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Paper-Scissors-Stone Game on Trees

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We study the cyclic advantage model with three states so-called Paper-Scissors-Stone (PSS) game on tree-structured lattice space with a fixed number of branches on each lattice site. Each particle on a site interacts only with the nearest neighboring particles. In general "Pair Approximation (PA)" as one of the analytical methods which introduces the information of nearest neighboring correlation is expected to be the more useful tool than "Mean-Field Approximation (MFA)". However, in the case of PSS game on two-dimensional square lattice it was reported that PA gives *the worse result* than MFA for the stability of internal equilibria by Tainaka[1]. In this paper we obtain the similar results about stability of internal equilibria of PSS game on trees contrary to the expectation.

Keywords: Tree, Lattice Model, Pair Approximation, Mean-Field Approximation

1 INTRODUCTION

In biological systems, we can recognize several kinds of relationships between species through competition or predation. Since Lotka[2] and Volterra[3] mathematical models are well known to be useful to study the dynamics of these biological systems. Their original models are too simple to describe real world, but have an important role to understand the basic mechanism for it.

May and Leonard[4] reveal that Lotka-Volterra competition model with three species shows the heteroclinic dynamics and cannot explain the mechanism of coexistence of them. As a competition between three species in nature, we can examplify that the stock abundance of pelagic fishes usually fluctuates and species replacement occurs forever. This cyclic advantage relationship was modeled by Matsuda et al.[5]~[6], Takeuchi *et al.*[7] and Takeuchi[8]. They tell us that only the introduction of refuges cannot avoid the extinction of some kind of species.

On the other hand, Tainaka[9] considered more simple cyclic advantage model, say, Paper-Scissors-Stone (PSS) game without density effect on population growth rate on two-dimensional square lattice space. He assumed that every individual belongs to either three species, and the weaker individual is replaced by the stronger at a constant rate for randomly chosen nearest neighboring two individuals sit on the lattice space. He showed that population dynamics gives the global stablity by Monte Carlo simulation. This result differs from that without spatial structure in population; when we choose two individuals for an interaction these two are not necessarily adjacent each other. The latter shows the neutral stablity.

Interacting particle systems (or lattice models) introduced mathematically by Harris[10] have been studied for various kinds of models, but we cannot get enough mathematical results because of much difficulties. Even in one of the simplest models, basic contact processes, for example, critical values or critical exponents for phase transition are not known (e.g. Konno[11]). Then we often depend on approximation methods; Mean-Field Approximation (MFA) as the first order or Pair Approximation (PA) as the second. MFA corresponds to the non-spatial structured model, namely, considers no correlation between sites. On the other hand, PA pursues the correlation only between two nearest neighboring sites. Therefore it is plausible that PA gives better results not only quantitatively but also qualitatively and various studies on various lattice models support it (Matsuda *et al.*[12]; Sato *et al.*[13]; Tainaka[14]; Satulovsky and Tomé[15]; Harada *et al.*[16]; Harada and Iwasa[17]; Sato and Konno[18]; Kubo *et al.*[19]; Nakamaru *et al.*[20]; Iwasa *et al.*[21]; Konno[22]). However, for PSS game PA and MFA conjecture that the internal equilibria behave unstably and neutrally stably, respectively (Tainaka[1]). If these both conjectures turn to be affirmative, the stability of internal equilibria will not be satisfied and PA will be worse than MFA.

In this paper we consider PSS game on trees in expectation of the improvement of the conjecture by PA. Tretyakov and Konno[23] studied basic contact processes on binary tree with an intial condition of only one particle on the root by Monte Carlo simulation. When we start the system with random distribution as an initial configuration, we should decide the boundary condition of the system. Unfortunately, however, we fail to meet our expectation by PSS game on trees. We can remark that the special assumption in PA on two-dimensional square lattice does not affect the results.

2 MODEL AND APPROXIMATIONS

PSS game on lattice includes the following model assumptions:
(1) The whole system has infinitely large particles each of which sits on

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each site of lattice space. In this section we assume two-dimensional square lattice space. The space offers the homogeneous environments to particles for interactions.
(2) The states of particles change to stronger particle at the proportional rate to the number of those stronger particles in z nearest neighbors. The strength between three states is defined cyclic such as PSS game, i.e. the state $n+1$ is stronger than n, where $n = 0, 1$ or 2 and $n+1$ is given by modulus 3. When we do Monte carlo simulations in a system large enough, this process corresponds to the procedure by choosing adjoint two particles spatially randomly and changing states. The rates of this process is defined by the expected number of events in unit time interval. In this paper we assume the same absolute values of strength in every interactive combination, so we can choose the transition rates of the processes such as $n(\sigma)/z$ in which $n(\sigma)$ means the number of the sites with $\sigma \in \{0,1,2\}$ in z nearest neighbors.

