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Abstract. Since community structure is an important feature of complex network, the study of community detection has attracted more and more attention in recent years. Despite most researchers focus on identifying disjoint communities, communities in many real networks often overlap. In this paper, we proposed a novel MCLC algorithm to discover overlapping communities, which using random walk on the line graph and attraction intensity. Unlike traditional random walk starting from a node, our random walk starts from a link. First we transform an undirected network graph to a weighted line graph, and then random walks on this line graph can be associated with a Markov chain. By calculating the transition probability of the Markov chain, we obtain the similarity between link pairs. Next the links can be clustered into "link communities" by a linkage method, and these nodes between link communities can be overlapping nodes. When converting the "link communities" into the "node communities", we make a definition of attraction intensity to control the overlapping size. Finally the detected communities are permitted overlapped. Experiments on synthetic networks and some real world networks validate the effectiveness and efficiency of the proposed algorithm. Comparing overlapping modularity Q_{ov} with other related algorithms, the results of this algorithm are satisfactory.

Keywords: Community detection · Random walk · Link community · Overlapping community

1 Introduction

Including social, biological, and technological systems, many systems in world can be described as complex networks whose elements are neither purely regular nor purely random [1, 2]. One of the most relevant features of complex networks is community or modular structure, which should have more internal than external connections [3–5]. Communities often refer to groups or clusters, and people or things in same community often have more similarities.

Nowadays people's life has been inseparable from the mobile network[6–8]. As mobile web evolved from proprietary mobile technologies and networks to

full mobile access to the Internet[9–12], much like web-based social networking, mobile social networking occurs in virtual communities. Finding and analyzing community structure provides invaluable help in deeply understanding the structure and function of a network, as widely demonstrated by several case studies in social science[13], biology[14], ecology[15], economics[16] etc.

A large quantity of approaches for detecting community has been proposed over the years [17]. Most early approaches, such as the Kernighan-Lin algorithm [18], spectral partitioning [19], hierarchical clustering [20] , and modularity optimizing [21] etc, focus on identifying disjoint communities. This type of detection put each node into one and only one community. However, communities are nested and overlapped in most real world networks. For example, a social network where each vertex represents a person and communities represent different groups of friends: one community for family, another community for co-workers, still one for friends in the same sports club.

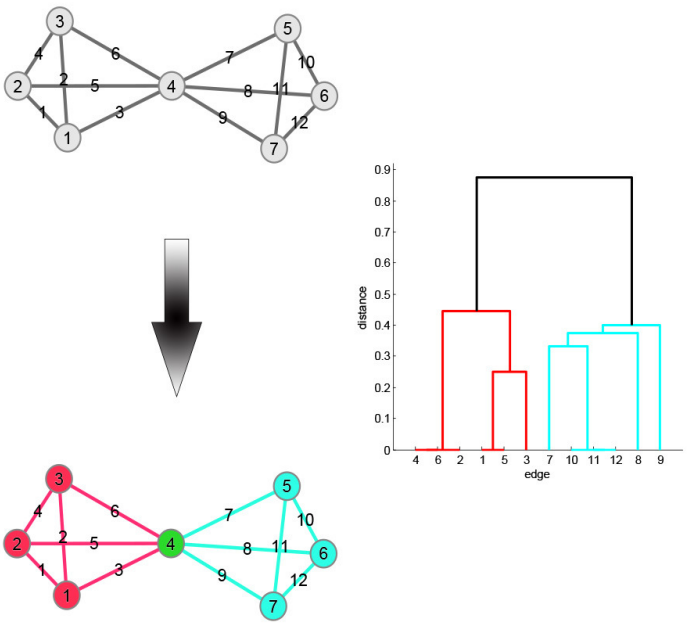


Fig. 1. Summary of the procedure for community analysis. From the network description (top panel) and a suitable definition of link distance, a hierarchical tree is derived by cluster analysis(left panel). The network finally divided into two link communities(bottom panel), overlapping node 4 is between two communities.

