遅延を持つ汎関数の偏微分方程式の解の安定性について

Mem. Muroran Inst. Tech.,48(1998)61～66

The Stability of the Solution of Partial Functional Differential Equation with Delay

Lixin DAI* and Shanjun ZHANG**

(Received 8 May 1998, Accepted 31 August 1998)

The dynamic status of a stable biological system can be described by a set of partial functional differential equations with delay. In this paper, the stabilities of the solutions of these kinds of partial differential equations are discussed by an inequality estimation method. With this method, we obtained a succinct criterion to describe the asympctic stabilities of the zero solutions.

Keywords: Delay, Partial functional differential equation, Stability.

1 Introduction

In the stable biological systems, the generation numbers of the species are function of time t and age x of the individuals of the species. Because of the life space restraint, food competition, and predating among different species, the distribution density $u(x,t)$ of a certain species varies with delay. The increasing (decreasing) status of the species is described by a set of partial functional differential equations known as "Logistic biological model with delay". In this paper, equation (1) and (31) provided typical examples of the kinds. In the past, the existence and uniqueness of the solutions of these partial functional differential equations are studied by using Liapunov function or Fourier transform [1 -4]. Here, we shall discuss another important problem: the stability of the solution of the kinds of functional differential equations. The origin of this paper is an attempt to settle the asymptotic distribution of the solution. We adopted an inequality estimation method, and obtained a succinct measure of the asymptotic stability of the $\mathbf{X}^{1,2}(\Omega)$ zero solution, and a judgment of the uniformly convergence to zero on $\bar{\Omega}$ for all solutions.

* Department of Mathematics, Jingzhou Teachers' College

** Department of Computer Science and Systems Engineering, Muroran Institute of Technology

2 Methods

In the discussion of this paper, we shall use as follows two function spaces $\mathbf{L}^p(\Omega)$ and $\mathbf{W}^{k,p}(\Omega)$:
Let $\mathbf{L}^p(\Omega)$, where $1 \le p \le \infty$, denote the whole measurable functions on Ω, which satisfy the following conditions:

$$\|u\|_{\mathbf{L}^p(\Omega)} = \|u\|_p = \{\int_\Omega |u(x)|^p dx\}^{\frac{1}{p}} < \infty \qquad (1 \le p < \infty)$$

$$\|u\|_{\mathbf{L}^\infty(\Omega)} = \|u\|_\infty = \operatorname*{ess\,sup}_{x\in\Omega} |u(x)| < \infty \qquad (p = \infty)$$

$\mathbf{L}^p(\Omega)$is a Banach space, specially, if $p = 2$, then $\mathbf{L}^2(\Omega)$ is a Hilbert space.

Let $\mathbf{W}^{k,p}(\Omega)$, where $1 \le p \le \infty$ and integer $k \ge 1$, denote the whole measurable functions which satisfy the following conditions

$$\mathbf{D}^\alpha u \in \mathbf{L}^p(\Omega),$$

$$\forall |\alpha| \le \mathbf{k} \quad \alpha = (\alpha_1, \cdots, \alpha_m), \alpha_i \ge 0, \sum_{i=1}^m \alpha_i = |\alpha|,$$

$$\mathbf{D}^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \cdots \partial x_m^{\alpha_m}}.$$

Its norm is

$$\|u\|_{w^{k,p}} = \{\sum_{|\alpha|\le k} \int_\Omega |\mathbf{D}^\alpha u|^p dx\}^{\frac{1}{p}}$$

or its equivalent form is

$$\|u\|_{w^{k,p}} = \sum_{|\alpha|\le k}\{\int_\Omega |\mathbf{D}^\alpha u|^p dx\}^{\frac{1}{p}} .$$

$w^{k,p}(\Omega)$ is a sobolev space, specially, if $u \in w^{k,p}(\Omega)$, then

$$\|u\|_{w^{k,p}} = \|u\|_p + \|\nabla u\|_p$$

where ∇ is a Hamilton operator.

First let we think the partial functional differential equation with constant delay.