We can describe this model by the following master equations:

$$\frac{d\rho_n}{dt} = \rho_{n-1,n} - \rho_{n,n+1}, \tag{1}$$

$$\begin{aligned}\frac{d\rho_{n,n+1}}{dt} &= \left(1-\frac{1}{z}\right)\rho_{n,n-1,n+1} + \left(1-\frac{1}{z}\right)\rho_{n,n,n+1} - \frac{1}{z}\rho_{n,n+1} \\ &- \left(1-\frac{1}{z}\right)\rho_{n+1,n,n+1} - \left(1-\frac{1}{z}\right)\rho_{n,n+1,n-1},\end{aligned} \tag{2}$$

where $\rho_\sigma, \rho_{\sigma\sigma'}, \rho_{\sigma\sigma'\sigma''}$ for $\sigma, \sigma', \sigma'' \in \{0,1,2\}$ are called as singlet densities, doublet densities and triplet densities, respectively, which are defined as the probabilities of randomly chosen one site of the state σ, randomly chosen one pair of nearest neighboring sites of the states σ-σ' and randomly chosen one triplet sequence of neighboring sites of the states σ-σ'-σ'', respectively.

In Eq.(1) the first term corresponds to the birth process, and the second to the death. For the occurence of the birth process the nearest neighboring site should have the weaker state, so the site of the state n needs the nearest neighboring site of the $n-1$. On the other hand, when the nearest neighboring site is the stronger state this site is absorbed to that stronger state; the site of the n changes by the effect of the nearest neighboring site of the $n+1$. The model does not assume the spatial direction of the interactions, and it means $\rho_{\sigma\sigma'} = \rho_{\sigma'\sigma}$.

Similarly Eq.(2) includes two birth terms and three death terms. There are triplet densities, and we suppose that the effect of the different configurations of the neighboring three sites (i.g. either straight or bent sequences) on population dynamics may not be serious on two-dimensional square lattice space. However, when the next nearest neighboring sites do not have unimportant roles comparative to nearest neighboring sites, we should treat that difference carefully.

The model requires the time evolution of higher order densities because Eqs.(1) and (2) does not constitute a closed set of equations; Eq.(2), the differential equation for doublet densities, includes triplet densities. However, when we describe the time evolution of triplet densities it must depend on the higher order densities, quartet densities. PA is an analytical method in order to close a set of equations by decoupling triplet densities such as $\rho_{\sigma\sigma'\sigma''} = \rho_{\sigma\sigma'}\rho_{\sigma''\sigma'}/\rho_{\sigma'}$. Equation (2) should be changed as:

$$\begin{aligned}\frac{d\rho_{n,n+1}}{dt} &= \left(1-\frac{1}{z}\right)\frac{\rho_{n,n-1}\rho_{n+1,n-1}}{\rho_{n-1}} + \left(1-\frac{1}{z}\right)\frac{\rho_{n,n}\rho_{n+1,n}}{\rho_n} \\ &- \frac{1}{z}\rho_{n,n+1} - \left(1-\frac{1}{z}\right)\frac{\rho_{n+1,n}\rho_{n+1,n}}{\rho_n} \\ &- \left(1-\frac{1}{z}\right)\frac{\rho_{n,n+1}\rho_{n-1,n+1}}{\rho_{n+1}}.\end{aligned} \tag{2'}$$

We can interpret PA as an approximation that the effect of the most far site can be most negligible by the notion of "local densities" or "environ densities" (Matsuda *et al.*[12]).

By solving a set of closed Eqs.(1) and (2') we can obtain interanal equilibrium values of singlet densities and doublet densities (Tainaka[1])

$$\rho_n = \frac{1}{3},$$

$$\rho_{n,m} = \begin{cases} \frac{z+1}{9(z-1)} & \text{if } n = m \\ \frac{z-2}{9(z-1)} & \text{if } n \neq m. \end{cases}$$

The result of doublet densities tell us the clumping property of this PSS game.