Indeed, overlap is quite a significant feature in real network. Therefore more and more researches focus on overlapping community detection, and many mea-

sures have been proposed. For instance, the CPM (Clique Percolation Method) algorithm [22] which is based on clique percolation , the LFM (Local Fitness Maximization) algorithm [23] which utilizes local expansion and optimization, the method based on a local definition of community strength [24], the link method which reinvents communities as groups of links rather than nodes [25], and some recent developed algorithms [26–29]. Among these measures, Evans et al. made a definition of line graph [30], and Ahn et al. proposed the notion of link community [25]. By coincidence, both of them divided network into small link groups and then mapped to node groups. As links reflect the relationship between node pairs, link based communities show the different features of groups in the whole network, and this paper focus on link based overlapping community detection method.

In addition, random walk has been a conventional method of detecting communities[31, 32]. Considering a network, there will be many links within a community, and fewer links between communities. If you were to start at a node, and then randomly travel to a connected node, you are more likely to stay within a community than travel between. By doing random walks upon the graph, it is possible to discover where the flow tends to gather, and therefore, where clusters are. There are many algorithms finding communities based on random walk, but most of random walks start from node and their detected communities are disjoint [33–35].

In the paper, we proposed an overlapping community detection algorithm based on Markov chain and link clustering (MCLC). The rough procedure of this method is shown in Fig.1. The distance (similarity) between link pairs can be obtained by computing the transition probability through random walks on the line graph. When the distance is smaller, namely the similarity is lager, the link pairs are more likely to be assigned into a same community. Clustering the link pairs according to the distance, thus link communities output. Finally, the link communities can be converted to node communities by setting an appropriate threshold of attention intensity. The last identified communities may have some overlapping nodes.

2 Related work

2.1 The weighted line graph model of network

Generally, a network can be simply regarded as a graph $G(V, E)$, where $|V| = N$ represents vertices and $|E| = M$ represents edges. In the most instances case the network is a directed weighted graph denoted by a $N \times N$ weight matrix $W = [w_{ij}]$, where $w_{ij} \geq 0$ is the weight of the link $i \to j$. The graph adjacency matrix $A = [a_{ij}]$ is a $N \times N$ binary matrix, where $a_{ij} = 1$ if $w_{ij} > 0$, and $a_{ij} = 0$ otherwise. In this paper, we consider the networks are undirected, namely $w_{ij} = w_{ji}$. If the network is also unweighted, then all weights equal to 1 and $W = A$.

Nodes and edges are two necessary roles of a network. Most of the network topology is based on the entities as the nodes, the relation as edges. However

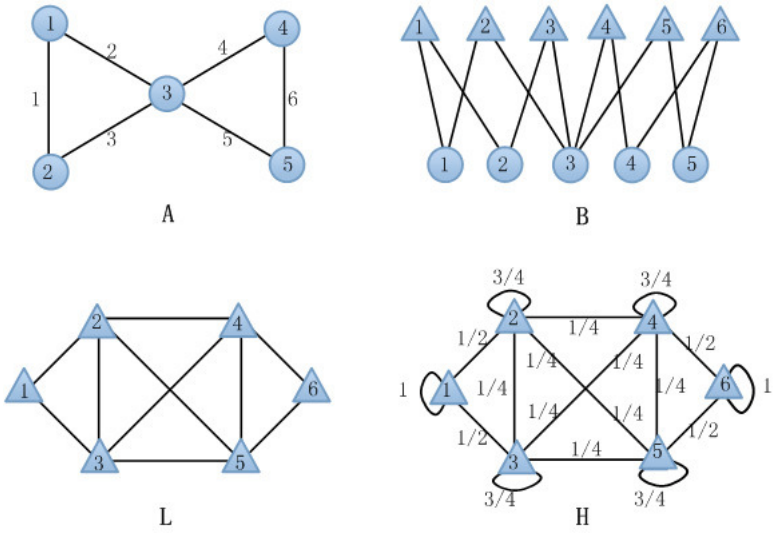


Fig. 2. Four different matrices

Evans et al.[30] proposed a definition of "line graph", which regards the relation as nodes and the entities as edges. The incidence matrix $B = [b_{i\alpha}]$ is an important bandage between node graph and line graph, which is a $N \times M$ matrix and the elements $b_{i\alpha}$ can be obtained by

$$b_{i\alpha} = \begin{cases} w_{ij}, \text{ node } i \text{ and } j \text{ are two ends of edge } \alpha; \\ 0, \quad \text{otherwise.} \end{cases} \tag{1}$$

