Numerically designed phase-mask for stellar coronagraph
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ABSTRACT
Phase-mask coronagraph holds the ability to detect exoplanets very close to their parent star. We report a new kind of phase mask that performs the contrast ratio of more than the tenth power of 10 for a circular aperture with shades of a secondary mirror and spiders. The phase distribution of the phase mask is numerically obtained by making the leaked light distribute outside the transparent part of the pupil. We applied the hybrid input-output algorithm, one of phase retrieval methods, to find the phase distribution of the phase mask. We show the characteristics of thus obtained phase mask.

Keywords: stellar coronagraph, high-contrast imaging, phase mask, exoplanet, telescope, phase retrieval

1. INTRODUCTION
Nearly 2000 exoplanets have been reported their detection and some of them have been directly detected [1]. Exoplanets are located near their parent star, and therefore, imaging of faint exoplanets near the bright star requires very high-contrast imaging scheme. In the direct detection of exoplanets a phase-mask coronagraph is one of promising methods. The phase-mask coronagraph holds the ability to detect exoplanets at a distance of around $1 \lambda/D$ from their parent star where $\lambda$ denotes an observational wavelength and D a diameter of a telescope. Phase masks like a four-quadrant [2], an eight-octant [3], and a vortex [4, 5] types are shown to completely extinguish starlight for circular aperture telescopes. The problem here is that ordinary telescopes do not have perfect circular apertures but have shades of a secondary mirror and spiders. It is known that shades of a secondary mirror and spiders deteriorate significantly the performance of phase-mask coronagraph. There have been proposed several ideas to overcome the deterioration. A pupil-remapping optical element is used to convert a secondary mirror obscured aperture to a clear circular one [6]. A multistage vortex coronagraph has been proposed to reduce the deterioration [7]. Apodization to an entrance pupil has been shown effective to avoid the deterioration [8, 9]. However, these ideas are based on using theoretically established phase-masks.

We report a new kind of phase mask that performs a contrast ratio of more than the tenth power of 10 for a circular aperture with shades of a secondary mirror and spiders. The phase distribution of the phase mask is numerically obtained by letting the leaked light outside the transparent part of the pupil. The Lyot filter employed here is the same shape of the entrance pupil without any margin. We applied the hybrid input-output algorithm [10] to find the phase distribution of the phase mask. The characteristics of thus obtained phase mask are shown.

2. NUMERICAL DESIGNE OF PHASE MASK
Here we consider an aperture with shades of a secondary mirror and spiders. We employ FFT with an array size of 1024 $\times$ 1024. The diameters of the outer and inner aperture rings are set to 64 and 20, respectively. The width of the spider is set to 5. When we place a vortex phase-mask with a vortex charge of 2 as shown its phase distribution with a modulo of
2π in Fig.1(a) on the focal plane, the re-imaged pupil becomes as shown in Fig. 1(b). Light is leaked into the transparent part of the pupil. When a Lyot filter with the same shape as the pupil image is placed on the re-imaged pupil plane, the PSF of an on-axis point source becomes as shown in Fig. 1(c). The maximum intensity is $8.1 \times 10^{-3}$ that of PSF without the vortex phase-mask. The extinction ratio is poor and the vortex phase-mask is not efficient for an aperture with shades of a secondary mirror and spiders. Figure 1(d) shows off-axis PSF of a point source located at $1 \lambda/D$ in a direction of $\theta = 45^\circ$. The normalized maximum intensity is 0.18.

![Figure 1](http://proceedings.spiedigitallibrary.org/)
distribution is shown in gray levels. (c) Square root of the PSF intensity distribution of an on-axis source through a Lyot-filter with the same shape as the entrance pupil. (d) Square root of the intensity distribution of PSF of an off-axis source located at $1 \lambda/D$ in a direction of $\theta=45^\circ$.