We compare the results between PA and MFA, whose dynamics can be obtained by the replacement of the doublet densities to the multiplication of two singlet densities (in other words, the replacement of the local densities to singlet densities, Matsuda *et al.*[12]) in Eq.(1):

$$\frac{d\rho_n}{dt} = \rho_{n-1}\rho_n - \rho_n\rho_{n+1}. \tag{3}$$

In equilibrium we get:

$$\rho_n = \frac{1}{3},$$

and doublet densities are equal to the multiplication between two singlet densities corresponding to two states because the spatial correlation between two nearest neighboring sites is negnected in MFA:

$$\rho_{n,m} = \rho_n\rho_m = \frac{1}{9} \quad \text{for any } n \text{ and } m.$$

Notice that there is no difference of singlet densities between two approximations. The Monte Carlo simulations suggest that this equilibrium value is asymptotically globally stable (Tainaka[9]). On the other hand, for doublet densites PA gives better equilibrium values than MFA (see the next section).

Next we calculate the stability around the above internal equilibria. Tainaka[1] showed that PA gives the instability of singlet densities analytically and by the numerical calculation. Local stability analyses also supports this results (Appendix). On the other hand, MFA has the conservative quantity such as $\rho_0\rho_1\rho_2$ which does not change in the time development, so we can say that the internal eqilibrium of MFA is neutrally stable.

3 PSS GAMES ON TREES

It is natural to consider the possibility that the above assumption in PA of the neglection of the configuration in triplet neighboring sites collapses the correct dynamics. Then we change lattice space on which each particle experiences state transition from two-dimensional square lattice to tree with four branches for each site (Fig. 1). One of the main characteristics of trees is that all the z sites at the edge of the branches from each site sits on the exactly the same spatial position, then we need not distinguish the configuration of three neighboring sites on trees and it seems to reflect the assumption of PA correctly (Fig. 2). So we can expect that the disagreement between PA and simulations can disappear.

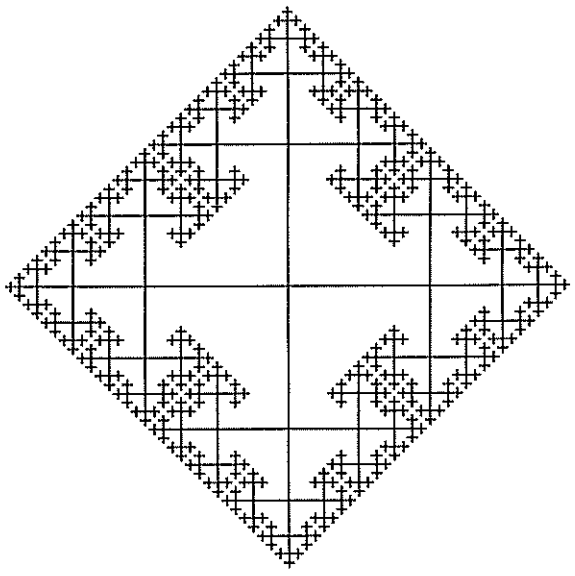

Fig.1. Tree-structured lattice space with $z = 4$.
Every branch elongates from the root infinitely in the model.