The adjacency matrix of line graph $L = [l_{\alpha\beta}]$ is a $M \times M$ binary matrix , where $l_{\alpha\beta} = 1$ if link α and link β have a common node, otherwise $l_{\alpha\beta} = 0$. When random walking on the line graph, the probability of next step should be different. Thus we depend on a weighted link matrix $H = [h_{\alpha\beta}]$, which can be obtained by matrix B:

$$H_{\alpha\beta} = \sum_i \frac{b_{i\alpha} b_{j\alpha}}{s_i}. \tag{2}$$

where s_i is the strength of node i and $s_i = \sum_j w_{ij} = \sum_\alpha b_{i\alpha}$. The weights in H indicate the strength of link pairs. As seen in Fig.2, it has displayed four different matrices and illustrated the transformation of a graph adjacency matrix A into a weighted matrix H. We find there are self-loops in H, that is to say, each link have connection in itself. Thus a random walker on line graph can stay in a link (or actually walk between two connected nodes).

2.2 The distance between links

The $M \times M$ link weighted matrix H can be associated to a M-state Markov chain, the transition matrix $P = [P_{\alpha\beta}]$ is defined by

$$p_{\alpha\beta} = \frac{h_{\alpha\beta}}{\sum_{\beta} h_{\alpha\beta}} \tag{3}$$

Consider a large number of repetitions of a random walk start from link α. $[P^t]_{\alpha\beta}$ is the probability that the walker start from α and stay in β after t steps. If random walks of length T are performed from α, the excepted probability of visits to β is $\sum_{t=1}^{T} [P^t]_{\alpha\beta}$ $(1 \leq t \leq T)$. Cluster analysis can be used to group "similar link pairs" into candidate link communities. We propose a (symmetric) similarity $\phi_{\alpha\beta}$ defined by

$$\phi_{\alpha\beta} = \phi_{\beta\alpha} = \sum_{t=1}^{T} ([P^t]_{\alpha\beta} + [P^t]_{\beta\alpha}) \tag{4}$$

Then the distance $d_{\alpha\beta}$ between link pair (α, β) can be obtained by complementing the similarity and normalizing the results from 0 to 1,

$$d_{\alpha\beta} = d_{\beta\alpha} = 1 - \frac{\phi_{\alpha\beta} - min\ \phi}{max\ \phi - min\ \phi} \tag{5}$$

Note that the choice of the time horizon T is potentially critical. Cluster analysis yields a different hierarchical tree (dendrogram) for each time horizon T, whose choice is thus nontrivial. At the two extremes, setting $T = 1$ restricts the pairs of links which are candidate to nonzero similarity to neighboring pairs only, whereas larger and larger values of T tend to make any link equally similar to any other.

2.3 The function of attractive intensity

Suppose a network with M links, $C_L = \{P_1, P_2, \ldots, P_q\}$ is a partition of the links into q link communities, with $\bigcup_c P_c = E$ and $P_c \cap P_d = \emptyset$ for all c, d. In order to convert link communities into node communities, we proposed a function of attractive intensity $I_i^{P_c}$, which is defined as

$$I_i^{P_c} = \frac{\sum_{(i,j) \in P_c} w_{ij}}{\sum_j w_{ij}} = \frac{s_i^{in}(P_c)}{s_i} \tag{6}$$

where (i, j) represents a link with two end nodes i and j, s_i is the strength of node i, $s_i^{in}(P_c)$ is the sum weight of i connected links in link community P_c.

