脆性物質の破壊形状成長に関する数値シミュレーション

Mem. Muroran Inst. Tech., 59（2009）149～154

Numeric Simulation of Damage Pattern Growth in Heterogeneous Brittle Material

Li WANG*, ShiBo LI** and Zheng-yi WANG*

(Received 27 May 2009, Accepted 20 November 2009)

The study of damage pattern growth in brittle material is of fundamental importance for understanding the gestation mechanism and occurring conditions of catastrophe of the great structures. It is especially important in understanding the dynamic catastrophe in rock, such as seismic, rock burst, shock bump, and the three kinds of outburst which are the outburst of water, gas and coal in coal underground mining. In the paper, damage pattern growth in a 2-D sample of heterogeneous brittle material is simulated, which is based on the principle that the damage evolution is irreversible, and the lattice finite element is used for the numeric model. The simulation showed that the complex damage pattern was formed by deduction, which was controlled by the simple dynamic law of the individual element and the strong interaction between elements. The damage pattern grows from dots to lines and to a fractal of which geometry dimension is between lines and a plane.

Keywords : damage pattern, numeric simulation, heterogeneous brittle material, lattice finite element, catastrophe prediction

1 INTRODUCTION

Failure in solids is a problem of scientific and technological importance and is a complex and fundamental problem dealing with a wide range of disciplines including mechanics, physics, and nonlinear science. Especially, the research of damage evolution induced catastrophe in brittle materials has great importance in disaster mitigation. And it is the theory foundation of prediction of dynamic disasters in great civil structures. In mechanics, damage evolution induced catastrophe was known as a problem that involves multi-scale mechanical problem. In the paper, a numerical method was suggested to simulate the course of damage evolution in a rock sample, and the damage pattern was displayed for a whole course which from the micro-damages distributed in random mode to fracture bands. We hope it could give some help in understanding the course of disaster gestated in the engineering structures.

Pattern can be regarded as a kind of space structure, such as geometry feature and symmetry. It was used in all kinds of physical phenomenon. In the paper, pattern was used to represent a time-space structure. The complicated mechanism of damage evolution can be described with pattern growth and formation, competition and choosing, gradual change and catastrophe. According to the principle of irrepressible damage evolution, the growth of damage pattern of a 2-D plate made of rock was simulated on the model of lattice finite elements, which was loaded in a self-adaptive mode. The result showed that the damage pattern grows in fractal which began with spotted damage and ended with run-through cracks. The mechanism of damage of meso-elements is based on the rule of max tensile strain, and numerical program was developed on the platform of ANSYS.

* Department of Civil Engineering of Henan Polytechnic University, Jiaozuo, 454000, China
** College of Resource and Environment of Hebei Polytechnic University, Tangshan, 063000)

2 MESCOPIC DISORDER AND LATTICE FINITE ELEMENT MODEL

The complicated and variable pattern of damage in brittle materials is rooted on the two fundamental factors[3]:

(1)The system of structure of brittle materials is disordered, trans-scale inhomogeneous and discontinuity, that is it's a multi-scale disordered structure.

(2)The system with damage and fracture usually is non-equilibrium, especially when external forces were loaded.

So the growth of the damage pattern in dynamic mode can be done by irreversible and non-equilibrium iterations of a disordered system. The disorder in mesoscopic is introduced like this: The plate was scattered into discrete elements of very small size (Fig.1), which was regarded as a RVE(Representive Volume Element) of the material. The elastic modulus E of the RVEs satisfied Weibull probability distribution function:

$$f(E)=\frac{m}{\hat{E}}\left(\frac{E}{\hat{E}}\right)^{m-1}e^{-(E/\hat{E})m} \tag{1}$$

In which $\hat{E}$ is scale parameter, *m* is shape factor of Weibull. The random values of the elastic modulus were produced by the method of Monte Carlo simulation.