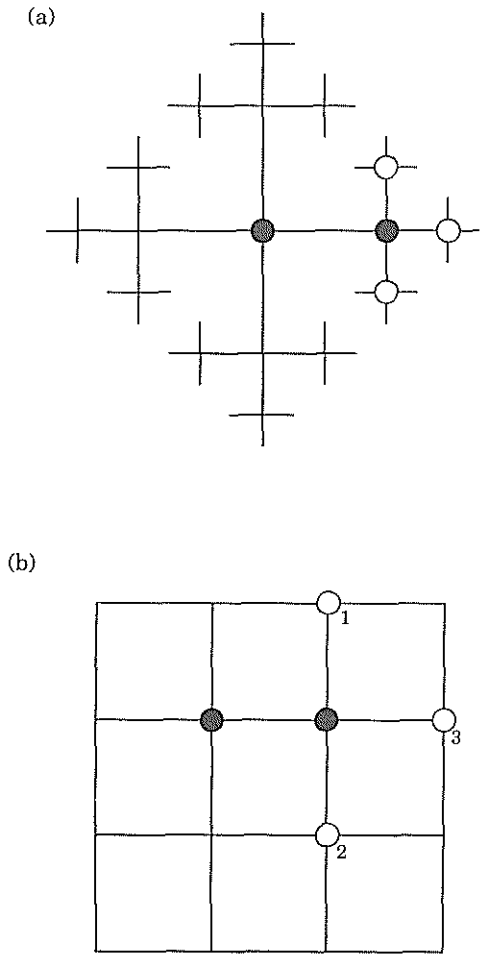

Fig.2. Comparison between two lattice spaces with $z = 4$.
(a) Tree-structured lattice, (b) Two-dimensional square lattice. When we consider the dynamics of nearest neighboring two black sites, the right black site has other three nearest neighboring white sites. In the case of (a) the left black site has only one path to reach either three white sites in the shortest distance. On the other hand, the left black site in (b) has two paths to go to the sites labeled '1' and '2' respectively but only one path to the site '3'.

We can change the number of branches for each site gradually in order to investigate the diection of the change of dynamics due to the number of nearest neighbors for each site. We do computer simulations by tree with about 10000 lattice sites (we cannot choose the same number of the sites for different z because of the characteristics of trees, Table 1). When the number of nearest neighboring sites z is equal to two, the situation is the same as in one-dimensional linear lattice space. Therefore throughout this paper we only concrentrate on the tree-structured lattice space with $z \geq 3$, which includes two-dimensional characteristics. While $z = 10000$ it indicates the every site connects each other so it agrees with the dynamics of MFA or no spatial structure.

z	Distance from the root	Total lattice size
3	13	12286
4	9	13121
10	5	8201
100	3	10001
10000	2	10001

Table 1. Total lattice size for various z's
Total lattice size for various tree-structured lattice spaces with the branching number of z are used for Monte Carlo simulations. Distance from the root is defined as the maximum number of sites in the system from the origin to the periphery along one path.

Notice that we adopt the following boundary condition for doing Monte Carlo simulations. Each site on the periphery has her "mother" site connected to her, which has $z-1$ "daughters". The k-th "daughter" ($k = 1, \cdots, z-1$) in $z-1$ "daughters" of every "mother" sits on the nearest neighbor of the k-th "daughters" of other "mothers" (Fig. 3). Our boundary condition does not satisfy spatial uniformity in periphery, but it may not be serious.

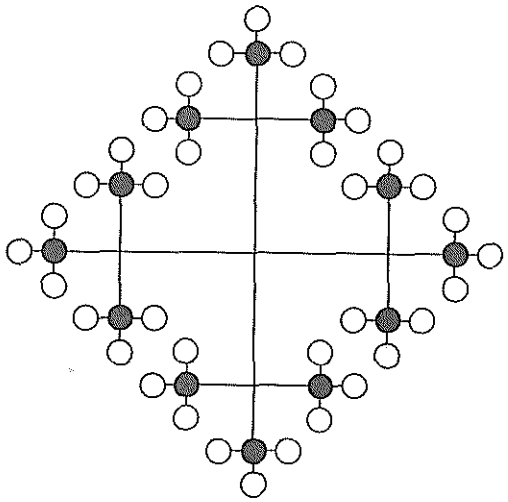

Fig.3. Boundary condition for Monte Carlo simulations.
For the explanation of the periodic boundary condition used in Monte Carlo simulations we depict a small system with $z = 4$. Each "mother" symbolized by black circle has three "daughters" on the periphery and hatched daughters are neareset neighbors each other.

Monte Carlo simulations with about 10000 sites are started by an initial random distribution in which each site is occupied by either three state with the equal probability 1/3. The differences between simulations with various z's are not so clear, but they suggest the possibility that the dynamics of PSS game on trees show stable for $3 \leq z < 10000$, and neutrally stable for $z = 10000$ (the reader can consider that limit cycle appears in the case of $z = 10000$, but it may be attributable to the finite size effect of the system and short term running; Fig. 4). This result indicates that the branching space has the effect of escaping from the extinction of either state and it is independent of the space either with loop or without loop. The increase of the number of connections between sites gives the more unstability, and in extremes the dynamics settles down to neutral stablility.