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = D(t)\Delta u(x,t) + \mathbf{A}(t)u(x,t) + \mathbf{B}(t)u(x,t-\tau), \\ \quad (x,t) \in \Omega \times \mathbf{R}^+ \\ u(x,t) = \varphi(x,t), \\ \quad (x,t) \in \Omega \times [-\tau, 0] \\ \frac{\partial u(x,t)}{\partial v} + \mathbf{C(x,t)u(x,t) = 0}, \\ \quad (x,t) \in \partial\Omega \times [-\tau, +\infty] \end{cases} \tag{1}$$

where $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are continuous $n \times n$ matrixes on $[0, +\infty)$, $D(t) = \text{diag }(d_1(t), \cdots, d_n(t))$, $C(x,t) = \text{diag }(C_1(x,t), \cdots, C_n(x,t))$ and $d_i(t) > 0$, $C_i(x,t) \ge 0$, $i = 1,2,\cdots,n$. φ is a proper smooth and known n-dimensional function on $\Omega \times [-\tau, 0]$. Δ is a Laplace operator on Ω, i.e., $\Delta = \sum_{i=1}^{m} \frac{\partial^2}{\partial x_i^2}$, and $\Omega = \{\, x = (x_1, \cdots, x_m)^\top ; |x_i| < \delta\}$, is a bounded open subaggregate in $\mathbf{R}^m$, whose border $\partial\Omega$ is smooth. $\mathbf{V}$ is a uniformizing outer normal vector on $\partial\Omega$, whose delay $\tau \ge 0$, where $\mathbf{R}^+ = [0, +\infty)$.

Definition 1. *For $\forall\varepsilon > 0$, $\exists\delta(\varepsilon) > 0$, and $N_1 > 0$, if $\sup_{-\tau\le t\le 0} \|\varphi(x,t)\|^2_{L^2(\Omega)} = M_1 < \delta(\varepsilon)$, We have*

$$\|u(x,t,\varphi)\|_{L^2(\Omega)} \le \varepsilon, \|\nabla u(x,t,\varphi)\|_{L^2(\Omega)} \le N_1\varepsilon,\ (t > 0),$$

then we call the zero solution of (1) is $\mathbf{X}^{1,2}(\Omega)$ stable. If again have

$$\lim_{t\to\infty} \|u(x,t)\|_{L^2(\Omega)} = \lim_{t\to\infty} \|\nabla u(x,t)\|_{L^2(\Omega)} = 0 \ ,$$

then we call the zero solution of (1) is $\mathbf{X}^{1,2}(\Omega)$ asymptotically stable.

Theorem 1. *If the equation (1) satisfies $b < r$, then its zero solution is $\mathbf{X}^{1,2}(\Omega)$ asymptotically stable, also, all solutions $u(x,t)$ of (1) satisfy $\lim_{t\to\infty} u(x,t) = 0$, which uniformly holds for any x, where $x \in \bar\Omega$, $b = \sup_{t\ge 0}\|B(t)\|$, $r = \inf_{t\ge 0}\{r(t)\} > 0$, and $-r(t)$ is the maximum eigenvalue of the matrix $A^*(t) = \frac{A^\top(t)+A(t)}{2} - l^{-2}\,\text{diag}(d_1(t), \cdots, d_n(t))$, $l = \frac{\delta}{\sqrt{m}}$.*

Proof. Multiplying the two sides of (1) by $u^\top(x,t)$, notice that for n-dimensional vectors A and B, we have $A^\top B = B^\top A$, then

$$\begin{aligned} \frac{1}{2}\frac{\partial}{\partial t}(u^\top(x,t)u(x,t)) &= u^\top(x,t)D(t)\Delta u(x,t) + u^\top(x,t) \\ &\quad A(t)u(x,t) + u^\top(x,t)B(t)u(x,t-\tau) \\ &= \sum_{i=1}^{n} d_i(t)u_i(x,t)\Delta u_i(x,t) + u^\top(x,t)\frac{A^\top(t)+A(t)}{2}u(x,t) \\ &\quad + u^\top(x,t)B(t)u(x,t-\tau) \ . \end{aligned} \tag{2}$$

Integrating the two sides of (2) about x, we get,

$$\begin{aligned} \frac{\partial}{\partial t}\int_\Omega u^\top(x,t)u(x,t)dx &= 2\sum_{i=1}^{n} d_i(t)\int_\Omega u_i(x,t)\Delta u_i(x,t)dx \\ &\quad + 2\int_\Omega u^\top(x,t)\frac{A^\top(t)+A(t)}{2}u(x,t)dx \\ &\quad + 2\int_\Omega u^\top(x,t)B(t)u(x,t-\tau)dx \ . \end{aligned} \tag{3}$$

