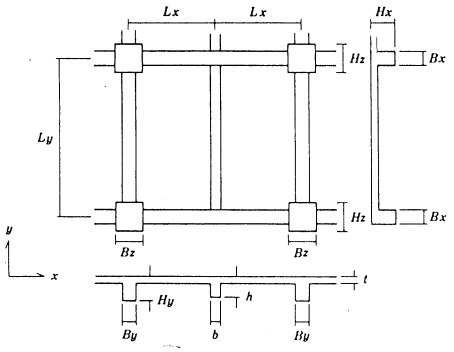


Fig. 6 Introduced Calculation Models with Identified Notation for their Dimensions.

Table 3 Dimensions of Current Calculation Models.

Item		Dimensions in cm	Note
Span	Lx	300, 375, 450	Measurements between Support Centrs.
	Ly	450, 600, 750, 900	
Girder without Beam, GY	By	40 + 2.5(Ly - 900)/150	Provided : Effective Widths are given AIJ Code Values for All Members in the Left Frames; h is made equal to Hx for h) Hx.
	Hy	0.1Ly + 5(600 - Ly)/150	
Girder with Beam, GX	Bx	40 + 5(2Lx - 750)/150	
	Hx	0.2Lx + 5(900 - 2Lx)/150	
Beam	b	40 + 2.5(Ly-900)/150	
	h	Ly/10, Ly/12, Ly/15	
Column	Bz Hz	60 + 5(Ly - 750)/150 〃	Bz = Hz
Storey Height	Lz	360	Both Figures Conventional or Assumably Most Frequent in Practice
Flange Depth	t	15	

Now let serviceability limit depflection be span/500 which is a justifiable standard as the common maximum in the majority of the available Western design codes.[4] Then it can be said that based on this value the above smallest depth will be determined at beam interior ends where reinforcement is to be designed for a customarily assumed rectangular section.

4. Beams as Elements of Floor System

4. 1 Assumption

The chosen calculation models consist of three types of beams in a slab-beam-girder flooor systems as illustrated inclusively in Fig. 6; where, except for a type lastly to be defined, an infinite multitude of identical bays are supposed to extend in the x- or y-direction.

Namely, depending on whether a floor system stretches outward at only one or both of the beam ends, any introduced beam is to be treated as what we call exterior or interior structure in the following. The said last type refers to a member with none to adjoin it at both its ends in a floor system longitudinally having only one span, henceforth being referred to as none-adjoining beam, for simplicity.

Sectional sizes of all model members are shown in Table 3, with the concurrent assumption of material properties and loading conditions in Table 2. Reinforcement is calculated with deformed bars D25 and D22 of nominal diameter respectively for girders and beams under gravity loads

alone.

Column tops and bottoms are assumed as before to be rigidly built-in at the considered floor level. The number of span subdivision is six for center-to-center span Lx for adopted square difference meshes.

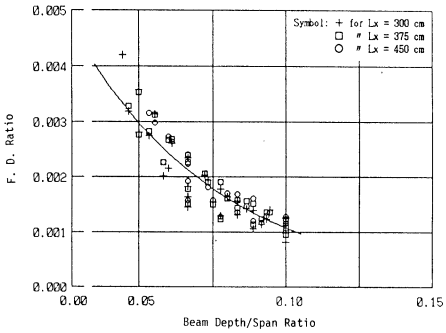


Fig. 7 F. D. Ratios plotted against Beam Depth/Span Ratios for None-Adjoining Beams.

4. 2 Calculation Result and Review

(1) None-Adjoining Beams

Final deflection ratios are calculated for beams assumed to be reinforced in one to two layers as beam depths get successively smaller than a maximum of Ly/10 at 5cm intervals and the results are plotted in relation with beam depth/span ratios in Fig. 7; where beam/span ratios corresponding to final deflection ratios exceeding 1/500 are more or less smaller than simply supported cases. This means that designing beams with stiffness ratios more than 0.075(=1/13.3) can seldom cause this serviceability limit deflection ratio to be exceeded. Major part of Figs. 8 shows how deflection orders change depending on spans and half bay widths delimiting load allocation to beams for differently assumed beam depth ratios of 1/10, 1/12 and 1/15.

For girders in the x-direction, to be labeled GX for convenience, relevant calculation results are not referred to here, in that we have only last reported their being found within the serviceability limit deflection ratio when the girders can be reinforced double layered and be treated as fixed at both ends due to their being interior structures in the defined context.

In Figs. 8(a) and 8(b) final deflection ratios are given as partly mentioned above for girders in the y-direction, called GY like above, and for beams, respectively.

With decreased beam depths negative moments at beam ends increase due to correspondingly larger relative stiffness of beam supporting girders GX's, concurrent with their torsional deformation, so as to bring about the noted slight increases in final deflection ratios for GY girders; admitting that these values are practically insignificant, being of the order of at most 0.0005 or so and far less than the serviceability limit deflection ratio.

On the other hand, beams with depths less than 1/15 of their spans which are 7m at the longest are noted in the pertinent data to be capable of deflection exceeding the above serviceability limit due to their half bay widths for load allocation being increased.

Also needed to be examined is another relative final deflection defined as the ratio to the di-

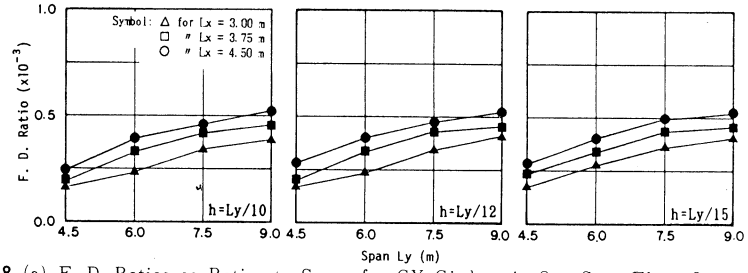


Fig. 8 (a) F. D. Ratios as Ratios to Spans for GY Girders in One-Span Floor Systems.