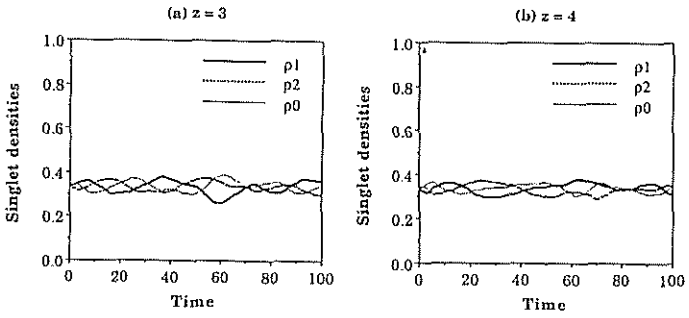

Fig.4. Time development of singlet densities by Monte Carlo simulations.
(a) $z = 3$, (b) $z = 4$, (c) $z = 10$, (d) $z = 100$, (e) $z = 10000$. The changes of singlet densities through time are shown. The fluctuations with small amplitudes are caused by the finite size effect of systems.

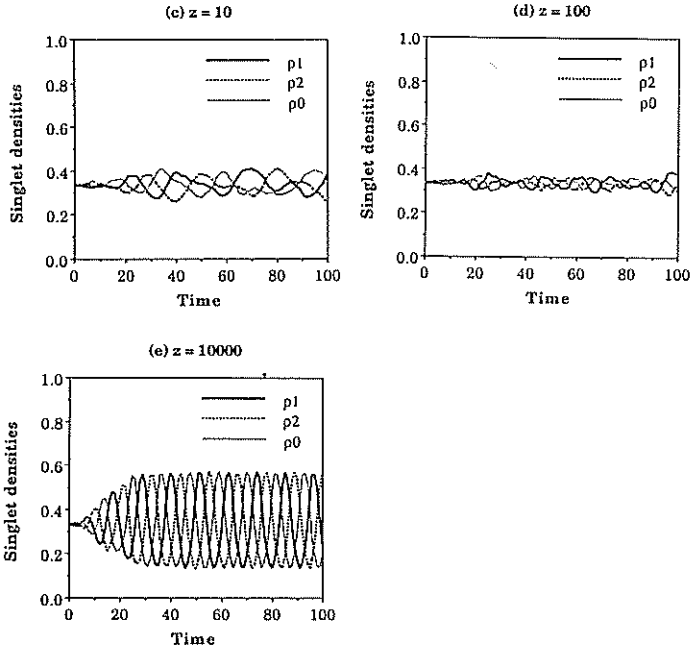

Fig.4. Time development of singlet densities by Monte Carlo simulations (cont.)

Simulations on the tree-structured lattice space do not support the stability analyses by PA, which shows unstability rather than neutral stability given by MFA. However, we can obtain better quantitative results for the equilibrium values of doublet densities by PA than by MFA (Figs.5 and 6).

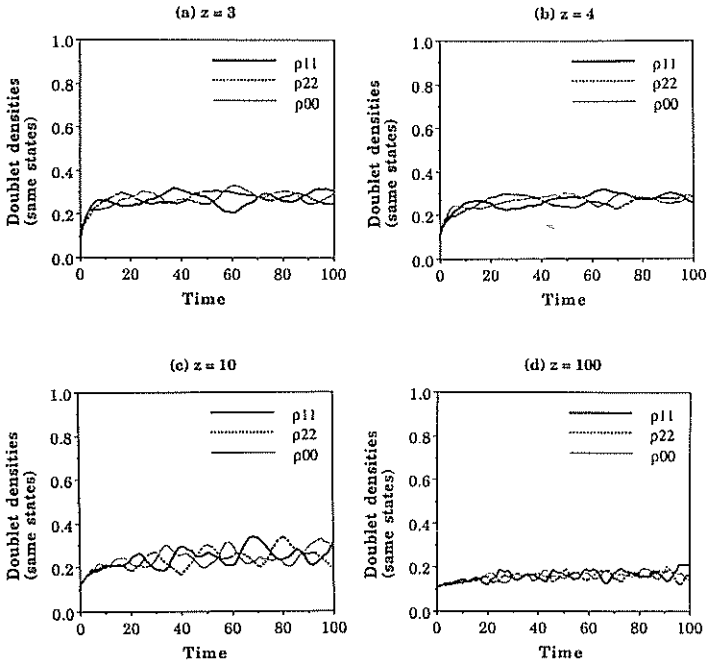

Fig.5. Time development of doublet densities with two same states by Monte Carlo simulations.
(a) $z = 3$, (b) $z = 4$, (c) $z = 10$, (d) $z = 100$, (e) $z = 10000$. The changes of doublet densities with two same states through time are shown.