Since $C_i(x,t) \ge 0$, by divergence theorem [5] we have

$$\begin{aligned} \int_\Omega (\nabla u_i \nabla u_i + u_i\Delta u_i)dx &= \int_{\partial\Omega} u_i\frac{\partial u_i}{\partial V}dx \\ &= -\int_{\partial\Omega} C_i u_i^2 dx \\ &\le 0 \ . \end{aligned} \tag{4}$$

Thus

$$\int_\Omega u_i\Delta u_i dx \le -\int_\Omega (\nabla u_i(x,t))^2 dx, \quad i = 1,2,\cdots,n. \tag{5}$$

According to Poincaré inequality [6] we have

$$\int_\Omega u_i^2(x,t)dx \le l^2\int_\Omega (\nabla u_i(x,t))^2 dx, \quad (l = \frac{\delta}{\sqrt{m}}) . \tag{6}$$

Since $b < r$, then it must exist enough small positive real number ε and σ such that

$$b(1 + e^{2\tau\sigma}) < 2(r - \frac{\varepsilon}{l^2} - \sigma) \ . \tag{7}$$

For the ε, then we have

$$\begin{aligned} d_i(t)\int_\Omega u_i\Delta u_i dx &\le -d_i(t)\int_\Omega (\nabla u_i(x,t))^2 dx \\ &= -\varepsilon\int_\Omega (\nabla u_i(x,t))^2 dx - (d_i(t) - \varepsilon)\int_\Omega (\nabla u_i(x,t))^2 dx \\ &\le -\varepsilon\int_\Omega (\nabla u_i(x,t))^2 dx - \frac{d_i(t)-\varepsilon}{l^2}\int_\Omega u_i^2(x,t)dx \ . \end{aligned} \tag{8}$$

Thus (3) can become

$$\frac{\partial}{\partial t}\int_\Omega \|u(x,t)\|^2dx \le -2\varepsilon\sum_{i=1}^n\int_\Omega(\nabla u_i(x,t))^2dx - \frac{2}{l^2}\sum_{i=1}^n(d_i(t)-\varepsilon)\int_\Omega u_i^2(x,t)dx + 2\int_\Omega u^\top(x,t)\frac{A^\top(t)+A(t)}{2}u(x,t)dx + 2\int_\Omega u^\top(t)(x,t)B(t)u(x,t-\tau)dx$$
$$= -2\varepsilon\sum_{i=1}^n\int_\Omega(\nabla u_i(x,t))^2dx + \frac{2\varepsilon}{l^2}\int_\Omega\sum_{i=1}^n u_i^2(x,t)dx + 2\int_\Omega u^\top(x,t)A^*(t)u(x,t)dx + 2\int_\Omega u^\top(x,t)B(t)u(x,t-\tau)dx \tag{9}$$
$$\le -2\varepsilon\sum_{i=1}^n\int_\Omega(\nabla u_i(x,t))^2dx - 2r\int_\Omega\|u(x,t)\|^2dx + 2\int_\Omega\|u(x,t)\|\|B(t)\|\|u(x,t-\tau)\|dx + \frac{2\varepsilon}{l^2}\int_\Omega\|u(x,t)\|^2dx\,.$$

Hence

$$\int_\Omega\|u(x,t)\|^2dx \le \int_\Omega\|\varphi(x,0)\|^2dxe^{-2(r-\frac{\varepsilon}{l^2})t} - 2\varepsilon\sum_{i=1}^n\int_0^t e^{-2(r-\frac{\varepsilon}{l^2})(t-s)}\int_\Omega(\nabla u_i(x,s))^2dxds + 2\int_0^t e^{-2(r-\frac{\varepsilon}{l^2})(t-s)}\int_\Omega\|u(x,s)\|\|B(s)\|\times \|u(x,s-\tau)\|dxds \tag{10}$$
$$\le M_1e^{-2(r-\frac{\varepsilon}{l^2})t} - 2\varepsilon\sum_{i=1}^n\int_0^t e^{-2(r-\frac{\varepsilon}{l^2})(t-s)}\int_\Omega(\nabla u_i(x,s))^2dxds + \int_0^t e^{-2(r-\frac{\varepsilon}{l^2})(t-s)}\|B(s)\|\times\int_\Omega[\|u(x,s)\|^2+\|u(x,s-\tau)\|^2]dxds\,.$$