The value of $I_i^{P_c}$ is belong to $[0, 1]$, which indicates the attraction intensity from link community P_c to node i. When $I_i^{P_c} = 1$, node i is attracted by link community P_c completely, namely i is contained by P_c; when $I_i^{P_c} = 0$, link community P_c has no attraction to node i, namely i is out of P_c. So we only

need to consider the attractive intensity of edge nodes between link communities. These edge nodes can be expressed by:

$$edge\_node = \{u|(u,v) \in P_c, (u,w) \in P_d, c \neq d\} \tag{7}$$

All the edge nodes can be regard as overlapping nodes, but the number of overlapping nodes is often large in this case. Here we set a threshold δ of the attractive intensity to control the overlapping size: if the maximal attractive intensity I_{max} to an edge node u satisfied

$$I_{max} = \max_{1 \leq c \leq q} \{I_u^{P_c}\} = I_u^{P_m} > \delta \tag{8}$$

and the link community P_m is unique, then the edge node u can be entirely absorbed into the link community P_m; otherwise the edge node u is an overlapping node. As a result, some of edge nodes can be brought in an appropriate link community, the rest of edge nodes are still overlapping nodes.

3 The algorithm

According to the three main idea given above, our MCLC algorithm can be summarized as three main stage as follows:

1. Given an undirected network $G(V, E)$, number each link, then compute the incidence matrix B and the weighted link matrix H, the pseudo-code of H generating is shown in Algorithm 1 ;
2. Calculate the distance(similarity) between link pairs by random walking on the weighted line graph. The pseudo-code of link distance calculating can be described in Algorithm 2. Then we use average-linkage clustering method to divide network into q link communities $C_L = \{P_1, P_2, \ldots, P_q\}$;
3. For internal nodes in a link community, assign the node to the same community; for edge nodes which are between link communities, assign some of the edge nodes to an appropriate link communities by setting a proper attraction intensity threshold δ. As a result, the link communities change to node communities allowed overlap.

Average-linkage hierarchical clustering builds a link dendrogram from the links distance. If you want to get a partition of q communities, cut the dendrogram at the maximal q clusters. We give the pseudo-code of our MCLC algorithm in Algorithm 3.

The attraction intensity threshold δ is also belong to $[0, 1]$. The lager δ , the more difficult to meet the condition, in other words, the less edge nodes can be completely absorbed into a community. Generally we set $\delta = 0.5$ in accordance with the notion of "community in a strong sense" put forward by Radicchi et al.[4], or simply: in an undirected and unweighted network, if the most and more than a half of u connected links are in a unique link community, then node u can be completely absorbed into the link community.

Algorithm 1 Atrans2H

Input: graph $G(V, E)$
$G(V, E) \rightarrow$ graph weighted matrix W
Sort each edge in G and save in matrix LG
$n = size(W, 1)$
$m = length(LG)$
$B = sparse(n, m)$;
for α in 1 to m **do**
    if node i connect to edge α in LG **then**
        $B(i, \alpha) = W(i, j)$
    end if
end for
Compute the strength of each node and save in vector S
$M = dig(1/S_1, 1/S_2, , 1/S_n)$
$H = B * M * B^T$
Output: H

Algorithm 2 LinkDistance

Input: H,T
$m = size(H, 1)$
$P = sparse(m, m)$;
for i in 1 to m **do**
    for j in 1 to m **do**
        $P(i, j) = H(i, j)/\sum_j H(i, j)$
    end for
end for
$D = sparse(m, m)$
for t_1 in 1 to T **do**
    $Ptot = P$
    $Pcurr = P$
    for t_2 in 2 to t_1 **do**
        $Pcurr = P * Pcurr$
        $Ptot = Ptot + Pcurr$
    end for
    $S = (Ptot + Ptot^T)/T$
end for
$S(i, i) = 0$
$D = 1 - \frac{S - minS}{maxS - minS}$
Output: D

Analyzing the complexity of MCLC algorithm , the first step run in time $O(k_{max}^2 n)$. In the second step, it takes (tm^2) to calculate the distance between links. In the third step, it takes $O(n)$ to transform link communities to node communities. Finally the total computing time is $O(k_{max}^2 n + tm^2 + n)$, the worst computing time is upper bound at most $O(m^2 n)$.