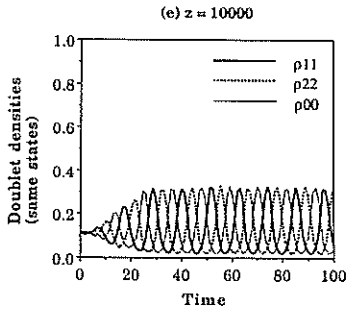

Fig.5. Time development of doublet densities with two same states by Monte Carlo simulations (cont.)

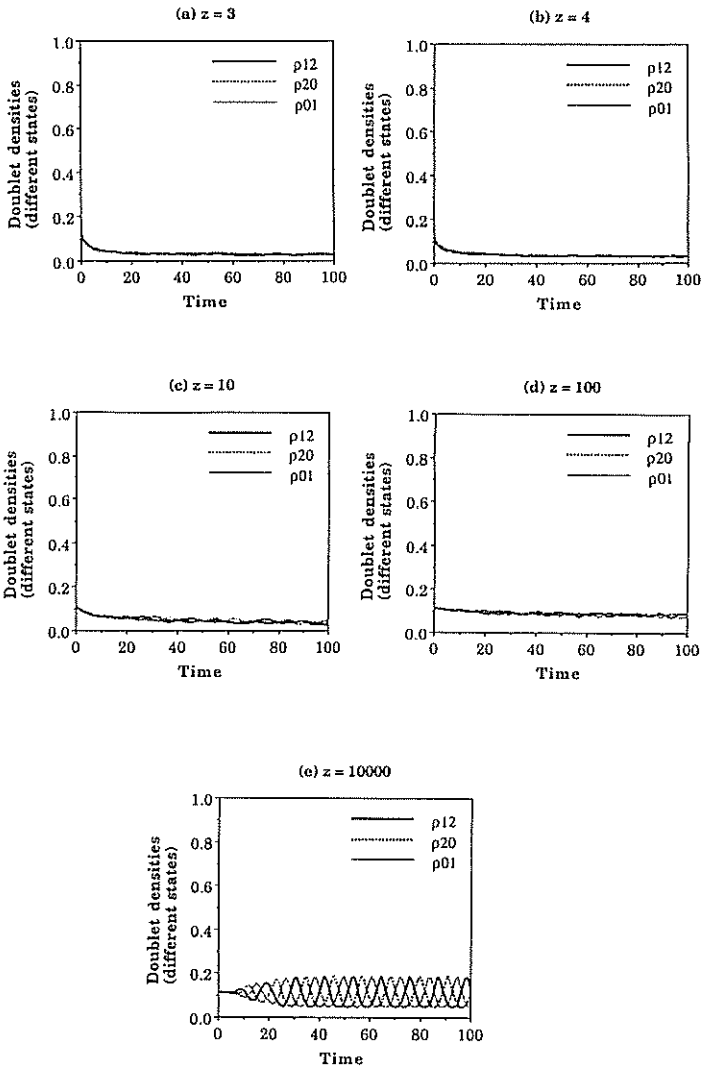

Fig.6. Time development of doublet densities with two different states by Monte Carlo simulations.
(a) $z = 3$, (b) $z = 4$, (c) $z = 10$, (d) $z = 100$, (e) $z = 10000$. The changes of doublet densities with two different states through time are shown.

4 DISCUSSIONS

Unfortunately it seems that we cannot get the strict result for stability by PA, and it may be attributed to the clumping property by interactions between nearest neighboring sites, which causes the necessity of the consideration in correlations on the far distanced sites (Iwasa *et al.*[21]). However, we can say that the neglection of the configration of triplet sequence of sites for two-dimensional square lattice space in PA does not give the different results.

We have another possibility that disagreement between PA and simulations will be settled by using the more proper boundary condition which has the spatial uniformity and reflects the feature of trees (i.e. no loop or exponentially increase of the number of sites from the origin to the periphery). We need some ideas for doing it.

PA exhibits the powerful usefulness for analyses in several lattice models. Our problem studied in this paper will be one of the trials to understand the properties of PA and improve PA as more applicable tools to various lattice models.