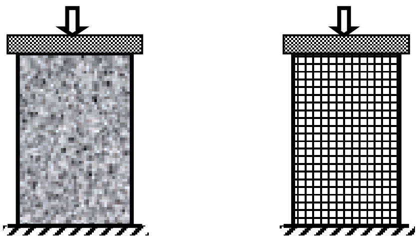

Fig. 1. (a)2-D sample of rock under uniaxial compression test , (b) the lattice finite model

3 CRITERION OF MESOSCOPIC DAMAGE AND IRREVERSIBLE EVOLUTION

Thought in macroscopic, the mode of fracture and post peak curve of elastic-plasto are variable, and so they are the whole behavior of the system. In mesoscopic, the damage mechanism of each element is simple. As for rock, the damage mechanism of maximum tensile strain should be reasonable. Consider a mesoscopic element under complex stress state of $\sigma(\sigma_1, \sigma_2, \sigma_3)$, the stress state can be divided into two cases according to twin-shear strength theory[9]:

When $\sigma_2 \le \frac{\sigma_1+\alpha\sigma_3}{1+\alpha}$, the elements was in a state of tensile deformation, the damage law is:

$$D=\begin{cases}0 & \varepsilon_I \le \varepsilon_{t0}\\ 1-\dfrac{\lambda\varepsilon_{t0}}{\varepsilon_I} & \varepsilon_{t0}\le\varepsilon_I<\varepsilon_{tu}\\ 1 & \varepsilon_I\ge\varepsilon_{tu}\end{cases} \tag{2}$$

When $\sigma_2 > \frac{\sigma_1+\alpha\sigma_3}{1+\alpha}$, the elements was in a state of compression, and the maximum strain is also still the variable controlling the damage development, the damage law is:

$$D=\begin{cases}0 & \varepsilon_I > \varepsilon_{t0}\\ 1-\dfrac{\lambda\varepsilon_{t0}}{\varepsilon_I} & \varepsilon_I\le\varepsilon_{t0}\end{cases} \tag{3}$$

In which, ε_I is the maximum principal strain of the element ; ε_{tu} is the strain threshold of damage ; λ is the factor of remain strength in tensile state ; ε_{tu} is the limit tensile strain.

Here, we assumed that the effect of the micro-damage on elastic modulus of the element is same in all directions, so the elastic modulus of the element after k steps iterative calculations is written as:

$$E(1-D^{(k)})=E(1-D_{(1)})(1-D_{(2)})\cdots(1-D_{(k)}) \tag{4}$$

The irreversible damage evolution is written as:

$$D^{(k)}=1-(1-D_{(1)})(1-D_{(2)})\cdots(1-D_{(k)}) \tag{5}$$

In which $D^{(k)}$ represents the accumulated damage after k steps iterative calculation;
$D_{(k)}$ represents the relative damage at step k, is determined by formula (3) and (4).

4 ITERATIONS OF NON-EQUILIBRIUM AND ADAPTIVE LOADING

At a certain load step, damage and fracture of elements result in bearing capacity decreases, which causes redistribution of stress in the structure. As a result, the stress around the damage zones increase, and further, the elements around the damage zones will damage and crack under the increased stress. This is a course of damage pattern and stress pattern coupling with each other. They are alternatively expanding and stopped. In numeric method, the stress redistribution is carried out by non-equilibrium iterations, and adaptive displacement load simulates the real load.

4．1 Non-equilibrium Iterations

At the load of step k , the finite element equation is:

$$K^{(k)}a^{(k)}=P^{(k)} \tag{6}$$

In which $a^{(k)}$ is the displacement array of step k; $P^{(k)}$ is node load array; $K^{(k)}$ is the overall stiffness matrix, it is assembled by element stiffness matrix:

$$K^{(k)} = \sum_e (K^{(e)})^{(k)} = \sum_e \int_{V_e} B^T (D^{(e)})^e B dV \tag{7}$$

In which B is strain matrix, $(D^{(e)})^{(k)}$ is element stiffness matrix of stepk , it is expressed in tensor as:

$$(D^{(e)}{}_{ijkl})^{(k)} = (2G\delta_{ik}\delta_{jl} + \lambda\delta_{ij}\delta_{kl}) \tag{8}$$

In which G, λ are Lamei constants.