For $\sigma>0$ in equation (7), we have

$$e^{2\sigma t}\int_\Omega\|u(x,t)\|^2dx \le M_1e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)t} - 2\varepsilon\sum_{i=1}^n\int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)}\int_\Omega e^{2\sigma s}(\nabla u_i)^2dxds + \int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)}be^{2\sigma s}\int_\Omega\|u(x,s)\|^2dxds + \int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)}be^{2\sigma\tau}e^{2\sigma(s-\tau)}\times\int_\Omega\|u(x,s-\tau)\|^2dxds$$
$$\le M_1 - 2\varepsilon\sum_{i=1}^n\int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)}\{e^{2\sigma s}\times\int_\Omega(\nabla u_i(x,s))^2dx\}ds + \int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)}\times b(1+e^{2\tau\sigma})\sup_{-\tau\le\theta\le s}\{e^{2\sigma\theta}\int_\Omega\|u(x,\theta)\|^2dx\}ds \tag{11}$$
$$\le M_1 - 2\varepsilon\sum_{i=1}^n\int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)}\{e^{2\sigma s}\times\int_\Omega(\nabla u_i(x,s))^2dx\}ds + \frac{1+e^{2\tau\sigma}}{2(r-\frac{\varepsilon}{l^2}-\sigma)}b\times\sup_{-\tau\le\theta\le t}\{e^{2\sigma\theta}\int_\Omega\|u(x,\theta)\|^2dx\}\,.$$

The property of the nonreducibility of $p(t)$, where $p(t)=\sup_{-\tau\le\theta\le t}\{e^{2\sigma\theta}\int_\Omega\|u(x,\theta)\|^2dx\}$, has been used in the last inequality of (11). So (11) can be deduced into

$$e^{2\sigma t}\int_\Omega\|u(x,t)\|^2dx \le M_1 + \frac{b(1+e^{2\tau\sigma})}{2(r-\frac{\varepsilon}{l^2}-\sigma)}\times\sup_{-\tau\le\theta\le t}\{e^{2\sigma\theta}\int_\Omega\|u(x,\theta)\|^2dx\}\,. \tag{12}$$

Since the right side of (12) does not reduce, then we have

$$\sup_{-0\le s\le t}\{e^{2\sigma s}\int_\Omega\|u(x,s)\|^2dx\} \le M_1 + \frac{b(1+e^{2\tau\sigma})}{2(r-\frac{\varepsilon}{l^2}-\sigma)}p(t) \tag{13}$$

and

$$\sup_{-\tau\le s\le t}\{e^{2\sigma s}\int_\Omega\|u(x,s)\|^2dx\} \le \sup_{-\tau\le\theta\le 0}\{e^{2\sigma s}\int_\Omega\|u(x,s)\|^2dx\} + \sup_{0\le s\le t}\{e^{2\sigma s}\int_\Omega\|u(x,s)\|^2dx\} \le M_1 + \sup_{0\le s\le t}\{e^{2\sigma s}\int_\Omega\|u(x,s)\|^2dx\}\,. \tag{14}$$

Thus

$$p(t) \le 2M_1 + \frac{b(1+e^{2\tau\sigma})}{2(r-\frac{\varepsilon}{l^2}-\sigma)}\sup_{-\tau\le s\le t}\{e^{2\sigma s}\int_\Omega\|u(x,s)\|^2dx\}\,. \tag{15}$$

From (7) we know

$$1-\frac{b(1+e^{2\sigma\tau})}{2(r-\frac{\varepsilon}{l^2}-\sigma)}=h>0$$

Then

$$p(t)=\sup_{-\tau\le s\le t}\{e^{2\sigma s}\int_\Omega\|u(x,s)\|^2dx\le\frac{2}{h}M_1 \tag{16}$$

Substituting (16) into (11), if $t>a>0$, then

$$\begin{aligned}e^{2\sigma t}\int_\Omega\|u(x,t)\|^2dx&\le M_1[1+\frac{2}{h}(1-h)]\\&-2\varepsilon\sum_{i=1}^n\int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)}\{e^{2\sigma s}\int_\Omega(\nabla u_i(x,s))^2dx\}ds\\&=M_1[1+\frac{2}{h}(1-h)]-2\varepsilon(\int_0^{t-a}+\int_{t-a}^t)e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)}\times\\&\sum_{i=1}^n\{e^{2\sigma s}\int_\Omega(\nabla u_i(x,s))^2dx\}ds\\&\le M_1[1+\frac{2}{h}(1-h)]-2\varepsilon\int_{t-a}^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)}\times\\&\sum_{i=1}^n\{e^{2\sigma s}\int_\Omega(\nabla u_i(x,s))^2dx\}ds\,.\end{aligned}\tag{17}$$