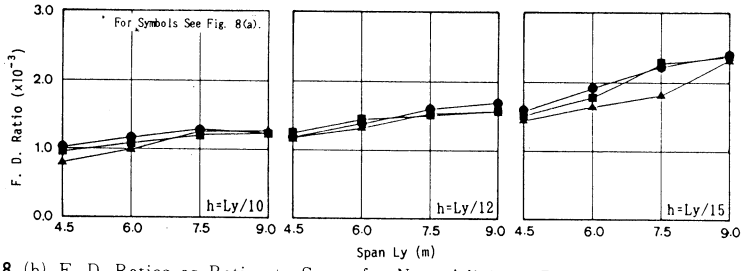


Fig. 8 (b) F. D. Ratios as Ratios to Spans for None-Adjoining Beams in One-Span Floor Systems.

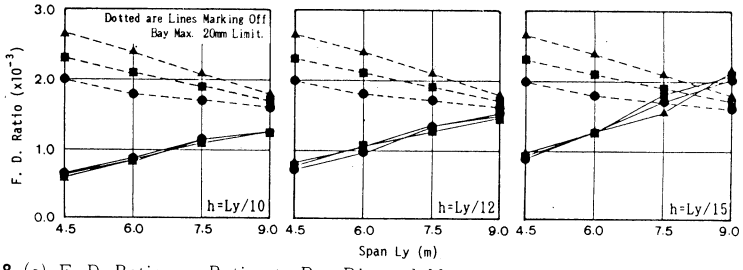


Fig. 8 (c) F. D. Ratios as Ratios to Bay Diagonal Measurements for None-Adjoining Beams.

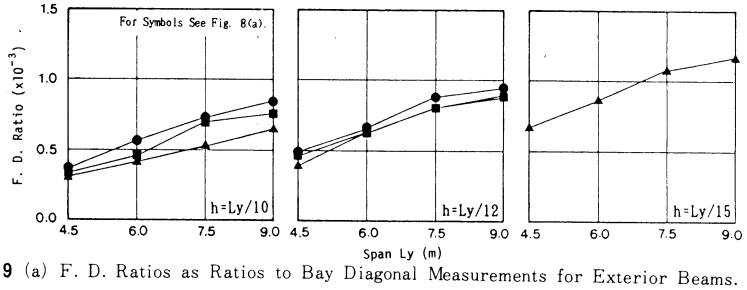


Fig. 9 (a) F. D. Ratios as Ratios to Bay Diagonal Measurements for Exterior Beams.

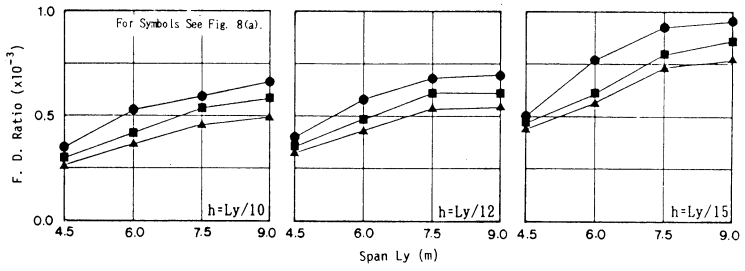


Fig. 9 (b) F. D. Ratios as Ratios to Bay Diagonal Measurements for Interior Beams.

agonal measurement of the considered bay of summed final deflections of pertinent beams and girders of a floor system. Namely in Fig. 8(c) distributions of that ratio are plotted. Reviewing them in comparison with an assumed total system deflection limit of 20mm, commonly adopted by most foreign building codes [4] shows any associated hazard of detrimental deflection for the beam depths less than 1/12 of the span and for the beam spans over 7.5m.

(2) Exterior and Interior Beams

Plotted distributions of the above total system final deflection ratio is represented as well for exterior and interior beams respectively in Figs. 9(a) and 9(b); wherein noted in all cases of beams capable of double layered reinforcement are the corresponding ratios being kept within the preceding deflection limit; provided that in the case of exterior beams, reinforcement has proved impossible for those assumed here to have depths of span/15 and any half bay widths of load allocation other than 3m.

5. Summary and Concluding Remark

The whole foregoing results of current observation are summarized into the following items. Namely, given an admissible limit of beam deflection ratio of 1/500 as customarily assumed we note at first for individual beams capable of double layered reinforcement:

(1) deflection check is always unneeded for beams across interior or exterior span; including respective extreme cases of their being fixed at both ends or being fixed at one and supported at the other;

(2) the above check is needed for beams across single-span floor systems or for simply supported beams, notably for cases with depths smaller than 1/12 or 1/13 respectively because of their being capable of causing detrimental deflections.

And secondly for total floor frame systems:

(3) when we assume a usual limit of 20mm on their deflection as totaled for their beams and girders, interior and exterior beams have no possibility of their deflection exceeding this limit, while none-adjoining structures have any even when their beam depth is more than 1/13 of the span, naturally to require a check.

However, the limiting sectional dimensions obtained at this time are based on idealized assumptions, tending toward criteria practically as conservative as may be expected from the actual incidental conditions, including the practically natural trapezoidal load allocation dictating some greater amount of load than is confirmed to be the case and also beam-supporting GX girders in one-span structures regarded in most cases as latently having appreciably larger stiffness than

here assumed due to sagging wall or spandrel elements then monolithically attached to them.

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