The non-equilibrium load result of element damage is

$$(K^{(k)} - \widetilde{K}^{(k)})a^{(k)} = \Delta P^{(k)} \tag{9}$$

In which $\widetilde{K}^{(k)}$ is the overall stiffness matrix because of damage at step k

$$\widetilde{K}^{(k)} = \sum_e (\widetilde{K}^{(e)})^{(k)} = \sum_e \int_{V_e} B^T (\widetilde{D}^{(e)})^{(k)} B dV \tag{10}$$

In which $(\widetilde{D}^{(e)}{}_{ijkl})^{(k)}$ is constitutive matrix of the damaged element, it is written as in tensors:

$$(\widetilde{D}^{(e)}{}_{ijkl})^{(k)} = (D^{(e)}{}_{ijkl})^{(k)}(1 - D^{(k)}) \tag{11}$$

Keep the Load and boundary unchanged, approach equilibrium by iterative calculation, the stress was redistributed:

$$\widetilde{K}^{(k)} \Delta a^{(k)} = \Delta P^{(k)} \tag{12}$$

The displacements was resolved as $\widetilde{a}^{(k)}$

$$\widetilde{a}^{(k)} \leftarrow a^{(k)} + \Delta a^{(k)} \tag{13}$$

The accumulated displacement is $\widetilde{\varepsilon}^{(k)}$

$$\widetilde{\varepsilon}^{(k)} = B\widetilde{a}^{(k)} \tag{14}$$

Use formula (3) and (4) to each element, and calculate their damage value, then enter the formula (12) ～ (14). Repeat this course, until no damage produces. Then the step k calculation ends and the program enter the next step calculation.

4．2 Determination the Step Size of Adaptive Load

In order to simulate fracture under quasi-static load, adaptive load was used. In the step k, load is written as:

$$P^{(k)} = P^{(0)} + \Delta P^{(k)} \tag{15}$$

$P^{(0)}$ is the initial displacement determined for a linear elastic phrase. $\Delta P^{(k)}$ is the increment of displacement load at step k. The method of adaptive load is to adjust the increment $\Delta P^{(k)}$ automatically as the displacement load increases so that the structure can not breakdown rapidly because of a large increment of displacement load, and also, the program will not do calculation repeatedly because of too small increment of displacement load.

The adaptive increment $\Delta P^{(k)}$ of load is determined as the following: at step k, a small initial increment $\Delta P_0^{(k)}$ of load is given, substitute it to (15) and (6), the initial stress corresponding to $\Delta P_0^{(k)}$ is resolved, further, calculate the maximum principal stress $(\sigma_I^{(e)})^{(k)}$ for each element, then the load factor is defined as [11]

$$\rho^{(k)} = \max_e \left\{ \frac{(\sigma_I^{(e)})^{(k)}}{\sigma_b^{(e)}} \right\} \tag{16}$$

The increment of load at step k is expressed as

$$\Delta P^{(k)} = \frac{\Delta P_0^{(k)}}{\rho^{(k)}} \tag{17}$$

In which $\sigma_b^{(e)}$ represents the tensile strength of element calculated by initial elastic modulus and limit tensile strain.

5 RESULT OF SIMULATION

A plate made of brittle rock, which size is 10m×5m, was divided by 120×60 into 7200 elements. The position rate is 0.29. The plate was fixed at one end and loaded at the other end (see Fig. 1). The damage parameters are: ε_{t0}=5×10^{-3}, ε_{tu}=8×10^{-3}, λ=0.5, α=0.1.

5．1 The Growth of Damage Pattern

The whole course of damage pattern growth is shown in Fig. 2, in which the Webull parameters are: $\hat{E} = 45GPa$， $m = 1.2$. The damage element was painted black. The whole course of pattern growth can be divided into four phases:

The first phase is microdamage emerge and growth. It is R6~R17 in the Fig. 2. The microdamage emerges in the plate randomly. The difference between their size is very small.

The second phase is the microdamage growth, connection into large size damage. It was showed in R18～R23 of Fig. 2. At this phase, multi-scale damages coexist, and the large size damage is superior in growth.