Algorithm 3 MCLC

Input: graph $G(V, E), T, q$
$H \leftarrow Atrans2H(G)$
$D \leftarrow LinkDistance(H, T)$
$Cluster(D, q) \rightarrow C_L = \{P_1, P_2, \ldots, P_q\}$
Find the edge nodes between C_L and save in set U
for i in U **do**
    for P_c in C_L **do**
        calculate attraction intensity $I_i^{P_c}$
    end for
    if $I_{max} = I_i^{P_m} > \delta$ and P_m is unique **then**
        node i only belong to P_m
    end if
end for
Update C_L to $C_N = \{P_1, P_2, \ldots, P_q\}$
Output: C_N

4 Experiments and results

To evaluate the performance of MCLC, we implement our method and design experiments in Matlab platform, running on a PC with 2.94 GHz, 4 GB memory and Win7 operating system. we have considered the algorithm in synthetic networks and some real world networks.

4.1 Synthetic networks

The LFR benchmarks is a widely accepted benchmark for testing community detection introduced by LancichinettiFortunato,and Radicchi [36, 37]. LFR benchmarks is a family of synthetically generated graphs, it contains several types and rich parameters to simulate various networks. In order to find the overlapping communities on undirected and unweighted networks, the parameters for the LFR benchmarks are given in Table 1.

Measuring Normalized Mutual Information (NMI)[38] is a common method to estimate the similarity between the true partition and the detected ones, which is defined as follows:

$$I(A,B) = \frac{-2\sum_{i=1}^{c_A}\sum_{j=1}^{c_B} N_{ij} log(N_{ij}N/N_{i.}N_{.j})}{\sum_{i=1}^{c_A} N_{i.} log(N_{i.}/N) + \sum_{i=1}^{c_B} N_{.j} log(N_{.j}/N)} \tag{9}$$

where the number of real communities is denoted c_A and the number of found communities is denoted c_B, the sum over row i of matrix N_{ij} is denoted $N_{i.}$ and the sum over column j is denoted $N_{.j}$. If the found partitions are identical to the real communities, then $I(A, B)$ takes its maximum value of 1. If the partition found by the algorithm is totally independent of the real partition, for example when the entire network is found to be one community, then I(A, B) = 0.

Table 1. The parameter settings of LFR benchmarks

Parameters	Values
Number of nodes N	1000
Average degree $< k >$	20
Maximum degree k_{max}	50
Degree distribution τ_1	2
Community size distribution τ_2	1
Mixing parameter μ	0.1, 0.3
Number of overlapping nodes O_n	{100 200 300 400 500}
Memberships of the overlapping nodes O_m	2
Maximum community size c_{max}	10
Minimum community size c_{min}	20

For overlapping communities, the overlapping NMI [23] is extended from the NMI in Ref.[38]. For partitions C' and C'' , the overlapping NMI is given as follows:

$$N(X|Y) = 1 - [H(X|Y) + H(Y|X)]/2 \tag{10}$$

where $X(Y)$ is the random variable associated to the partition $C'(C'')$, $H(X|Y)$ is the normalized conditional entropy of a cover X with respect to cover Y, which is defined as:

$$H(X|Y) = \frac{1}{|C'|} \sum_k \frac{H(X_k|Y)}{H(X_k)} \tag{11}$$

We test our MCLC algorithm on these LFR benchmarks by setting four different attraction intensity threshold $\delta = \{0, 0.5, 0.7, 1\}$. The results of experiments can be seen in Fig.3. The maximum value of NMI is close to 0.9 when $\mu = 0.1$ and $\delta = 0$. When the number of overlapping nodes O_n become lager, the value of NMI is diminish. We find both $\mu = 0.1$ and $\mu = 0.3$, the value of NMI is smaller when δ become lager. If δ is fixed, the value of NMI is larger when $\mu = 0.1$. So we conclude MCLC algorithm can find suitable communities through setting proper attraction intensity threshold.