We employ the hybrid input-output algorithm to find a phase mask that makes light distribute outside of the transparent portion of the pupil. When we denote the phase-mask function at the $k$-th iteration as $m_k(x, y)$, the amplitude distribution at the $k$-th iteration as $P_k(u, v)$, the $k+1$-th phase-mask function is derived as follows:

\begin{align}
P_k(u, v) &= F.T.\{PSF \times m_k(x, y)\} \\
P_{k+1}(u, v) &= \left\{ \begin{array}{ll}
P_k(u, v) & |P_k(u, v)| \geqslant \gamma \\
\alpha P_k(u, v) & |P_k(u, v)| < \gamma \\
\end{array} \right. \\
\alpha &= \left( \frac{|P_k(u, v)| - \beta |P_k(u, v)|}{|P_k(u, v)|} \right) \\
m_{k+1}(x, y) &= \text{Phase of } F.T.\{P_{k+1}(u, v)\}
\end{align}

where $\gamma$ designates the transparent portion of the pupil and $\beta$ is a feedback constant.

When an aperture holds a symmetric property like four-time symmetry in the aperture treated here, the symmetric constraint is inserted at each iteration. It is noticed that symmetry is sometimes automatically revealed without the symmetric constraint.

3. RESULT

We show one example of our results. Figure 2(a) shows a phase distribution of the resultant phase mask. The initial phase values are set all to zeroes. The iteration is repeated 10000 times and $\beta = 0.19$. A four-time symmetry constraint is imposed at each iteration. The gray tone shows $0 \sim 2 \pi$ from black to white. The phase distribution is rather complex. When this phase-mask is placed on the focal plane, the re-imaged pupil becomes as shown in Fig. 2(b) where square root of the intensity distribution is displayed in a linear gray scale. Compared with Fig. 1(b) almost all light is outside of the transparent portion of the pupil. The fractional energy inside the transparent portion of the pupil is $4.3 \times 10^{-10} \%$ of the total energy.

Figure 2(c) shows the PSF of an on-axis point source when a Lyot filter with the same shape as the pupil image is placed on the re-imaged pupil plane. There square root of the intensity distribution is displayed in a linear gray scale. The maximum intensity is $2.6 \times 10^{-13}$ that of PSF without the phase-mask. Extinction for on-axis light is highly performed with the phase mask shown in Fig. 2(a). The angular averaged intensity of the PSF is shown in Fig. 3. The normalized maximum intensity is less than $10^{-14}$. The extinction is high around the center and it is expected that exoplanets near their parent star can be detected. Figure 2(d) shows off-axis PSF of a point source located at $1 \lambda/D$ in a direction of $\theta=45^\circ$. The normalized maximum intensity is 0.17. On the other hand the normalized maximum intensity of PSF is 0.14 for a point source located at $1 \lambda/D$ in a direction of $\theta=0^\circ$. The intensity contrast ratio of the on-axis to an off-axis source is less than $10^{-10}$ with thus obtained phase-mask. It should be noted that there are no margins for the Lyot filter (exactly same shape of the entrance pupil), namely no loss of incident light by the Lyot filter.
The phase mask shown above is not a unique solution in the numerical design. There are other phase masks that perform similar extinction of light depending on parameters in the numerical calculation. It seems that some constraint on the iterative process will lead to more favorable phase-masks.

Figure 2. (a) Phase distribution of a phase-mask that makes light distribute outside of the transparent portion of the pupil. Phases are displayed from 0 to $2\pi$ with gray levels. (b) Re-imaged pupil through the phase-mask placed on the focal plane. Square root of the intensity distribution is shown in gray levels. (c) Square root of PSF intensity distribution of an on-axis source through a Lyot-filter with the same shape as the entrance pupil. (d) Square root of the PSF intensity distribution of an off-axis source located at $1\lambda/D$ in a direction of $\theta=45^\circ$. 
4. CONCLUSION

We show that a new kind of phase mask performs a contrast ratio of more than the tenth power of 10 at $1 \lambda /D$ for a circular aperture with shades of a secondary mirror and spiders. One of effective ways to directly image exoplanets may be to use phase masks designed numerically as described above. There need several strategies toward achromatization. Compensation of chromatic difference of PSF can be realized by using Wynne’s corrector [11]. One of promising ways to fabricate phase masks is use of liquid crystals [12] and multilayer liquid crystal achromatization technique has been proposed [13].

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