The third phase the crack is formed, and connected quickly. It is R24～27 in the Fig. 2.

The fourth phase is the crack connection and run-through. It is R28～R35 in the Fig. 2.

The correspondence between the number of micro-damage and bearing stress during damage evolution was shown in Fig. 3. Each microdamage could be regarded as a sonic emission. The correspondence between accumulated number of micro-damage and bearing stress was shown in Fig. 4.

5．2 The Affection of Disorder On Macro Mechanical Quantity

The Fig. 5 showed that the bearing capacities are greatly different because of the Weibull parameter m difference, though the samples look the same.

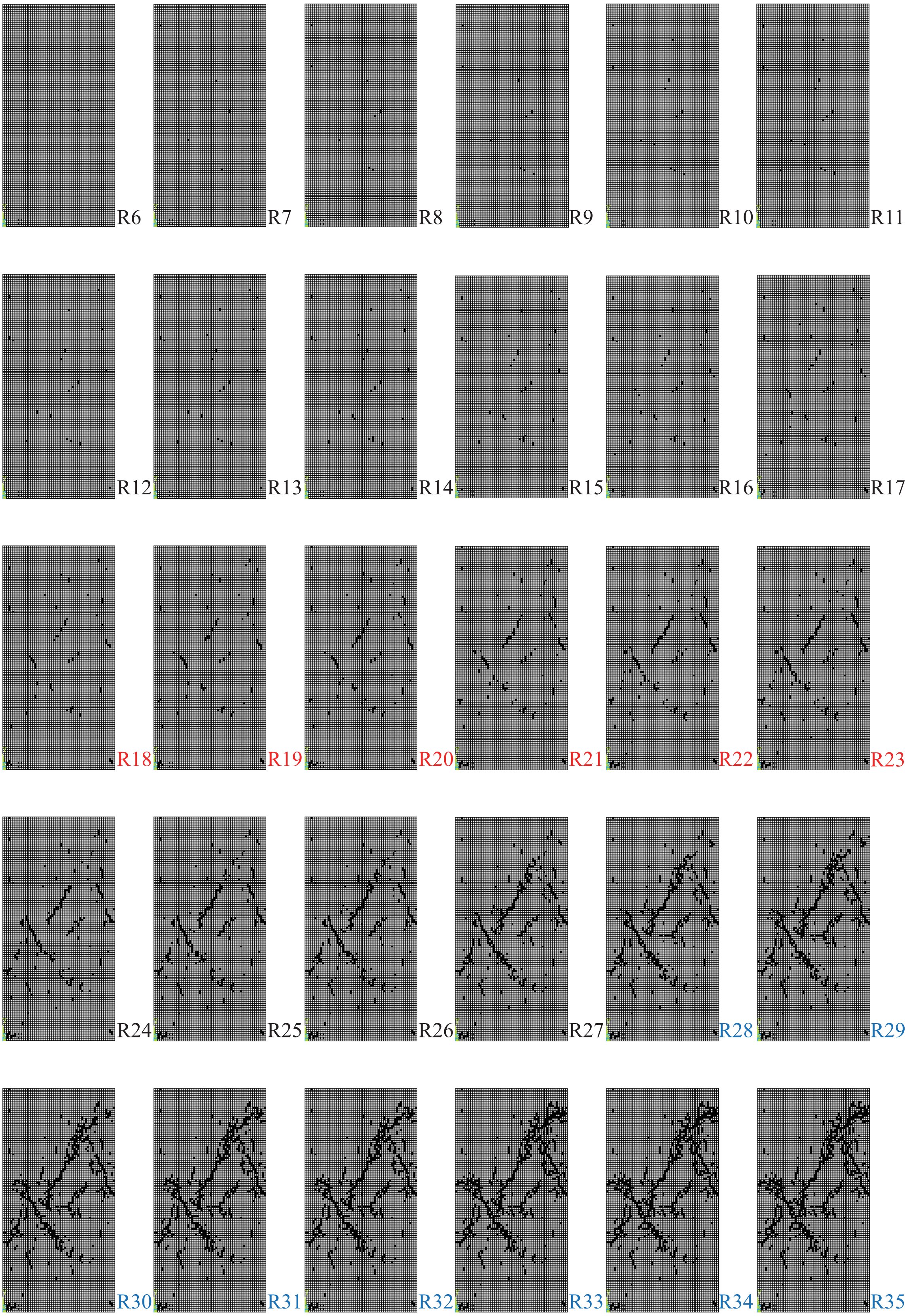

Fig. 2. Demonstration of the course of growth of damage pattern

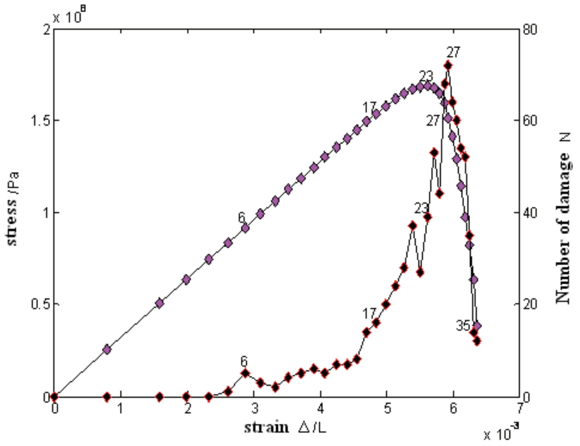

Fig. 3. The curves of stress and number of micro-damage related to strain on the top of the sample

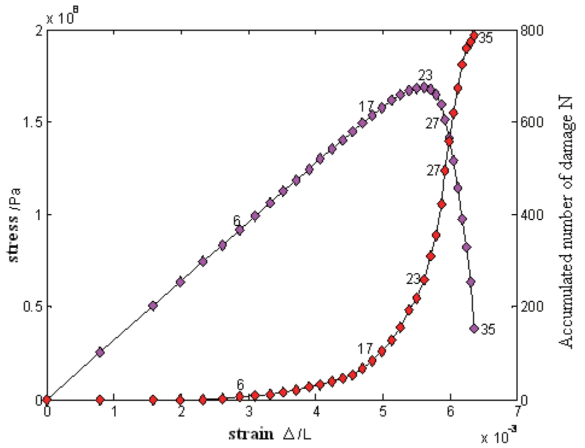

Fig. 4. The curves of stress and accumulated number of micro-damage related to strain

6 CONCLUSIONS