4.2 Real world networks

Zachary's karate club network Zachary's karate club network [39] is a real social network, which is a widely used network for testing communitys algorithm. There are 34 individuals and 78 links in the network, and the links represent friendships between individuals in the karate club. Later, the club split in two as a result of the contradiction between the administrator and the instructor. When we apply the MCLC algorithm to this network, the results can be shown in Fig. 4 .

We divide the karate club network into two communities using our MCLC algorithm with $T = 1$. First we set $\delta = 1$, all the edge nodes (number 1,2,3, and 34) are overlapping nodes, the remain nodes are absorbed into two link communities, as seen in Fig. 4.(a). The maximal attractive intensities of the four

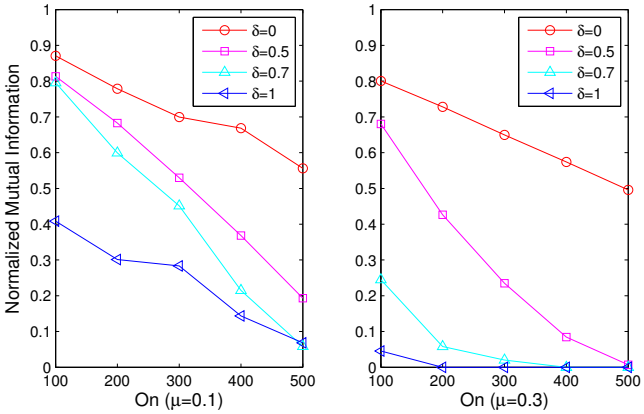
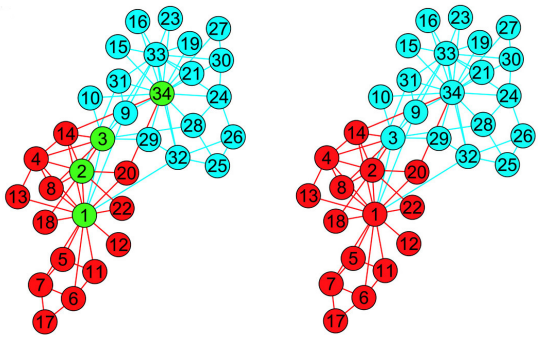


Fig. 3. The results on LFR benchmarks.

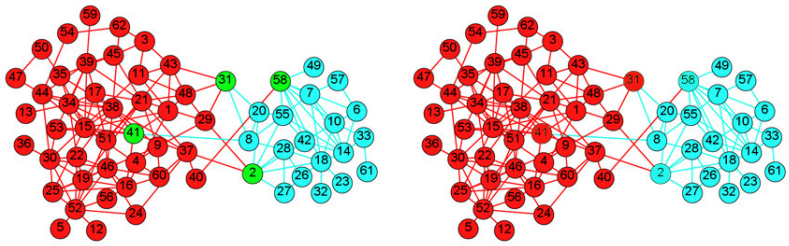


(a) (b)

Fig. 4. Zachary's karate club network.(a)Two communities colored with red and blue respectively are generated by MCLC method when $\delta = 1$, and the green nodes represent the overlapping parts; (b)Two communities generated by MCLC method when $\delta < 0.6$.

edge nodes are 0.8125, 0.8889, 0.6000 and 0.8824 respectively. So when we set $\delta < 0.6$, all edge nodes can be absorbed into the related link communities, the result can be seen in Fig. 4.(b). Except the node 3 is assigned error, other nodes are consistent with the real nodes partitions.

Dolphins network Dolphins network [40] is an undirected social network of frequent associations between 62 dolphins in a community living off Doubtful Sound, New Zealand, as compiled by Lusseau et al. (2003). In the 7-years observation, the original community naturally divided two big group, then the larger group separated into four small group later.