APPENDIX: LOCAL STABILITY ANALYSES

For the analyses of our closed dynamical system we need only five independent equations in Eqs.(1) and (2), so we choose the following five equations:

$$\frac{d\rho_0}{dt} = f_1(\rho_{01},\rho_{20}) = \rho_{20}-\rho_{01}, \tag{A1}$$

$$\frac{d\rho_1}{dt} = f_2(\rho_{01},\rho_{12}) = \rho_{01}-\rho_{12}, \tag{A2}$$

$$\begin{aligned}\frac{d\rho_{01}}{dt} &= f_3(\rho_0,\rho_1,\rho_{01},\rho_{12},\rho_{20})\\ &= \left(1-\frac{1}{z}\right)\frac{\rho_{20}\rho_{12}}{1-\rho_0-\rho_1}+\left(1-\frac{1}{z}\right)\frac{(\rho_0-\rho_{20}-\rho_{01})\rho_{01}}{\rho_0}\\ &\quad -\frac{1}{z}\rho_{01}-\left(1-\frac{1}{z}\right)\frac{\rho_{01}^2}{\rho_0}-\left(1-\frac{1}{z}\right)\frac{\rho_{01}\rho_{12}}{\rho_1}\end{aligned} \tag{A3}$$

$$\begin{aligned}\frac{d\rho_{12}}{dt} &= f_4(\rho_0,\rho_1,\rho_{01},\rho_{12},\rho_{20})\\ &= \left(1-\frac{1}{z}\right)\frac{\rho_{01}\rho_{20}}{\rho_0}+\left(1-\frac{1}{z}\right)\frac{(\rho_1-\rho_{01}-\rho_{12})\rho_{12}}{\rho_1}\\ &\quad -\frac{1}{z}\rho_{12}-\left(1-\frac{1}{z}\right)\frac{\rho_{12}^2}{\rho_1}-\left(1-\frac{1}{z}\right)\frac{\rho_{12}\rho_{20}}{1-\rho_0-\rho_1}\end{aligned} \tag{A4}$$

$$\begin{aligned}\frac{d\rho_{20}}{dt} &= f_5(\rho_0,\rho_1,\rho_{01},\rho_{12},\rho_{20})\\ &= \left(1-\frac{1}{z}\right)\frac{\rho_{12}\rho_{01}}{\rho_1}+\left(1-\frac{1}{z}\right)\frac{(1-\rho_0-\rho_1-\rho_{12}-\rho_{20})\rho_{20}}{1-\rho_0-\rho_1}\\ &\quad -\frac{1}{z}\rho_{20}-\left(1-\frac{1}{z}\right)\frac{\rho_{20}^2}{1-\rho_0-\rho_1}-\left(1-\frac{1}{z}\right)\frac{\rho_{20}\rho_{01}}{\rho_0}\end{aligned} \tag{A5}$$

where we use the relation $\rho_{00}=\rho_0-\rho_{20}-\rho_{01}, \rho_{11}=\rho_1-\rho_{01}-\rho_{12}, \rho_{22}=\rho_2-\rho_{12}-\rho_{20}$ and $\rho_2=1-\rho_0-\rho_1$. Local stability of internal equilibria, $\bar\rho_n=1/3, \bar\rho_{n,n+1}=(z-2)/9(z-1)$, can be investigated by the five independent linealized equations derived from Eqs.(A1)$\sim$(A5). When we put $\rho_n=\bar\rho_n+x_n$ for $n=0,1, \rho_{n,n+1}=\bar\rho_{n,n+1}+y_n$ for $n=0,1,2$, linearlized equations for small x_n and y_n are:

$$\frac{dx_0}{dt} = \frac{\partial f_1}{\partial\rho_{01}}(\bar\rho_{01},\bar\rho_{20})\,y_0+\frac{\partial f_1}{\partial\rho_{20}}(\bar\rho_{01},\bar\rho_{20})\,y_2, \tag{A6}$$

$$\frac{dx_1}{dt} = \frac{\partial f_2}{\partial\rho_{01}}(\bar\rho_{01},\bar\rho_{12})\,y_0+\frac{\partial f_2}{\partial\rho_{12}}(\bar\rho_{01},\bar\rho_{12})\,y_1, \tag{A7}$$