The simulation showed that the complex damage pattern was formed by deduction, which was controlled by the simple dynamic law of the individual element and the strong interaction between elements. According to the theory of complex system[13], the system consisted of a great amount of mesoelements spontaneously evolves to self-organized criticality. Appropriate to the evolution, the growth of damage pattern was called dynamics of fractal growth of Laplace., which the boundary of damage pattern moves outward in fractal.

The damage pattern grows from dots to lines and to a fractal which its geometry dimension is between lines and a plane. At the end of a large crack, the microcrack distributes in clusters, and its domain direction controls the direction of the large crack. At a certain mode of load, the cracks parallel arrange just like the flying array of wild gooses, this can be proved in many paper such as Art[14].

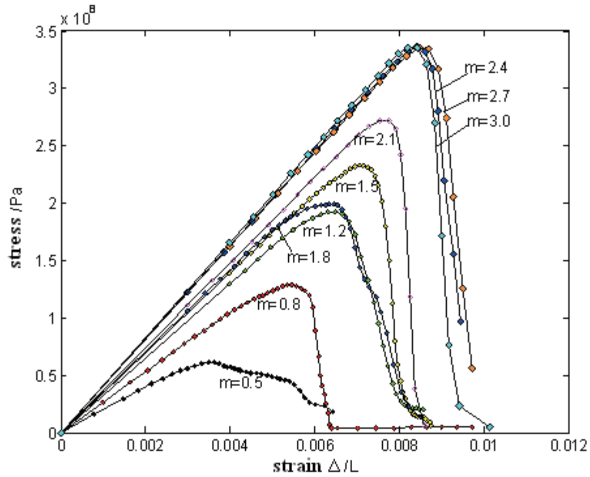

Fig. 5. Curves of stress-strain on the top of the sample at different parameter, *m*

REFERENCES

(1) Yu Shouwen, Feng xiqiao. Damge Mechanics[M]. Beijing : Tsinghua University Press, 1997.