(a) (b)

Fig. 5. Dolphins network. (a)Two communities generated by MCLC method, the red and the blue edges represent two link communities respectively, the green nodes represent the edge nodes; (b) When $\delta < 0.6$, every edge node between link communities is distributed to node community respectively.

From Fig. 5, the network is divided to two large groups by our method with $T = 1$. Fig. 5(a) indicates two identified link communities using MCLC algorithm, and the green nodes (number 2,31,41,58) are edge nodes. The maximal attractive intensities of the four edge nodes are 0.7500, 0.6000, 0.8750, 0.8889 respectively. If we set $\delta < 0.6$, all edge nodes can be absorbed into one neighbor community, as seen in the Fig. 5(b). Fortunately, the modified partition of dolphins is consist with the natural partition completely.

4.3 Comparison with other community detection methods

In this section, we compare our MCLC algorithm with CPM [22], Link [25] and UEOC [41] algorithms on the five real-world networks listed in Table 2.

Table 2. Five real-world networks

No.	Network	No.vertices	No.edges
1	Karate	34	78
2	Dolphins	62	159
3	Polbooks	105	441
4	Football	115	613
5	Email	1133	5451

Here we choose the overlapping modularity Q_{ov} [42], an extension of modularity Q[43], to test community structure. It is defined as following:

$$Q_{ov} = \frac{1}{2m} \sum_{l} \sum_{\{i,j\} \in C_l} \frac{1}{O_i O_j} \left[a_{ij} - \frac{k_i k_j}{2m} \right] \tag{12}$$

where m represents the number of edges in the network, $O_i(O_j)$ represents the number of communities to which vertex $i(j)$ belongs, a_{ij} is the element of adjacency of the network $k_i(k_j)$ is the degree of vertex $i(j)$.

Overlapping modularity Q_{ov} is an extension of modularity Q, which is the fraction of the edges that fall within the given groups minus the expected such fraction if edges were distributed at random, and it have considered the influence of overlapping nodes. If each vertex i satisfied $O_i = 1$, namely each vertex only belong to one community, then Q_{ov} will reduce to modularity Q.

We have made many experiments on these five real networks, and compared the maximum Q_{ov} with different algorithms. To reduce the time cost, we set $T = 1$ and $\delta = 0.5$. From Fig. 6, the results of MCLC are very good among those methods, and it is the best on the four real world networks except Football.

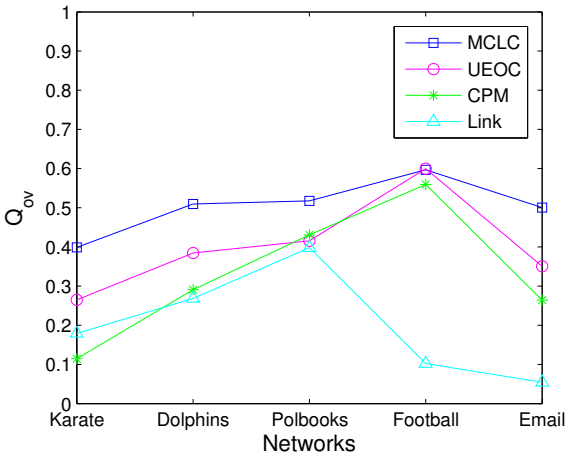


Fig. 6. Comparing Q_{ov} with different methods on networks

5 Conclusion

The MCLC algorithm, which can detect the overlapping community structure of complex networks, is proposed in this paper. First, it should generate the weighted line graph from the original network graph. Next, set M members walk on the line graph randomly. After walk T steps, and calculate the distance between links. Then the average-linkage method is adopted for producing the link partition. At last, we assign the edge nodes between link communities to neighboring communities by setting the threshold of attraction intensity. Results of the experiments on synthetic networks and the real-world networks are very good and validate the efficiency of the proposed algorithm. Comparing with some other algorithms, our method is as good as or even better. However, we have not done the experiments on the larger scale networks due to the limitation of devices, and the time complexity is higher than expected, which may be improved in the future.

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