Note that

$$\begin{aligned}&-2\varepsilon\int_{t-a}^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)}\sum_{i=1}^n\{e^{2\sigma s}\int_\Omega(\nabla u_i(x,s))^2dx\}ds\\&=-2\varepsilon a e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-\eta)}\sum_{i=1}^n\{e^{2\sigma\eta}\int_\Omega(\nabla u_i(x,\eta))^2dx\}\\&\le-2\varepsilon a e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)a}\sum_{i=1}^n\{e^{2\sigma\eta}\int_\Omega(\nabla u_i(x,\eta))^2dx\}\\&\qquad\eta\in[t-a,t]\,.\end{aligned}\tag{18}$$

Thus

$$e^{2\sigma t}\int_\Omega\|u(x,t)\|^2dx+2\varepsilon a e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)a}\times\sum_{i=1}^n\{e^{2\sigma\eta}\int_\Omega(\nabla u_i(x,\eta))^2dx\}\le M_1[1+\frac{2}{h}(1-h)]\,. \tag{19}$$

Since the right side of (19) is independent of t, for $t>a$, we have

$$\begin{aligned}&\sup_{t-a\le s\le t}\{e^{\sigma s}\int_\Omega\|u(x,s)\|^2dx\}\\&+2\varepsilon a e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)a}\sum_{i=1}^n\sup_{t-a\le s\le t}\{e^{2\sigma s}\int_\Omega(\nabla u_i(x,s))^2dx\}\\&\le M_1[1+\frac{2}{h}(1-h)]\,.\end{aligned}\tag{20}$$

Such that

$$\begin{aligned}&\int_\Omega\|u(x,t)\|^2dx+2\varepsilon a e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)a}\sum_{i=1}^n\int_\Omega(\nabla u_i(x,t))^2dx\\&=\{e^{2\sigma t}\int_\Omega\|u(x,t)\|^2dx+2\varepsilon a e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)a}\times\\&\sum_{i=1}^n\int_\Omega e^{2\sigma t}(\nabla u_i(x,t))^2dx\}e^{-2\sigma t}\\&\le[\sup_{t-a\le s\le t}\{e^{2\sigma s}\int_\Omega\|u(x,s)\|^2dx\}+2\varepsilon a e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)a}\times\\&\sum_{i=1}^n\sup_{t-a\le s\le t}\{e^{2\sigma s}\int_\Omega(\nabla u_i(x,s))^2dx\}]e^{-2\sigma t}\\&\le M_1[1+\frac{2}{h}(1-h)]e^{-2\sigma t}\end{aligned}\tag{21}$$

From formula (21), it is easy to conclude that the zero solution of (1) is $\mathbf{X}^{1,2}(\Omega)$ asymptotic stable.

In the following, we shall prove that $\lim_{t\to\infty}u(x,t)=0$ holds uniformly for $x\in\bar\Omega$.

From (21) we know,

$$\lim_{t\to\infty}\|u_i(x,t)\|_{L^2}=\lim_{t\to\infty}\|\nabla u_i(x,t)\|_{L^2}=0,\quad i=1,2,\cdots,n. \tag{22}$$

At the mean time, there must exists a positive constant M that,

$$|u_i(x,t)|\le M,\ \text{and}|\nabla u_i(x,t)|\le M \tag{23}$$

holds for any (x,t), where $(x,t)\in\bar\Omega\times R^+$.