(2) Xia Mengfeng, HanWensheng, Ke Fjiu etal.. Statistical Mesoscopic Damage Mechanics and Damage Evolution Induced catastrophe(I)[J]. *Advanced Mechanics*, 1995, Vol25(1)1~24

(3) Xia Mengfeng, HanWensheng,Ke Fjiu etal.. Statistical Mesoscopic Damage Mechanics and Damage Evolution Induced catastrophe(II)[J]. *Advanced Mechanics*, 1995, Vol25(2)145~159

(4) Xia Mengfeng. Pattern dynamics—The special report (9) of nonlinear science [J] *Physics Report*, 1999, No 4 : 3~6

(5) Aidun J B, Lo D C S, Trucano T G, Fye R M. Representative volume size : a comparison of statistical continuum mechanics and statistical physics. SAND99-1176, 1999

(6) Xia M F, Song Z Q, Xu J B, Zhao K H,et al. Sample-specific behavior in failure models of disordered media. *Commun Theor Phys*, 1996, 25(1) : 49~54

(7) Bai Yilong Wang Haiying, Xia Mengfeng.The Statistical Mesomechanics of Solids-−−Connecting multi-couping scales of time-space [J]. *Advanced Mechanics*, 2006, Vol.36(2) : 286~305

(8) Xia mengfen, Ke fujiu, Bai Jie. Ensemble statistics of catastrophe induced by the phenomenon of failure coupling pattern evolution [J]. Chinese Science Bulletin, 1999, Vol.44(6) : 562~572

(9) Yu Maohong. Twin Shear Theory and Its Applications Beijing:Science Press, 1998

(10)Tang C A, Kaiser P K. Numerical simulation of cumulative damage and seismic energy release during brittle rock failure-Part I : Fundamentals. *Int J Rock Mech Min Sci,*35(2) : 113~121

(11)ChenYongqiang, Zheng Xiaoping, Yao Zhenhan. Numerical simulation of failure processes in 3-D heterogeneous brittle material[J]. *Acta Mechanica Sinica*, 2002, Vol.34(3) : 351～361

(12)SHENG B, KING A, GUO H. Displacement, stress and seismicity in roadway roofs during mining-induced failure[J]. International Journal of Rock Mechanics and Mining Sciences, 2008, 45(5) : 672~688

(13)Yu Chongwen. Complexity of Geological System [M]. Beijing : Geological Press, 2003

(14)Scholz C H. Experimental study of fracturing process in brittle rock [J]. *J. Geophys.Res,1*968, 73 : 1447~1486

脆性物質の破壊形状成長に関する数値シミュレーション

Li WANG*, ShiBo LI** and Zheng-yi WANG*

概要
脆性物質の破壊形状成長に関する研究は，その形成の仕組みと崩壊の生じる条件を理解する上で基礎的な重要性を有する。特に，岩盤で生じる動的な激変，たとえば，地震，破裂，衝撃隆起，水，ガス，石炭による爆発などの理解は重要である。本論文では，不均質の脆性物質の 2 次元サンプルにおける破壊形状の成長が，破壊の進展が非可逆であるという原理に基づいてシミュレーションされる。シミュレーションでは，格子有限要素が数値モデルとして用いられ，複雑な破壊形状を，単一要素の単純な力学法則と要素間の強い相互作用によって統制されていると見なして，推論によって求めている。破壊形状成長は，点から線，そして幾何学的次元が線と面の間であるフラクタル形状にいたる。

キーワード : 破壊パターン, 数値シミュレーション, 不均質なもろい物質, 格子状有限要素,
カタストロフィー予測

*Department of Civil Engineering of Henan Polytechnic University, Jiaozuo, 454000, China,
**College of Resource and Environment of Hebei Polytechnic University, Tangshan, 063000)