$$\begin{aligned}\frac{dy_0}{dt} &= \frac{\partial f_3}{\partial\rho_0}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,x_0\\ &\quad+\frac{\partial f_3}{\partial\rho_1}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,x_1\\ &\quad+\frac{\partial f_3}{\partial\rho_{01}}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,y_0\\ &\quad+\frac{\partial f_3}{\partial\rho_{12}}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,y_1\\ &\quad+\frac{\partial f_3}{\partial\rho_{20}}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,y_2,\end{aligned} \tag{A8}$$

$$\begin{aligned}\frac{dy_1}{dt} &= \frac{\partial f_4}{\partial\rho_1}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,x_0\\ &\quad+\frac{\partial f_4}{\partial\rho_1}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,x_1\\ &\quad+\frac{\partial f_4}{\partial\rho_{01}}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,y_0\\ &\quad+\frac{\partial f_4}{\partial\rho_{12}}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,y_1\\ &\quad+\frac{\partial f_4}{\partial\rho_{20}}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,y_2,\end{aligned} \tag{A9}$$

$$\begin{aligned}\frac{dy_2}{dt} &= \frac{\partial f_5}{\partial\rho_0}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,x_0\\ &\quad+\frac{\partial f_5}{\partial\rho_1}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,x_1\\ &\quad+\frac{\partial f_5}{\partial\rho_{01}}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,y_0\\ &\quad+\frac{\partial f_5}{\partial\rho_{12}}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,y_1\\ &\quad+\frac{\partial f_5}{\partial\rho_{20}}(\bar\rho_0,\bar\rho_1,\bar\rho_{01},\bar\rho_{12},\bar\rho_{20})\,y_2,\end{aligned} \tag{A10}$$

where notice that, for example, the function f_1 depends only on two variables ρ_{20} and ρ_{01}, so the partial derivatives of f_1 on other variables are equal to zero. The above linearlized Eqs.(A6)$\sim$(A10) can be written down as the form of the following matrix:

$$\frac{d}{dt}\begin{bmatrix}x_0\\x_1\\y_0\\y_1\\y_2\end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 & 1\\ 0 & 0 & 1 & -1 & 0\\ \frac{4(z-2)^2}{9z(z-1)} & \frac{2(z-2)^2}{9z(z-1)} & -\frac{z-2}{z} & 0 & 0\\ -\frac{2(z-2)^2}{9z(z-1)} & \frac{2(z-2)^2}{9z(z-1)} & 0 & -\frac{z-2}{z} & 0\\ -\frac{2(z-2)^2}{9z(z-1)} & -\frac{4(z-2)^2}{9z(z-1)} & 0 & 0 & -\frac{z-2}{z}\end{bmatrix}\begin{bmatrix}x_0\\x_1\\y_0\\y_1\\y_2\end{bmatrix}.$$

The maximum real part of the eigenvalues of this matrix is

$$\lambda_{max}(z) = \frac{-(z-2)+\frac{(z-2)\sqrt{-4\sqrt{3}z+\sqrt{49z^2+6z+9}}\left(-3-z+4\sqrt{3}z+\sqrt{49z^2+6z+9}\right)}{2\sqrt{3(z-1)(z+3)}}}{2z},$$

which is positive for any $z\geq 3$, so the internal equilibria is locally unstable for $z\geq 3$. Notice that $\lambda_{max}(z)\to 0$ as $z\to\infty$, which indicates the necessary condition of neutral stability.

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ツリー上でのじゃんけんゲーム

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概　要

各格子点から等しい数だけの枝が延びているようなツリー構造をもつ格子空間上でのじゃんけんゲームの研究を行なった。格子点上の各粒子はすぐ隣合った格子点としか相互作用することはできない。隣合った格子点との相関を考慮した解析手法のひとつであるペア近似は、一般的に平均場近似よりも有効な手法であると思われる。しかし、２次元正方格子空間上でのじゃんけんモデルの場合には、内的平衡点における安定性に対して、ペア近似は平均場近似よりも悪い結果を与えることが泰中[1]によって報告されている。ツリー上のじゃんけんモデルのダイナミクスはペア近似を正しく反映するように思われるが、この論文で、内的平衡点の安定性については、２次元正方格子空間上での結果と同じであることを示す。

キーワード：　ツリー、格子モデル、ペア近似、平均場近似

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