From (23) we know, $\|u_i(x,t)\|_\infty$ and $\|\nabla u_i(x,t)\|_\infty$ are bounded functions. Cite the inequality in reference [5]

$$\|\mathbf{Z}\|_p\le\|\mathbf{Z}\|_\infty^{(p-2)/p}\|\mathbf{Z}\|_2^{2/p},$$

then if $p\ge2$, we have

$$\|u_i(x,t)\|_p\le\|u_i(x,t)\|_\infty^{(p-2)/p}\|u_i(x,t)\|_2^{2/p},\quad(i=1,2,\cdots,n)\,. \tag{24}$$

and

$$\|\nabla u_i(x,t)\|_p\le\|\nabla u_i(x,t)\|_\infty^{(p-2)/p}\|\nabla u_i(x,t)\|_2^{2/p},\quad(i=1,2,\cdots,n)\,. \tag{25}$$

Again from (22) we get

$$\lim_{t\to\infty}\|u_i(x,t)\|_p=\lim_{t\to\infty}\|\nabla u_i(x,t)\|_p=0,\quad(i=1,2,\cdots,n)\,. \tag{26}$$

According to Sobolev inequality [7], if $m<p$, we have

$$\|V\|_\infty\le C\{\|V\|_p+\|\nabla V\|_p\}\equiv C\|V\|_{W^{1,p}}\,. \tag{27}$$

Where $C=C(\Omega,m,p)$ is a positive constant.

Hence,

$$\|u_i(x,t)\|_\infty\le C\{\|u_i(x,t)\|_p+\|\nabla u_i(x,t)\|_p\}\,. \tag{28}$$

From (26) we have

$$\lim_{t\to\infty} \|u_i(x,t)\|_\infty = 0, \quad i = 1, 2, \cdots, n. \qquad (29)$$

From (29) we know the next equation will uniformly hold for $x \in \bar{\Omega}$.

$$\lim_{t\to\infty} u_i(x,t) = 0, \; i = 1, 2, \cdots, n\,. \qquad (30)$$

Next, we consider the equation with boundless delay

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = D(t)\Delta u(x,t) + \mathbf{A}(t)u(x,t) \\ \quad + \int_{-\infty}^{t} \mathbf{B}(t,s)u(x,s)ds, \\ \quad (x,t) \in \Omega \times \mathbf{R}^+ \\ u(x,t) = \varphi(x,t), \\ \quad (x,t) \in \Omega \times \mathbf{R}^- \\ \frac{\partial u(x,t)}{\partial v} + \mathbf{C}(\mathbf{x},\mathbf{t})\mathbf{u}(\mathbf{x},\mathbf{t}) = \mathbf{0}, \\ \quad (x,t) \in \partial\Omega \times \mathbf{R} \end{cases} \qquad (31)$$

, where $B(t,s)$ is a continuous $\mathbf{n} \times \mathbf{n}$ matrix function for t and s,$-\infty < s \le t < +\infty$, $\mathbf{R}^- = (-\infty, 0]$, $\mathbf{R} = (-\infty, +\infty)$. The others are the same with those in (1).

□

Theorem 2. *When* $\|B(t,s)\| \le be^{-h(t-s)}$, $b < rh$, *the zero solution of equation (31) is* $\mathbf{X}^{1,2}(\Omega)$ *asymptotic stable, therefore, all the solutions* $u(x,t)$ *uniformly holds for any* $x \in \bar{\Omega}$ *that* $\lim_{t\to\infty} u(x,t) = 0$, *where* r *is the same with that of Theorem 1.*

Proof. Since $b < rh$, it must exist enough small positive real number ε and σ, that

$$\frac{b}{2(r - \frac{\varepsilon}{l^2} - \sigma)}(\frac{1}{h} + \frac{1}{h - 2\sigma}) < 1$$

similar to Theorem 1 we can get

$$\begin{aligned} &\int_\Omega \|u(x,t)\|^2 dx \le M_2 e^{-2(r-\frac{\varepsilon}{l^2})t} - \\ &2\varepsilon \sum_{i=1}^{n} \int_0^t e^{-2(r-\frac{\varepsilon}{l^2})(t-s)} \int_\Omega (\nabla u_i(x,s))^2 dxds \\ &+ 2\int_0^t e^{-2(r-\frac{\varepsilon}{l^2})(t-s)} \int_\Omega \|u(x,s)\| \times \\ &\int_{-\infty}^{s} be^{-h(s-\theta)} \|u(x,\theta)\| d\theta dxds \\ &\le M_2 e^{-2(r-\frac{\varepsilon}{l^2})t} - 2\varepsilon \sum_{i=1}^{n} \int_0^t e^{-2(r-\frac{\varepsilon}{l^2})(t-s)} \times \\ &\int_\Omega (\nabla u_i(x,s))^2 dxds + \frac{b}{h} \int_0^t e^{-2(r-\frac{\varepsilon}{l^2})(t-s)} \times \\ &\int_\Omega \|u(x,s)\|^2 dxds + \int_0^t e^{-2(r-\frac{\varepsilon}{l^2})(t-s)} \times \\ &\int_\Omega \int_0^\infty be^{-h\theta} \|u(x,s-\theta)\|^2 d\theta dxds\,, \end{aligned} \qquad (32)$$

where $M_2 = \sup_{-\infty\le s\le t} \int_\Omega \|\varphi(x,s)\|^2 dx$. For $\sigma > 0$, we have

$$\begin{aligned} &e^{2\sigma t} \int_\Omega \|u(x,t)\|^2 dx \le M_2 e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)t} \\ &- 2\varepsilon \sum_{i=1}^{n} \int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)} \int_\Omega e^{2\sigma s} (\nabla u_i(x,s))^2 dxds \\ &+ \frac{b}{h} \int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)} \int_\Omega e^{2\sigma s} \|u(x,s)\|^2 dxds \\ &+ b\int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)} \int_\Omega \int_0^\infty e^{-(h-2\sigma)\theta} \times \\ &e^{2\sigma(s-\theta)} \|u(x,s-\theta)\|^2 d\theta dxds \\ &\le M_2 - 2\varepsilon \sum_{i=1}^{n} \int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)} \int_\Omega e^{2\sigma s} (\nabla u_i(x,s))^2 dxds \\ &+ \frac{b}{h} \int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)} \int_\Omega \sup_{-\infty\le\theta\le s} \{e^{2\sigma\theta} \|u(x,\theta)\|^2\} d\theta dxds \\ &+ b\int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)} \int_\Omega \int_0^\infty e^{-(h-2\sigma)\theta} \times \\ &\sup_{-\infty\le\theta\le s} \{e^{2\sigma\theta} \|u(x,\theta)\|^2\} d\theta dxds \\ &\le M_2 - 2\varepsilon \sum_{i=1}^{n} \int_0^t e^{-2(r-\frac{\varepsilon}{l^2}-\sigma)(t-s)} \times \\ &\int_\Omega e^{2\sigma s} (\nabla u_i(x,s))^2 dxds \\ &+ \frac{b}{2(r - \frac{\varepsilon}{l^2} - \sigma)}(\frac{1}{h} + \frac{1}{h - 2\sigma}) \times \\ &\sup_{-\infty\le s\le t} \{e^{2\sigma s} \int_\Omega \|u(x,s)\|^2 dx\}\,. \end{aligned} \qquad (33)$$

Similar to the inferences of (11) - (30), we can get the conclusion of theorem 2. □

3 Conclusion

In this paper, we studied the stability of the solution of the Logistic biological model with delay. We proposed two theorems to provide a succinct criterion of asymptotic stability of the zero solution through inequality value evaluating method.

References

[1] Arditc, A. & Riceidri, P. Nonlimear Anal. 1980, 2: 411 - 421.

[2] Mengxing he, Abstract Functional Differential Equations VII Bounded, Periodic and Almost Periodic Solutions, stable and unstable Manifolds, ACTA mathematica Sinica, Peking, 1990, 33(2): 205-231.

[3] Siaoling Xiang, Existence of Periodic Solutions for Delay Partial Differential Equation, ACTA Mathematica Application Sinica Peking, 1991, 14(1):73-81.

[4] Shengli Sie, Stability of Solution for Parabolic Partial Functional Differential Equation with Delay. Notification for General Information of science, Peking, 1991, 36(18): 1435-1436.

[5] Gilabarg, D. & Trudinger, N.S. Elliptic Partial Differential Equations of Second order, Springer-Verlag. Heidelbeg, New York, 1977.

[6] Bers, L. Joho, F. and Schechter, M. Partial Differential Equations, Amer. Math. Soc. Providence, Rhode Island, 1964.

[7] Adams, R. A. Soblev Spaces. new York: Academic Press. 1975.

遅延を持つ汎関数の偏微分方程式の解の安定性について

代　立新*・張　善俊**

概　要

生態系のダイナミックな状態は、しばしば遅延を持つ汎関数の偏微分方程式で説明される。本論文では、このような汎関数の偏微分方程式の解の安定性を不等式で推定し、ゼロ解の漸近収束の簡潔な判定基準を提示した。

キーワード：遅延、汎関数偏微分方程式、解の安定性

＊荊州師範学院　数学系、＊＊情報工学科　(知識工学講座)
